Preprint typeset in JHEP style. - HYPER VERSION

DAMTP-2001-44

Branes on the Horizon

Anne-Christine Davis[*] **Christophe Rhodes**[†] **Ian Vernon**[‡]

Department of Applied Mathematics and Theoretical Physics,
Centre for Mathematical Sciences,
Wilberforce Road, University of Cambridge, Cambridge, CB3 0WA, UK

ABSTRACT: Models with extra dimensions are often invoked to resolve cosmological problems. We investigate the possibility of apparent acausality as seen by a brane-based observer resulting from signal propagation through the extra dimensions. Null geodesics are first computed in static and cosmological single-brane models, following which we derive the equations of motion for the inter-brane distance in a two-brane scenario, which we use to examine possible acausality in this more complex setup. Despite observing significant effective acausality in some situations there is no *a priori* solution to the horizon problem using this mechanism. In the two-brane scenario there can be significant late time violation of gravitational Lorentz invariance, resulting in the gravitational horizon being larger than the particle horizon, leading to potential signals in gravitational wave detectors.

KEYWORDS: Extra Large Dimensions, Cosmology of Theories beyond the SM.

[*]a.c.davis@damtp.cam.ac.uk
[†]c.s.rhodes@damtp.cam.ac.uk
[‡]i.r.vernon@damtp.cam.ac.uk

Contents

1. Introduction

Recently there has been considerable interest in the novel suggestion that we live in a Universe that possesses more than four dimensions. The standard model fields are assumed to be confined to a hypersurface (or 3-brane) embedded in this higher dimensional space, in contrast the gravitational fields propagate through the whole of spacetime [1–10]. In order for this to be a phenomenologically relevant model of our universe, standard four-dimensional gravity must be recovered on our brane. There are various ways to do this, the most obvious being to assume that the extra dimensions transverse to our brane are compact. In this case gravity can be recovered on scales larger than the size of the extra dimensions [5–7]. This is different from earlier proposals since the restrictions on the size of the extra dimensions from particle physics experiments no longer apply, as the standard model fields are confined to the brane. The extra dimensions only have to be smaller than the scale on which gravity experiments have probed, of order 1mm at the time of writing. Another way to recover four-dimensional gravity at large distances is to embed a positive tension 3-brane into an AdS_5 bulk [9,10]. In this scenario four-dimensional gravity is obtained at scales larger than the AdS radius. Randall and Sundrum showed that this could produce sensible gravity even if the extra dimension was not compact.

The cosmology of these extra dimension scenarios has been investigated and the Friedman equation derived and shown to contain important deviations from the usual 4-dimensional case [11–13]. Some inflationary models have been investigated [14], as have brane world phase transitions, topological defects and baryogenesis [15].

The possibility that various cosmological problems could be solved in an extra-dimensional scenario has been examined by several authors; however a novel suggestion that could possibly resolve the horizon problem was made in [16], where Chung and Freese used a variety of simple metrics to demonstrate that gravity signals, propagating either purely in the extra dimension or via a second hidden brane, could connect regions of our four-dimensional universe naïvely thought to be causally unconnected. Such a mechanism would have important cosmological consequences as it could greatly alter the size of the particle horizon as well as possibly providing a method of experimentally confirming the existence of extra dimensions.

The aim of this present work is to fully investigate both the one and two brane mechanisms proposed by Chung and Freese, in a much more general, physically acceptable brane world scenario: that of a non-Z_2 symmetric cosmological model with a non-zero Weyl tensor component [17–22].

In the single brane case we examine numerically the nature of null geodesics leaving our 3-brane at various times in the universe: whether they return to our 3-brane or freeze out at the horizon; their apparent speed relative to that of light as viewed by a four-dimensional observer; and how these attributes depend on the initial velocity of the geodesics. The effect on the geodesics of breaking the Z_2 symmetry and of varying the Weyl tensor component is also investigated fully. We have therefore considerably generalised the previous work done on the subject [23–25], showing that there are only significant deviations from Lorentz invariance at early times.

The second case, where signals travel along a hidden brane, has not previously been investigated in any realistic scenario: the metrics used in [16] were not solutions to Einstein's equations. In order to examine the two brane case fully, the non-linear equation of motion of the inter-brane distance (otherwise known as the cosmological radion $\mathcal{R}$ [26]) is concisely derived and is used to evaluate the relative speeds of signals on the second brane compared to the first, quantitatively for the stationary case and qualitatively for the moving brane case. It is found that although signals can in some situations propagate along the second brane significantly faster than along ours, it is not a large enough effect to solve the horizon problem; on the other hand we do get significant violation of Lorentz invariance at late times such that the gravitational horizon is larger than the particle horizon. This could lead to signals in future gravitational wave detector experiments.

The paper is organised as follows: the general setup and geodesic equations are derived in section 2); section 3 contains a brief discussion of geodesics in the normal Randall–Sundrum model [23]. We start, in section 4, by investigating non-cosmological (i.e. static) solutions of Einstein's equations, with the Randall–Sundrum model extended by admitting [19] the possibility of Z_2-symmetry breaking across the brane, as well as perturbations from perfect tuning; the behaviour of null geodesics in the five-dimensional theory is examined both analytically and numerically. We then extend the investigation in section 5 to fully cosmological solutions of Einstein's equations without mirror symmetry, numerically investigating the possibilities for solving the horizon problem in a similar fashion to [25]. In section 6 we consider the possible acausality in a two brane scenario where in general the second brane is moving. Our conclusions are discussed in section 7.

2. Geodesics in 5-dimensional Einstein Gravity

Throughout this paper we investigate the behaviour of geodesics in a variety of different brane world scenarios. In this section we describe both the general setup and the assumptions that have been made, as well as giving the equations from which all the results in this paper are derived.

In each scenario that we investigate, it is assumed that the five-dimensional spacetime satisfies three-dimensional homogeneity and isotropy as we are mainly interested in realistic cosmological models. We choose to work in a 'brane-based' coördinate system as it will facilitate physical interpretation of the results. We therefore assume that the four-dimensional universe in which we reside is situated on a 3-brane which is chosen to be at the origin of the extra dimension ($y = 0$). This implies that the most general metric must have the form

$$\mathrm{d}s^2 = -n^2(t,y)\mathrm{d}t^2 + a^2(t,y)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j + \mathrm{d}y^2, \tag{2.1}$$

where t is the cosmic time on our brane, x^i represents the three spatial dimensions of our brane, and y is the coördinate of the extra dimension. γ_{ij} is the maximally symmetric three-dimensional metric with $k = -1, 0, 1$ parameterizing the spatial curvature, although throughout most of this paper k will be set to zero. The bulk is assumed to be empty except for a cosmological constant Λ, and therefore the metric is obtained by solving the five-dimensional Einstein's equations[4],

$$G_{AB} - \Lambda g_{AB} = \kappa^2 T_{AB}, \tag{2.2}$$

where we define $\kappa^2 \equiv 1/\widetilde{M}_5^3$, $\widetilde{M}_5$ being the fundamental (reduced) five-dimensional Planck Mass. In the single brane scenario, the energy-momentum tensor T_{AB} takes the form,

$$T_{AB} = \delta(y)\mathrm{diag}(\rho_0, P_0, P_0, P_0, 0), \tag{2.3}$$

[4]We use the standard brane world convention in that lower-case Roman indices (such as i, j) run across the normal space dimensions (1 to 3); Greek indices (μ, ν) run across time and normal space (0 to 3), while capital Roman indices (A, B) cover all space and time dimensions (0 to 3 and 5).

and for the two brane scenario,

$$T_{AB} \;=\; \delta(y)\mathrm{diag}(-\rho_0, P_0, P_0, P_0, 0) \;+\; \delta(y-\mathcal{R}(t))\mathrm{diag}(-\rho_2, P_2, P_2, P_2, 0). \tag{2.4}$$

where we follow the notation used in [26] by defining ρ_0 and ρ_2 to be the energy density of our brane and of the second brane respectively, and by defining the pressures P_0 and P_2 similarly. The position of the second brane given by $y = \mathcal{R}(t)$, is in general time dependent.

In Sections (4) and (5) we use solutions of Einstein's equations that do not possess a Z_2 symmetry (or mirror symmetry) across the brane. For a full discussion of this topic see [17–22].

2.1 Geodesic Equations

Here we derive the necessary geodesic equations corresponding to the metric given by equation (2.1). In what follows the notation is such that dots indicate differentiation with respect to the affine parameter and dashes with respect to the fifth dimension, y, leaving $\frac{\partial}{\partial t}$ as the differentiation operator with respect to coördinate time. Starting from the variational principle

$$\delta S = \delta \int \mathcal{L} \mathrm{d}s = 0, \tag{2.5}$$

with

$$\mathcal{L} = \sqrt{g_{AB}\dot{x}^A\dot{x}^B}, \tag{2.6}$$

we can derive the equations of motion for test particles. We shall principally be concerned here with lightlike geodesics, which means that we can consider the variation of $\int \mathcal{L}^2 \mathrm{d}s$, so that the effective Lagrangian density is $-n^2(t,y)\dot{t}^2 + a^2(t,y)\dot{x}_i\dot{x}_i + \dot{y}^2$, giving Euler-Lagrange equations:

$$\begin{aligned} \ddot{t} &= \frac{1}{n}\left(\frac{\partial n}{\partial t}\dot{t}^2 - 2\dot{n}\dot{t} - \frac{\partial a}{\partial t}\frac{\theta_i\theta_i}{a^3 n}\right) \\ \dot{x}_i &= \frac{\theta_i}{a^2} \\ \ddot{y} &= -\frac{\theta_i\theta_i}{a^2}\left(\frac{n'}{n} - \frac{a'}{a}\right) - \frac{n'}{n}\dot{y}^2, \end{aligned} \tag{2.7}$$

where θ_i are integration constants. For null geodesics, with which we shall be exclusively concerned in this study, we have the additional constraint that the test particle must move at the speed of light; this tells us that the first integral of the t equation above must be of the form

$$n^2\dot{t}^2 = \dot{y}^2 + \frac{\theta_i\theta_i}{a^2}. \tag{2.8}$$

These equations will be used extensively throughout sections 3, 4 and 5 to evaluate possible shortcuts through the bulk.

3. Geodesics in the Randall-Sundrum Model

3.1 The Randall-Sundrum Metric

The Randall-Sundrum model, as initially presented [9,10] is not a cosmological one; rather, it is included here as a simple model to develop the reasoning employed in rather more complicated cases. It may be derived from the above, general, metric Ansatz given in equation (2.1) by taking $n(\cancel{t}, y) = a(\cancel{t}, y) = a(y)$, and then solving Einstein's equations while assuming a Z_2 symmetry across each brane. Provided that the bulk cosmological constant Λ is less than zero, and that energy densities of each of the branes are tuned such that

$$\kappa^2 \rho_0 \; = \; -\kappa^2 \rho_2 \; = \; \sqrt{-6\Lambda}, \tag{3.1}$$

the familiar Randall-Sundrum metric is recovered:

$$\mathrm{d}s^2 = \mathrm{e}^{-2\mu|y|}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{d}y^2, \tag{3.2}$$

which implies that

$$n(y) = a(y) = \mathrm{e}^{-\mu|y|}. \tag{3.3}$$

μ is the inverse of the AdS$_5$ curvature radius and is given by $\mu = \sqrt{-\Lambda/6}$.

3.2 RS Geodesics

We can immediately see that the Randall-Sundrum model is not going to help us solve the horizon problem. The local speed of light is everywhere the same, because $n(y) = a(y)$, and so even if off-brane null geodesics can return to the brane, they will return with an effective sub-luminal velocity.

However, the Randall-Sundrum model does not exhibit even this kind of behaviour; as shown in [23], the geodesic follows the path given by

$$\mathrm{e}^{2\mu y} = \mathrm{e}^{2\mu y_0} + 2\mu \dot{y}_0 \mathrm{e}^{2\mu y_0} t - v^2 \mu^2 t^2, \tag{3.4}$$

with $v^2 = \eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu$.

As a side note, $v^2 = 0$ does not necessarily correspond to a lightlike geodesic, as claimed in section 2 of [23], as the condition for lightlike behaviour is $g_{AB}\dot{x}^A\dot{x}^B = 0$, which includes an extra $\dot{y}^2$.

4. Geodesics in Other Static Braneworld Models

4.1 Obtaining the Metric

We relax some of the assumptions made in the Randall-Sundrum model; specifically, we no longer impose strict Z_2 symmetry, nor do we fine-tune the bulk cosmological constant. Further to the relaxation of fine-tuning and Z_2 symmetry, we must also adapt the Ansatz so that $n(y)$ is no longer equal to $a(y)$ to obtain a self-consistent solution.

The G_{00} equation from [11] yields

$$a^2(y) = \cosh(2\mu y) + A \sinh(2\mu y), \tag{4.1}$$

where we have implicitly scaled so that $a_0 = a|_{y=0} = 1$. To consider the loss of Z_2 or mirror symmetry, when we apply the Israel junction condition

$$[a'(y)] = -\frac{\kappa^2}{3}\rho, \tag{4.2}$$

where $[f] = f(0^+) - f(0^-)$, we make the assumption that $a'(0^+) = -a'(0^-) + d_a$, giving

$$\mu A = -\frac{\kappa^2}{6}\rho + \frac{d_a}{2}; \tag{4.3}$$

$d_a = 0$ for Z_2-symmetric braneworlds. Then, parameterizing the lack of fine-tuning by

$$r = \frac{\kappa^2 \rho}{6\mu} \tag{4.4}$$

and the Z_2 symmetry breaking by

$$d = \frac{d_a}{2\mu}, \tag{4.5}$$

we obtain

$$a(y) = \sqrt{\cosh(2\mu|y|) + (d-r)\sinh(2\mu|y|)}. \tag{4.6}$$

The G_{55} equation then yields

$$n(y) = \frac{\sinh(2\mu|y|) + (d-r)\cosh(2\mu|y|)}{(d-r)\sqrt{\cosh(2\mu|y|) + (d-r)\sinh(2\mu|y|)}}, \tag{4.7}$$

which has again been scaled so that $n(y=0) = 1$; this solution may then be substituted into the other Einstein equations and junction conditions to check that it is self-consistent.

4.2 Geodesics in Static Non-Z_2 Brane Worlds

Since the Lagrangian density $\mathcal{L}$ given by equation (2.5) does not depend explicitly on coördinate time t, we may obtain a further first integral of the form

$$\dot{t} = \frac{\theta_0}{n^2(y)}, \tag{4.8}$$

and so straight from the Lagrangian constraint for lightlike geodesics we have

$$\dot{y}^2 = \frac{\theta_0^2}{n^2(y)} - \frac{\theta_i\theta_i}{a(y)^2}. \tag{4.9}$$

This is effectively a conservation of energy equation, with a kinetic term proportional to $\dot{y}^2$ and a potential of the form

$$V(y) = -\frac{\theta_0^2}{n^2(y)} + \frac{\theta_i\theta_i}{a(y)^2}. \tag{4.10}$$

We may therefore reason that firstly if a geodesic starts its life on our brane (at $y = 0$) then we have, since $\dot{y}^2 \geq 0$,

$$\theta_0^2 \geq \theta_i\theta_i. \tag{4.11}$$

Further, if the geodesic is ever to return, then $\dot{y}$ must at some point be zero, so

$$\theta_0^2\left(\frac{1}{n^2(y)} - \frac{\gamma^2}{a^2(y)}\right), \tag{4.12}$$

where $0 < \gamma^2 = \frac{\theta_i \theta_i}{\theta_0^2} < 1$, must be zero at some positive value of y. Rearranging and substituting the above solution for the metric, we find that the positions at which $\dot{y} = 0$, denoted by $y_t^{\pm}$, satisfies

$$\cosh(2\mu y_t^{\pm}) + (d-r)\sinh(2\mu y_t^{\pm}) = \pm\gamma \frac{\sinh(2\mu y_t^{\pm}) + (d-r)\cosh(2\mu y_t^{\pm})}{(d-r)}, \tag{4.13}$$

or

$$y_t^{\pm} = \frac{1}{2\mu}\tanh^{-1}\left[\frac{(d-r)(\pm\gamma - 1)}{(d-r)^2 \mp \gamma}\right]. \tag{4.14}$$

However, there could be a horizon intervening between $y = 0$ and this point; a horizon will occur if either of $a(y)$ or $n(y)$ is equal to zero. We split the analysis into two parts, one for $(d-r) > -1$ and one for $(d-r) < -1$, reminding the reader that $(d-r) = -1$ corresponds to the Randall-Sundrum braneworld. Below, subscript t refers to the geodesic turning point, while subscript h to the position of the horizon.

4.2.1 The $(d-r) > -1$ Case

For $(d-r) > -1$, $a(y)$ (equation 4.6) is never zero because $\cosh(2\mu y) > \sinh(2\mu y)$ for all y. $n(y) = 0$ gives (from equation 4.7)

$$y_h^{>} = \frac{1}{2\mu}\tanh^{-1}(r-d). \tag{4.15}$$

Then y_t^- (equation 4.14) is clearly greater than $y_h^>$ (as $(d-r)^2 < 1$); also y_t^+ is either greater than $y_h^>$ or is negative (the latter when $(d-r)^2 < \gamma$). Thus a geodesic leaving our brane in the positive y direction for $(d-r) > -1$ will always reach a horizon at $y = y_h^>$, and will for an observer on the brane therefore never return. See figures 1 and 2 for illustrations of a sample geodesic in this régime.

4.2.2 The $(d-r) < -1$ Case

For $(d-r) < -1$, $n(y)$ is never zero, and $a(y) = 0$ gives

$$y_h^{<} = \frac{1}{2\mu}\tanh^{-1}\frac{1}{(r-d)}. \tag{4.16}$$

This is always greater than y_t^- which is the smaller of the two turning points, and so for $(d-r) < -1$, the geodesic will turn around at a y value of

$$y_t = \frac{1}{2\mu}\tanh^{-1}\left[\frac{(r-d)(1-\gamma)}{(d-r)^2 - \gamma}\right]. \tag{4.17}$$

Figures 3 and 4 similarly show a sample geodesic for $(d-r) < -1$.

5. Cosmological Models

5.1 The non-Z_2 Symmetric Cosmological Metric

In this Section we investigate the behaviour of geodesics in the full cosmologically realistic, non-Z_2 symmetric brane world scenario (see [19] for more details). In order to obtain

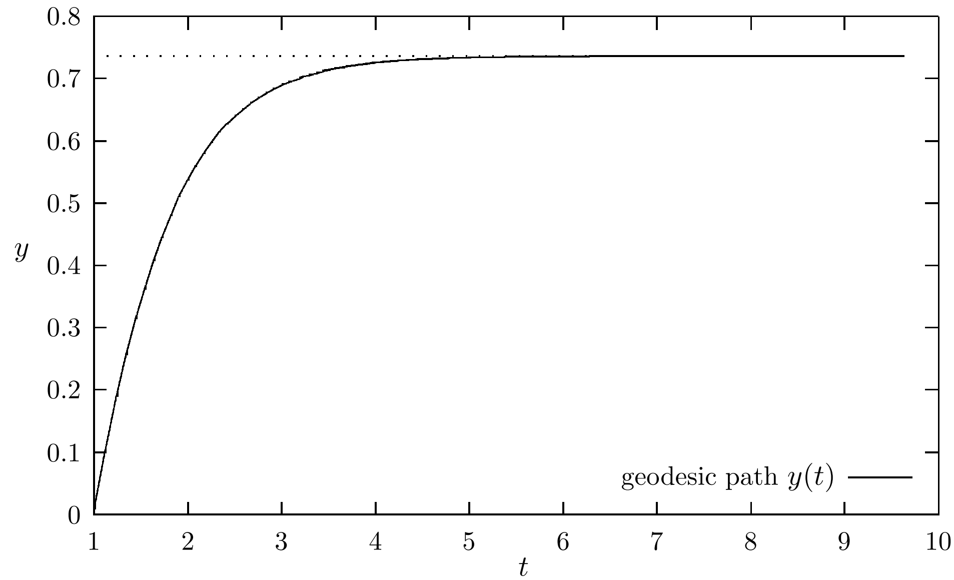

Figure 1: Graph of y against coördinate time for a geodesic in a static untuned ($d - r = -0.9$) braneworld with $\dot{x}_0 = -0.5$. The dotted line represents the calculated $y_h^>$ horizon position.

standard cosmology at late times we make the usual assumption, first noted by [12], that our brane possesses an energy density which is the sum of the brane tension σ and a physical energy density ρ, and we assume that the brane tension and bulk cosmological constant are tuned to ensure that the effective four-dimensional cosmological constant is zero. The Friedman equation is then given by [19],

$$H_0^2 = \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{\kappa^4\sigma}{18}\rho + \frac{\kappa^4}{36}\rho^2 - \frac{k}{a_0^2} + \frac{\mathcal{C}}{a_0^4} + \frac{F^2}{(\rho+\sigma)^2 a_0^8}, \tag{5.1}$$

where $a_0(t) = a(t,0)$, $\mathcal{C}$ is the dark radiation term corresponding to a non-zero Weyl tensor component, and F represents the extent to which the Z_2 symmetry is broken. Motivated by previous work on geodesics [24] which suggests that more interesting effects are to be seen at earlier times, we assume a radiation dominated universe and therefore set

$$\rho = \frac{\lambda}{a_0(t)^4}, \tag{5.2}$$

where λ is some constant. We can now use the bulk solutions to the metric Ansatz (2.1) which have been found for a single brane without reflection symmetry in an infinite fifth dimension [19], and are given by

$$a^2(t,y) = a_0^2(t)\left(A(t)\cosh 2\mu y + B(t)\sinh 2\mu y + C(t)\right), \tag{5.3}$$

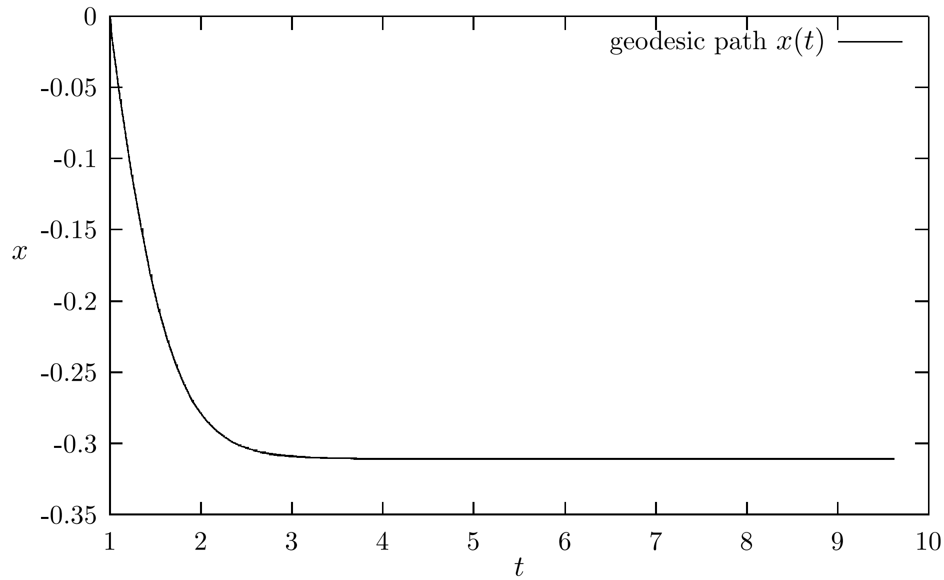

Figure 2: Graph of x against coördinate time for a geodesic in a static untuned ($d - r = 0.9$) braneworld with $\dot{x}_0 = -0.5$. Note that as the geodesic goes towards the horizon its x-velocity decreases.

where μ as before, is defined in terms of the bulk cosmological constant Λ as $\mu = \sqrt{-\Lambda/6}$. The purely time dependent constants $A(t)$, $B(t)$, and $C(t)$ are given by

$$A(t) = 1 + (1+w)\frac{\rho}{\sigma} + \frac{1}{2}\frac{\rho^2}{\sigma^2} + \frac{f^2\rho^2}{2(\sigma+\rho)^2} \tag{5.4}$$

$$B(t) = -(1+\frac{\rho}{\sigma}) \pm \frac{f\rho}{(\sigma+\rho)} \tag{5.5}$$

$$C(t) = -(1+w)\frac{\rho}{\sigma} - \frac{1}{2}\frac{\rho^2}{\sigma} - \frac{f^2\rho^2}{2(\sigma+\rho)^2}. \tag{5.6}$$

Here w and f are dimensionless constants defined by $w \equiv 18\mathcal{C}/\kappa\sigma\lambda$ and $f \equiv 6F/\kappa^2\sigma\lambda$, and the $\pm$-signs in the expression for $B(t)$ give the two different solutions on either side of the brane. The solution for $n(t, y)$ is given in terms of $a(t, y)$ as:

$$n(t, y) = \frac{\dot{a}(t, y)}{\dot{a}_0(t)}, \tag{5.7}$$

where from now the dots denote differentiation with respect to coördinate time.

To find $a_0(t)$, we must solve the Friedman equation (5.1); however, for $f \neq 0$, there is no analytical solution, and so it must be solved numerically; this presents a problem, however, due to the initial singularity. Therefore, to solve this equation we approximate

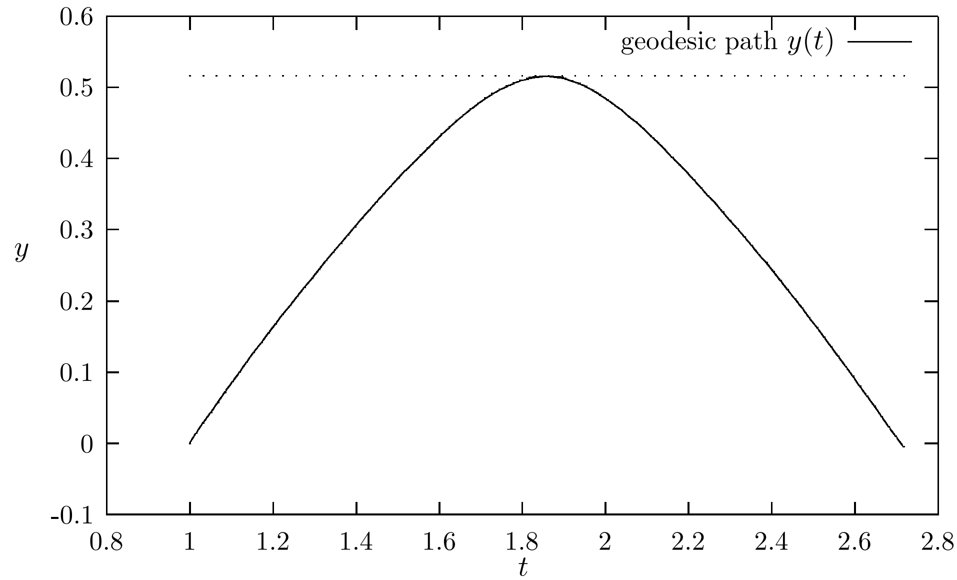

Figure 3: Graph of y against coördinate time for a geodesic in a static untuned ($d - r = -1.1$) braneworld with $\dot{x}_0 = -0.5$. The dotted line represents the calculated y turnaround position.

the f^2 term for early times, when $\rho \gg \sigma$, as $f^2\sigma^2$; the solution is then, as in [19],

$$a_0 = \left\{ \gamma f \left(\frac{1+w}{f\sigma} \left[\cosh \frac{2\sigma f}{3\tilde{M}_5^3} t - 1 \right] + \frac{1}{\sigma} \sinh \frac{2\sigma f}{3\tilde{M}_5^3} t \right) \right\}^{\frac{1}{4}}. \tag{5.8}$$

This solution will be approximately valid to some time t_0; as an estimate of that time, we find when $\rho = 4\sigma$, giving

$$\frac{1+w}{f} \left[\cosh \frac{2\sigma f}{3\tilde{M}_5^3} t_0 - 1 \right] + \sinh \frac{2\sigma f}{3\tilde{M}_5^3} t_0 = \frac{f}{4}, \tag{5.9}$$

or

$$t_0 = \frac{3\tilde{M}_5^3}{2\sigma f} \log \left\{ \frac{\frac{1+w}{f} + \frac{f}{4} + \sqrt{(\frac{1+w}{f} + \frac{f}{4})^2 - (\frac{1+w}{f})^2 + 1}}{\frac{1+w}{f} + 1} \right\}. \tag{5.10}$$

Over the range of values of interest ($0 < w < 0.2$, $0 < f < 60$), $t_0 > \frac{1}{20} \frac{3\tilde{M}_5^3}{2\sigma f}$, (see figure 5) and so we may safely take the initial conditions for numerical integration of the Friedman equation as our approximate solution at $t = \frac{1}{40} \frac{3\tilde{M}_5^3}{2\sigma f}$. This can then be numerically integrated using the full Friedman equation (5.1) to generate the solutions of the Einstein equations.

5.2 Geodesics

We would expect at late times for geodesics to be similar to the static case, assuming that the dynamical timescale is much smaller than the cosmological one. This appears broadly

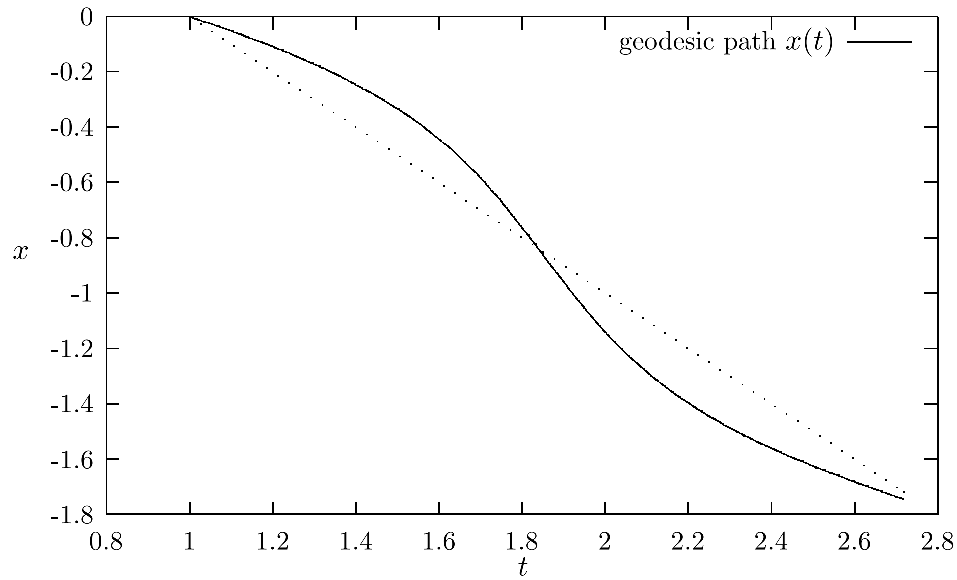

Figure 4: Graph of x against coördinate time for a geodesic in a static untuned $(d - r = -1.1)$ braneworld with $\dot{x}_0 = -0.5$. The dotted line represents a light ray moving on the brane. Note that the true null geodesic has covered a greater distance on return to the brane than the light ray in the same time t; lower magnitudes of $\dot{x}_0$ lead to higher superluminosities.

to be the case, though the position or cross-section of the horizon would need to be tuned for an adequate comparison. The main difference is that for large dynamical times there will be a certain amount of asymmetry in the geodesic's path, due to the changing scale factor $a = a(t, y)$.

Consider figure 6, which shows null geodesics leaving the brane at a time t_0 such that $a_0(t_0) = 0.5$, when the ρ^2 term dominates in the Friedman equation[5]. In that figure, both geodesics have an initial $\dot{y}$ of 0.975 (where the speed of light is 1); the difference is that the dotted geodesic is in a braneworld with broken Z_2 symmetry ($f = 0.1$, in the notation of [19]). The graph has been cut off so that the path of the returning geodesic is clear; the $f = 0.1$ geodesic asymptotically (as t tends to infinity) reaches the horizon in the bulk. It is perhaps slightly clearer when viewed in conjunction with figure 7, which shows that when the geodesic in the $f = 0.1$ world reaches x of about 290 it is at the horizon, at which point it will move with the horizon in the y direction.

Let us next examine figures 8 and 9, in which we have taken a number of geodesics as in figures 6 and 7, and examined the superluminosity of the returning signal; in other words, how much faster than light, which is constrained to travel along the brane with speed $\frac{1}{a_0(t)}$, is the true null geodesic.

The results are perhaps surprising at first sight. Conceptually there are two separate

[5]Though extensive tests have shown that there is no particular difference in between the régimes.

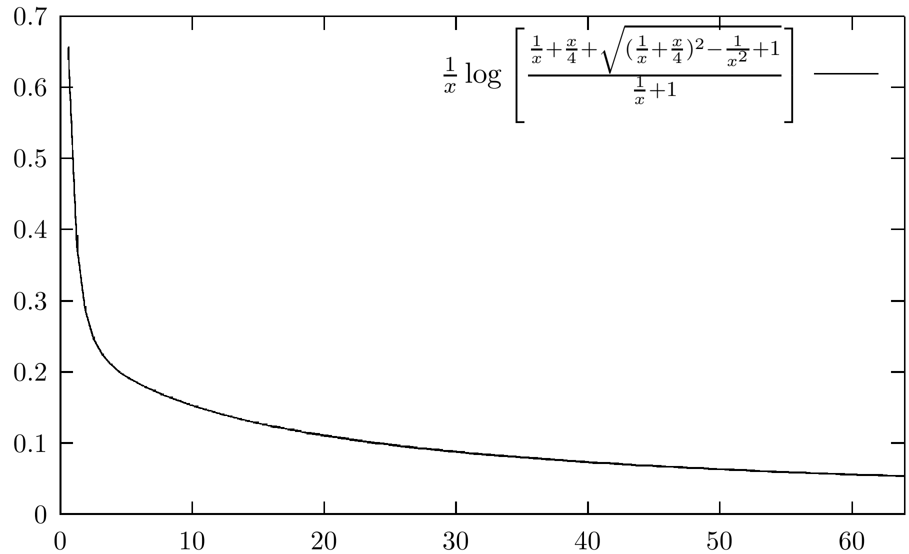

Figure 5: A graph of t_0 (for $w = 0$) over a range of values of f; this is used to determine the point at which numerical integration of the Friedman equation must begin.

effects happening here. One is that space is 'warped'; loosely, there is an $e^{\mu y}$ factor warping space in the bulk, meaning that null geodesics can cover effectively much greater distances. Thus, the farther into the bulk that the geodesic penetrates, the more superluminal the geodesic will be. However, there is a competing effect due to the presence of a horizon in the bulk; the horizon is only a coördinate singularity, but since it is a singularity in the physical brane coördinates it is nevertheless of physical relevance, in that geodesics that reach the horizon never return from the point of view of an observer on the brane.

Null geodesics that have too large a velocity component in the fifth dimension will not be able to escape the horizon. Thus, at very high $\dot{y}_0$, the geodesics will not return to the brane before the end of the universe. Null geodesics with small initial $\dot{y}$, however, will be confined to the brane.

In between these two régimes, what will happen? At some critical angle, the effect of the horizon will begin to dominate over the effect of the warped spacetime, and consequently there will be a maximum in the ratio of gravitational to electromagnetic effective speeds.

As discussed in [24], the maximal superluminal effect will be obtained when the hierarchy is the most pronounced experimentally-allowed value, and the null geodesic is set off at the earliest physical time possible (of the order of M_5^{-1}). However, it is not because of any particularly stronger warping at this earlier time, but rather because the geodesic can spend much 'longer' in the warped area before returning to the brane. Thus there is no discrepancy in principle[6] between the relatively low superluminal values shown here and

[6] Unfortunately it is not possible without extensive code writing to numerically investigate this figure in

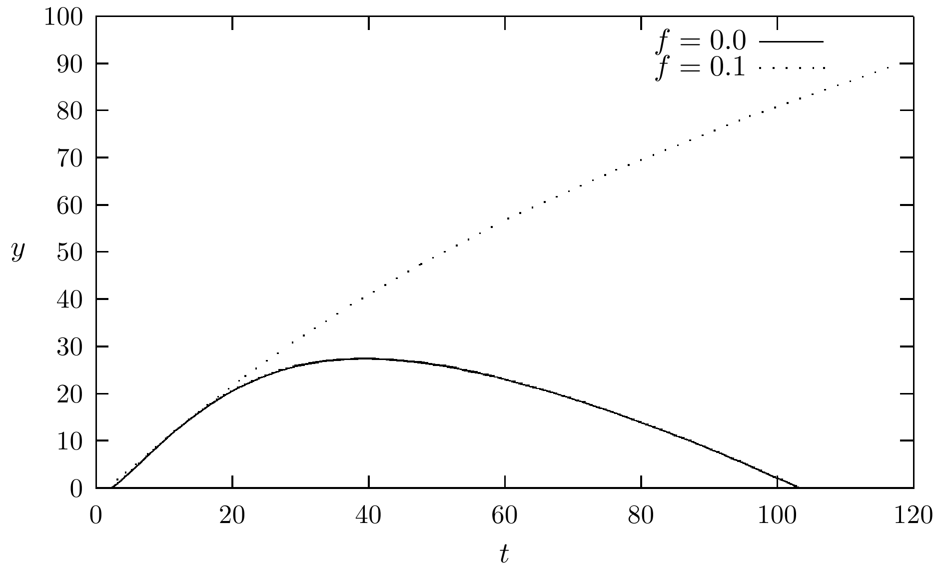

Figure 6: Graph of y against coördinate time for a geodesic in a cosmological braneworld with $\dot{y}_0 = 0.975$. The returning geodesic is in a Z_2-symmetric braneworld, while the one tending towards the horizon has $f = 0.1$. The geodesics were set off at $a_0 = 0.5$

the figure of 10^3 quoted in [24].

It appears from figures 8 and 9 that the effect of including asymmetry[7] is to increase the horizon cross-section, for geodesics leaving at equivalent times (taken as equal redshifts, or, what amounts to the same thing, equal energy densities), and also to decrease the ratio of the speeds even for geodesics that do not go anywhere near the horizon.

The effect of the start time on the superluminosity is such that when the ρ^2 term dominates, at early times the superluminosity is increased (see figure 10). A corresponding late-time graph is not shown, as the cross-section of the horizon has increased to such an extent that almost all geodesics hit the horizon and do not return, from which we note that the acausality in this model is maximal at early time, with energy leakage from the brane at later times.

Including dark radiation likewise dampens the superluminosity; figure 11 shows the effect of including a dark radiation term $w = 2$ in the calculation. Note that this value of w is well outside the nucleosynthesis bound [27], and is tested in our investigation only so as to exaggerate any possible visible effect.

this framework, as the maximal superluminosity will be obtained with an initial $\dot{y}$ of somewhere between $1 - 10^{-15}$ and 1.

[7]We have shown here results with positive f only, as results with $f < 0$ are physically similar.

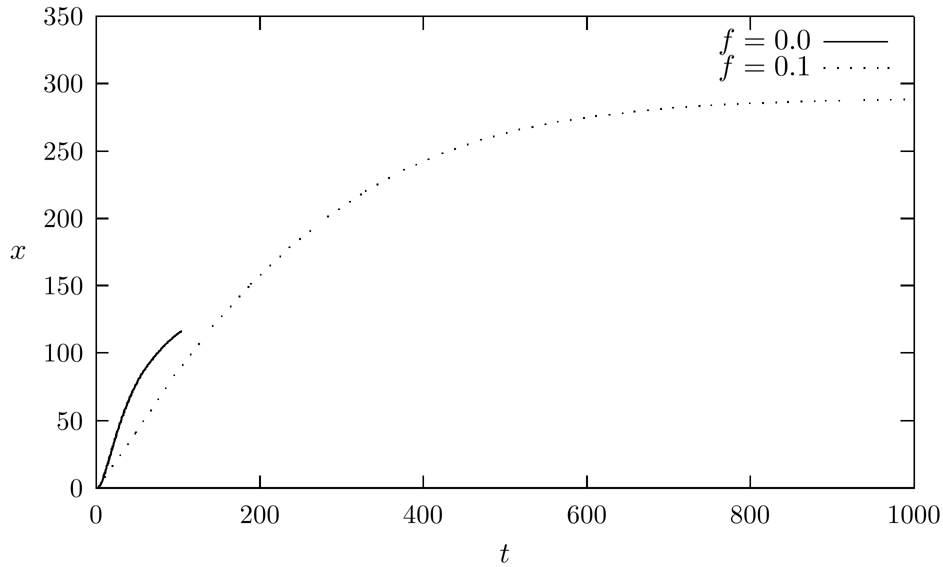

Figure 7: Graph of x against coördinate time for a geodesic in a cosmological braneworld with $\dot{y}_0 = 0.975$. The geodesics are the same as in Figure 6. Note that the $f = 0.1$ geodesic asymptotically tends to a maximal $x = x(\tau_H)$, where τ_H is the value of the affine parameter when the geodesic crosses the horizon.

6. The Two Brane Model

It would seem from the above results, that the possibility of null signals taking shortcuts through the bulk in the single brane scenario would not solve the well known cosmological horizon problem as the apparent speeds of such gravity signals are not significantly greater than the speed of light signals confined to the brane. Due to this, we now investigate an alternative suggestion made by [16], which was that a 2-brane scenario could provide a solution to the horizon problem. By investigating a variety of 'toy' metrics, Chung and Freese showed that apparently acausal signals could be sent between different points on our brane, via the second hidden brane. These signals would, as before, bring in to causal contact regions of our Universe that would have otherwise been unable to communicate.

In order for this method to work, a metric of the following form was assumed:

$$\mathrm{d}s^2 \quad = \quad \mathrm{d}t^2 - e^{-2\mu y} a^2(t) \mathrm{d}x^2 - \mathrm{d}y^2, \tag{6.1}$$

where y corresponds to one extra spatial dimension. Our brane and the hidden brane were assumed to be at $y = 0$ and $y = \mathcal{R}$ respectively. The distance D_{AE} travelled by a null signal on our brane in time t_f is then (see figure 12)

$$D_{AE} \quad = \quad \int_0^{t_f} \frac{\mathrm{d}t}{a(t)}. \tag{6.2}$$

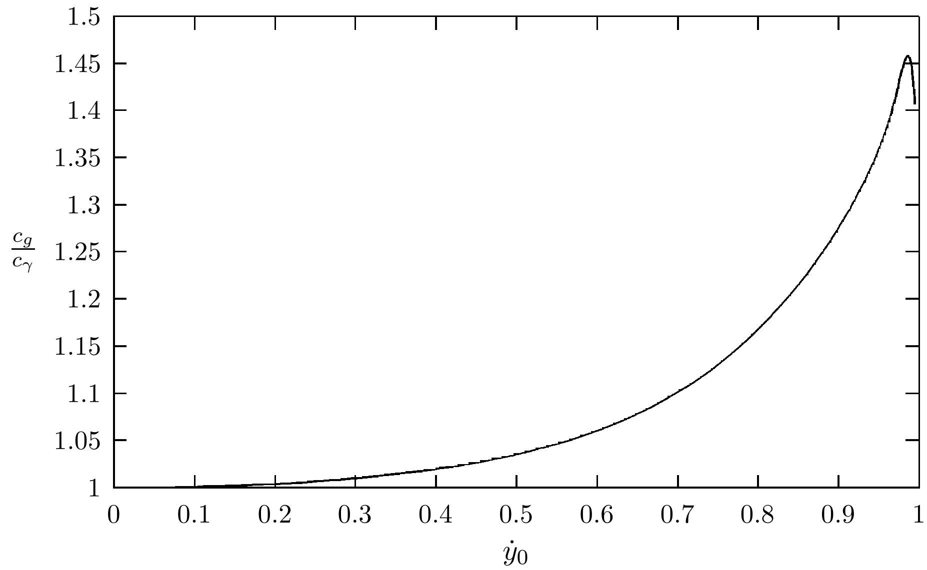

Figure 8: The ratio of gravitational to electromagnetic speeds, as a function of the initial component of velocity in the fifth dimension, in a Z_2-symmetric braneworld. $a_0(t_0) = 0.5$.

However, if it is possible for a null signal to leave our brane, interact with and therefore travel along the hidden brane, before eventually returning to our world, we can then ask how far it would have appeared to have travelled D_{AD}, in time t_f. If it takes a time t_c to cross between the branes then this distance would be given by

$$D_{AD} \quad = \quad e^{\mu\mathcal{R}} \int_{t_c}^{t_f - t_c} \frac{\mathrm{d}t}{a(t)}. \tag{6.3}$$

Therefore if $e^{\mu\mathcal{R}} \gg 1$ then $D_{AD} \gg D_{AE}$ and the horizon problem will be solved. The major problem with this suggestion is that it involves metrics that are not realistic. It relies upon there being a conformal factor that affects the spatial part of the four-dimensional metric only. In typical Randall-Sundrum Brane World models the metric is usually of the form

$$\mathrm{d}s^2 \quad = \quad e^{-2\mu y}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu - \mathrm{d}y^2, \tag{6.4}$$

and hence the horizon problem will remain unsolved.

6.1 A Cosmologically Realistic Metric

The above metrics (6.1) and (6.4) are obviously not realistic descriptions of our Universe; it is therefore worthwhile to consider if the above proposed solution to the horizon problem will work for a cosmological metric. If w, f and k are set to zero, the solutions to Einstein's equations given by (5.3) can be greatly simplified. Writing $a(t, y)$ in terms of exponentials gives

$$a(t,y)^2 \quad = \quad \frac{a_0^2 e^{-2\mu y}}{4}\left[\frac{\rho}{\sigma}e^{2\mu y} - (\frac{\rho}{\sigma} + 2)\right]^2, \tag{6.5}$$

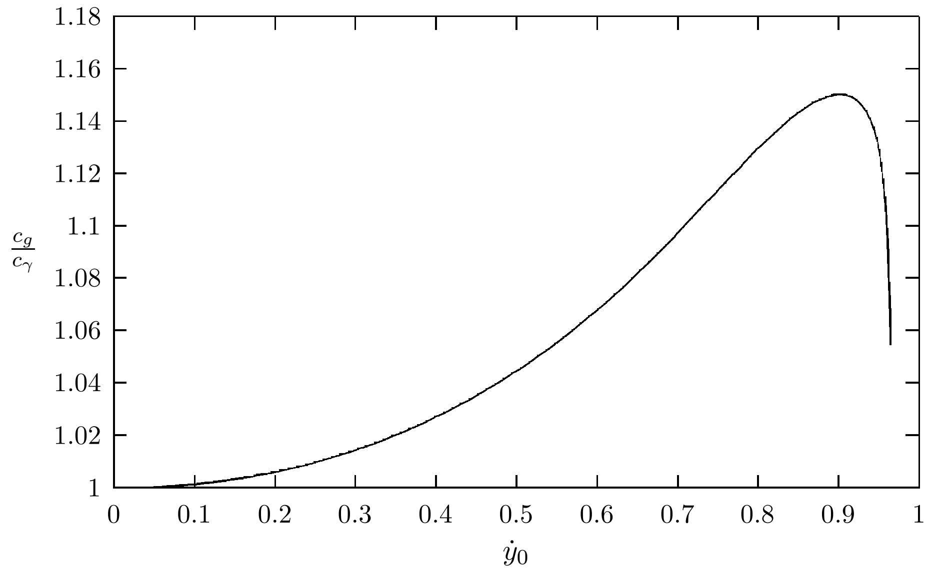

Figure 9: The ratio of gravitational to electromagnetic speeds, as a function of the initial component of velocity in the fifth dimension, in a non-Z_2-symmetric braneworld ($f = 0.1$). Note firstly that the maximum has moved leftwards, and also that some geodesics from our samples no longer return.

This can be further simplified to give

$$a(t,y) \quad = \quad a_0\left(\cosh\mu y - \eta_0 \sinh\mu y\right), \tag{6.6}$$

which can be used with equation (5.7) to give

$$n(t,y) \quad = \quad \left(\cosh\mu y - \tilde{\eta}_0 \sinh\mu y\right), \tag{6.7}$$

We have defined

$$\eta_0 \quad = \quad 1 + \frac{\rho}{\sigma}, \qquad \tilde{\eta}_0 \quad = \quad \eta_0 + \frac{\dot{\eta}_0}{H_0}. \tag{6.8}$$

Here, H_0 is the Hubble constant on the brane at $y = 0$, given by equation 5.1 and can now be written in the form

$$H_0^2 \quad = \quad \frac{\kappa^4}{36}\rho_0^2 - \frac{\Lambda}{6} \quad = \quad \mu^2\left(\eta_0^2 - 1\right). \tag{6.9}$$

In order for there to be a significant difference in the relative 'speeds' of null signals travelling along the two branes, we require that n/a is larger for some y than it is for $y = 0$, i.e. that

$$\frac{n(t,y)}{a(t,y)} \gg \frac{n(t,0)}{a(t,0)} \quad \Leftrightarrow \quad \frac{\cosh\mu y - \tilde{\eta}_0\sinh\mu y}{a_0(\cosh\mu y - \eta_0 \sinh\mu y)} \gg \frac{1}{a_0}. \tag{6.10}$$

An acceptable equation of state on our brane demands that $\rho \propto a_0^{-q}$ and this implies that $\tilde{\eta}_0 = \eta_0 - q\rho/\sigma$ and therefore that $\tilde{\eta}_0 < \eta_0$. Because of this the ratio n/a will diverge as

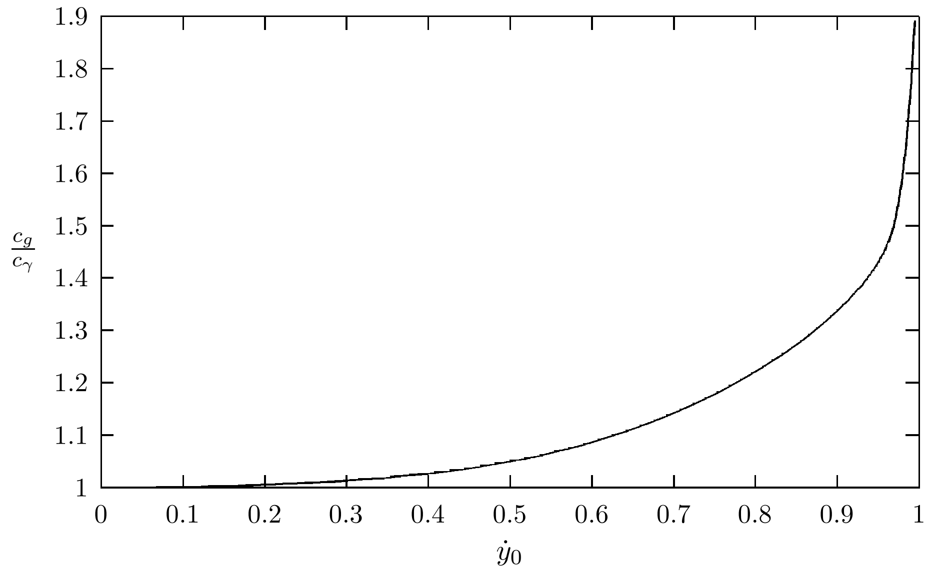

Figure 10: The ratio of gravitational to electromagnetic speeds, as a function of the initial component of velocity in the fifth dimension, in a Z_2-symmetric braneworld. $a_0(t_0) = 0.125$ (earlier time than Figure 8).

$y \to y_h$, where y_h is the position of the horizon defined by $a(t, y_h) = 0$ and is given by

$$\tanh \mu y_h = \frac{1}{\eta_0}. \tag{6.11}$$

This suggests that null signals may travel along the second brane at much greater 'speeds' than on our brane, provided the second brane is close to the horizon. Unfortunately, to correctly calculate the difference in speeds, we need to know the motion of the second brane with respect to the one at $y = 0$.

6.2 General Behaviour of the Inter-brane Distance

In this section we use a simple non-perturbative method to derive the general equation of motion for the inter-brane distance otherwise known as the cosmological radion and denoted by $\mathcal{R}(t)$, which was first derived by [26]. For alternative approaches to this topic see [28–31].

Noting first that if the second brane follows a trajectory given by $y = \mathcal{R}(t)$, then the induced 4-dimensional metric on the second brane is given by

$$\begin{aligned} \mathrm{d}s^2 &= -\left[n^2(t, \mathcal{R}(t)) - \dot{\mathcal{R}}^2\right] \mathrm{d}t^2 + a^2(t, \mathcal{R}(t))\mathrm{d}x^2 && (6.12) \\ &= -\mathrm{d}\tau^2 + a_2^2(\tau)\mathrm{d}x^2, && (6.13) \end{aligned}$$

where τ has been defined as the proper time as seen by an observer on the second brane. The expansion rate of the second brane as seen by an observer on our brane is simply

$$H_2(t) = \frac{1}{a_2}\frac{\mathrm{d}a_2}{\mathrm{d}t} = \left(\frac{\dot{a}}{a} + \frac{a'}{a}\dot{\mathcal{R}}\right)_2, \tag{6.14}$$

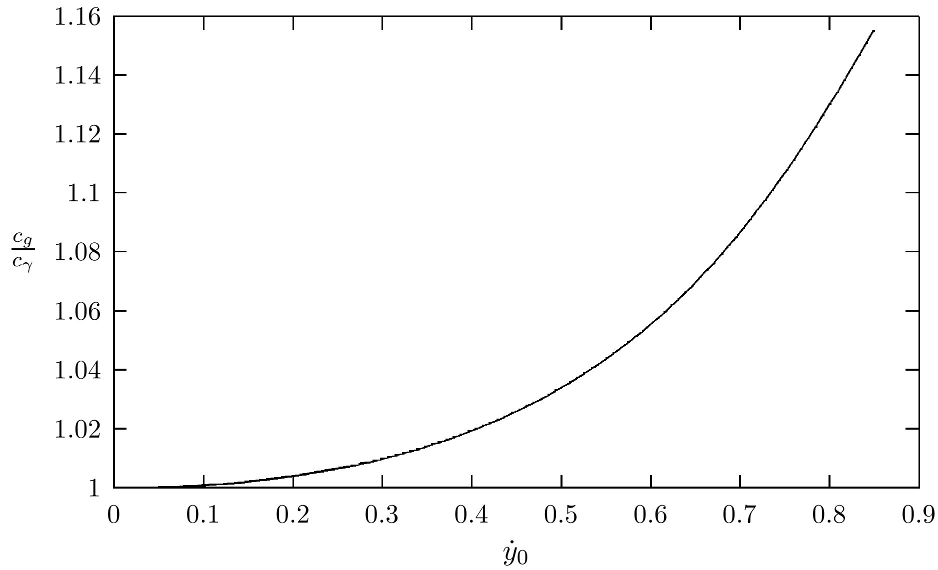

Figure 11: The ratio of gravitational to electromagnetic speeds, as a function of the initial component of velocity in the fifth dimension, in a Z_2-symmetric braneworld. $w = 2.0$ (same time as Figure 8). Note that some previously-returning geodesics no longer return.

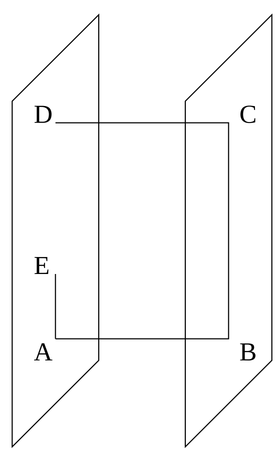

Figure 12: A Null signal that remains on our brane goes from A to E in time t_f, one which leaves the brane could travel between A, B, C and then return to our brane at D in the same time as the first signal reaches E.

and as seen by an observer on the second brane itself

$$\mathcal{H}_2(\tau) = \frac{1}{a_2}\frac{\mathrm{d}a_2}{\mathrm{d}\tau} = H_2(t)\frac{\mathrm{d}t}{\mathrm{d}\tau} \tag{6.15}$$

$$= \left(\frac{\dot{a}}{a} + \frac{a'}{a}\dot{\mathcal{R}}\right)\left(n^2 - \dot{\mathcal{R}}^2\right)^{-1/2}. \tag{6.16}$$

This equation for $\mathcal{H}_2$ is important as it relates the expansion rate of the second brane (with respect to proper time) to $\mathcal{R}$ and $\dot{\mathcal{R}}$. It does not seem so useful at first, as calculating $\mathcal{H}_2$ could be difficult; however, we know that the brane world Friedman equation given by (6.9) is derived from a purely local analysis and that should we have chosen the second brane to be stationary and at $y = 0$, we would have derived the equivalent Friedman equation with ρ replaced by ρ_2. This means that $\mathcal{H}_2(\tau)$ must have the form

$$\mathcal{H}_2^2(\tau) \quad = \quad \frac{\kappa^4}{36}\rho_2^2 - \frac{\Lambda}{6} \quad = \quad \mu^2\left(\eta_2^2 - 1\right). \tag{6.17}$$

Where we have defined $\eta_2 = \rho_2/\sigma$. Equation (6.17) ensures that the second brane is Z_2 symmetric and that it evolves according to the junction conditions of the extrinsic curvature tensor, just as equation (6.9) ensures similar behaviour for our brane. At this point we use the following identity[8] which is obtained from the Einstein equations and is derived in [27]:

$$\left(\frac{\dot{a}}{na}\right)^2 \quad = \quad \frac{a'^2}{a^2} \; - \; \mu^2, \tag{6.18}$$

giving

$$\mathcal{H}_2^2 \quad = \quad \left(\frac{\dot{a}}{na}\right)^2 - \frac{a'^2}{a^2} + \mu^2\eta_2^2. \tag{6.19}$$

Substituting (6.19) into (6.15) and rearranging finally gives the following first order equation for $\dot{\mathcal{R}}$:

$$\dot{\mathcal{R}} \quad = \quad n\left(-\frac{a'\dot{a}}{a^2 n} \;\pm\; \mu^2\eta_2\sqrt{\eta_2^2 - 1}\right)\left(\frac{\dot{a}^2}{n^2a^2} + \mu^2\eta_2^2\right)^{-1}. \tag{6.20}$$

This can be rewritten in the form found by [26]

$$\frac{a'}{a} \;+\; \frac{\dot{a}}{n^2 a}\dot{\mathcal{R}} \quad = \quad \mu\eta_2\left(1 - \frac{\dot{\mathcal{R}}}{n^2}\right)^{1/2}. \tag{6.21}$$

In order to investigate acausal signals we will be interested in a stationary second brane, and therefore need an equation for $\ddot{\mathcal{R}}$. [26] have shown that by differentiating (6.21) with respect to time, considering the three-dimensional symmetries of the bulk energy momentum tensor, and the Z_2-symmetry of the branes, it is possible to derive the following equation for $\ddot{\mathcal{R}}$

$$\frac{\ddot{\mathcal{R}}}{n^2} \;+\; \frac{n'}{n}\left(1 - 2\frac{\dot{\mathcal{R}}^2}{n^2}\right) \;-\; \frac{\dot{n}}{n}\frac{\dot{\mathcal{R}}}{n^2} \quad = \quad -(2\mu\eta_2 + 3\mu p_2)\left(1 - \frac{\dot{\mathcal{R}}^2}{n^2}\right)^{3/2}. \tag{6.22}$$

Equations (6.20) and (6.22) govern the evolution of $\mathcal{R}$ and will be used in the next two sections to evaluate the difference in the speeds of null signals travelling along each brane.

[8] Note that although the derivation is done here for $f = k = w = 0$, this method generalizes to braneworlds with non-zero values of these parameters with the use of the appropriate Friedman equation for equation 6.17 and identity for equation 6.18.

6.3 Acausal Signals: Stationary Branes

The simplest solutions to equations (6.20) and (6.22) are those describing a stationary second brane; in this section we will investigate the appropriate conditions and the ratio of distances travelled on each brane in a certain time interval. Setting $\dot{\mathcal{R}} = \ddot{\mathcal{R}} = 0$ in (6.21) and (6.22) trivially gives the well known conditions on η_2 and $p_2 \equiv P_2/\sigma$ which ensure that the second brane does not move:

$$\eta_2 = \frac{\sinh\mu\mathcal{R} - \eta_0\cosh\mu\mathcal{R}}{\cosh\mu\mathcal{R} - \eta_0\sinh\mu\mathcal{R}}, \tag{6.23}$$

$$p_2 = -\frac{1}{3}\frac{\sinh\mu\mathcal{R} - \tilde{\eta}_0\cosh\mu\mathcal{R}}{\cosh\mu\mathcal{R} - \tilde{\eta}_0\sinh\mu\mathcal{R}} - \frac{2}{3}\eta_2. \tag{6.24}$$

Note that if we demand standard cosmology on our brane at late times then this requires that both η_0 and $\tilde{\eta}_0$ tend to 1 (not zero as [26] have suggested). This results in the equation of state of the second brane becoming $-\eta_2 = p_2 = 1$ which is equivalent to $-\rho_2 = P_2 = 6\mu/\kappa^2$, the conditions in the original Randall-Sundrum two brane scenario. Assuming the above form for the energy and pressure densities on the second brane, we now just have to evaluate the distances travelled by null signals between times t_1 and t_2 on either brane:

$$D_1 = \int_{t_1}^{t_2}\frac{1}{a_0}\mathrm{d}t, \qquad D_2 = \int_{t_1}^{t_2}\frac{\sqrt{n^2(t,\mathcal{R}(t)) - \dot{\mathcal{R}}^2}}{a^2(t,\mathcal{R}(t))}\mathrm{d}t, \tag{6.25}$$

however since we have assumed a stationary second brane, D_2 simplifies to become

$$D_2 = \int_{t_1}^{t_2}\frac{n(t,\mathcal{R}(t))}{a(t,\mathcal{R}(t))}\mathrm{d}t = \int_{t_1}^{t_2}\frac{\cosh\mu\mathcal{R} - \tilde{\eta}_0\sinh\mu\mathcal{R}}{a_0(\cosh\mu\mathcal{R} - \eta_0\sinh\mu\mathcal{R})}\mathrm{d}t. \tag{6.26}$$

As mentioned before, a standard equation of state on our brane leads to $\tilde{\eta}_0$ having the form $\tilde{\eta}_0 = \eta_0 - q\rho/\sigma$ and therefore,

$$D_2 = \int_{t_1}^{t_2}\left[\frac{1}{a_0} + \frac{(q\rho/\sigma)\sinh\mu\mathcal{R}}{a_0(\cosh\mu\mathcal{R} - (1+\rho/\sigma)\sinh\mu\mathcal{R})}\right]\mathrm{d}t \tag{6.27}$$

$$= D_1 + \Delta D. \tag{6.28}$$

Rewriting the Friedman equation for our brane in terms of ρ/σ gives $H_0^2 = \mu^2 q^2((\rho/\sigma)^2 + 2\rho/\sigma)$. This, and the fact that $\rho = \lambda/a_0^q$ allows the ΔD integral to be converted into the following form:

$$\Delta D = \frac{A(\mathcal{R})}{\mu(\sigma\lambda)^{1/q}}\int_{u_1}^{u_2}\frac{\mathrm{d}u}{u^{1/q}\sqrt{2u+1}(u - A(\mathcal{R}))} \tag{6.29}$$

where $u = \sigma/\rho$ and all the information on the second brane's position has been grouped into $A(\mathcal{R}) = \sinh\mu\mathcal{R}/(\cosh\mu\mathcal{R} - \sinh\mu\mathcal{R})$. The integral in (6.25) can be similarly converted, and we then obtain the ratio of the extra distance travelled on the second brane, divided by the distance travelled on our brane in the same time,

$$\frac{\Delta D}{D_1} = qA(\mathcal{R})\left(\int_{u_1}^{u_2}\frac{\mathrm{d}u}{u^{1/q}\sqrt{2u+1}(u - A(\mathcal{R}))}\right)\left(\int_{u_1}^{u_2}\frac{\mathrm{d}u}{u^{1/q}\sqrt{2u+1}}\right)^{-1}. \tag{6.30}$$

We now need to determine the possible limits on u_1 and u_2. Obviously the contribution of the integral to ΔD for values of u greater than $(q+1)A(\mathcal{R})$ will be less than the corresponding contribution to D_1 and so we choose the upper limit on u_2 to be $(q+1)A(\mathcal{R})$, as a larger u_2 is of no interest. The minimum possible value of u_1 is slightly harder to find as it is constrained by requiring that the energy densities of both branes remain below the 5-dimensional Planck mass limit

$$\rho \;\leq\; M_5^4 \quad\Rightarrow\quad u \;=\; \frac{\sigma}{\rho} \;\;\leq\;\; \frac{6M_5^2}{M_4^2} \;=\; \frac{6}{M}, \tag{6.31}$$

where we have defined M to be the dimensionless ratio of the squares of the four- and five-dimensional Planck masses $M \equiv M_4^2/M_5^2$. Demanding a similar constraint on ρ_2 and using the stability condition given by (6.23), results in the following condition

$$|\rho_2| \;\leq\; M_5^4 \quad\Rightarrow\quad |\eta_2| \;=\; \frac{|\rho_2|}{\sigma} \;\;\leq\;\; \frac{M}{6} \tag{6.32}$$

$$\Rightarrow\quad \frac{\eta_0\cosh\mu\mathcal{R} - \sinh\mu\mathcal{R}}{\cosh\mu\mathcal{R} - \eta_0\sinh\mu\mathcal{R}} \;\;\leq\;\; \frac{M}{6}. \tag{6.33}$$

This can be rearranged and expressed in terms of u using the fact that $\eta_0 = 1 + 1/u$, to give the constraint on u that ensures that $|\rho_2|$ is always less than M_5^4:

$$u \;\;\geq\;\; \frac{M\tanh\mu\mathcal{R} + 6}{(M-6)(1-\tanh\mu\mathcal{R})}. \tag{6.34}$$

In the next two sections we will use this constraint to determine u_1 which corresponds to the earliest possible time that a null signal can set off along the second brane. We examine both the near brane ($\mu\mathcal{R} \ll 1$) and the far brane ($\mu\mathcal{R} \gg 1$) limits.

6.4 Near Brane Limit

We will now evaluate the integral expression for $\Delta D/D_1$ given by (6.30), in the small $\mu\mathcal{R}$ limit. The minimum possible value of the inter-brane distance $\mathcal{R}$ will be the inverse of the 5D Planck mass: $\mathcal{R} \geq 1/M_5$ and for a cosmologically realistic brane μ is given by $\mu = M_5^3/M_4^2$, leading to the relation

$$\mu\mathcal{R} \;\;\geq\;\; \frac{M_5^2}{M_4^2} \;\;=\;\; \frac{1}{M}. \tag{6.35}$$

If we therefore assume $1/M \leq \mu\mathcal{R} \ll 1$, we then find that in this limit $A(\mathcal{R}) \simeq \mu\mathcal{R}$ and therefore that $u_1 < u_2 = (q+1)A(\mathcal{R}) \ll 1$. Equation (6.30) can then be approximated by

$$\frac{\Delta D}{D_1} \;\;\simeq\;\; qA(\mathcal{R})\left(\int_{u_1}^{(q+1)A(\mathcal{R})} \frac{\mathrm{d}u}{u^{1/q}(u - A(\mathcal{R}))}\right)\left(\int_{u_1}^{(q+1)A(\mathcal{R})} \frac{\mathrm{d}u}{u^{1/q}}\right)^{-1}. \tag{6.36}$$

If we now assume that our brane is undergoing radiation dominance by setting $q = 4$, the first integral on the right hand side of (6.36) can be integrated to give

$$A^{\frac{1}{4}}\int_{u_1}^{u_2} \frac{\mathrm{d}u}{u^{1/4}(u - A(\mathcal{R}))} \;\;=\;\;$$

$$2\arctan\left[\left(\frac{u_2}{A}\right)^{\frac{1}{4}}\right] \;-\; 2\arctan\left[\left(\frac{u_1}{A}\right)^{\frac{1}{4}}\right] \;+\; \ln\left[\frac{(\frac{u_2}{A})^{\frac{1}{4}} - 1}{(\frac{u_1}{A})^{\frac{1}{4}} - 1}\cdot\frac{(\frac{u_1}{A})^{\frac{1}{4}} + 1}{(\frac{u_2}{A})^{\frac{1}{4}} + 1}\right],$$

and the second integral is trivially given by

$$\int_{u_1}^{u_2} \frac{\mathrm{d}u}{u^{1/4}} \quad = \quad \frac{4}{3}(u_2^{\frac{3}{4}} - u_1^{\frac{3}{4}}). \tag{6.37}$$

From above we take $u_2 = (q+1)A(\mathcal{R}) = 5A(\mathcal{R})$, and therefore we just need to evaluate $u_1/A(\mathcal{R})$ in the small $\mu\mathcal{R}$ limit using equation (6.34)

$$\frac{u_1}{A(R)} \quad = \quad \frac{M\tanh\mu\mathcal{R} \;+\; 6}{(M \;-\; 6)\tanh\mu\mathcal{R}} \quad \simeq \quad 1 \;+\; \frac{3}{2M\mu\mathcal{R}}. \tag{6.38}$$

Replacing the values for u_1 and u_2 into the expression for $\Delta D/D_1$ and ignoring all sub-dominant terms, finally gives the ratio of the maximum extra distance travelled by a null signal on the second brane compared to the first in the near brane limit as

$$\frac{\Delta D}{D_1} \quad \simeq \quad 1.28\ln(M\mu\mathcal{R}). \tag{6.39}$$

The value of M_5 is constrained by nucleosynthesis such that $M_5 \geq 30$TeV. This leads to the corresponding constraint on the dimensionless ratio M to be

$$M \quad = \quad \frac{M_4^2}{M_5^2} \quad < \quad 10^{29}. \tag{6.40}$$

In order for the above approximations to be valid $\mu\mathcal{R}$ must satisfy $\mu\mathcal{R} < 10^{-2}$, therefore (6.39) becomes

$$\frac{\Delta D}{D_1} \quad \simeq \quad 80. \tag{6.41}$$

This shows that any seemingly acausal effects due to signals travelling on the second brane as opposed to our brane are too small to solve the cosmological horizon problem in the $(\mathcal{R} \ll 1/\mu)$ limit. Another important point is that ΔD is only significantly greater than D_1 for a very brief time, shortly after the big bang. If the signals were to travel for longer periods, say until $u_2 = 1$ then the distance ratio would become much less than 1: $\Delta D/D_1 \simeq \mu\mathcal{R}\ln(M\mu\mathcal{R})$ as can be seen from (6.36).

6.5 Far Brane Limit

In this Section we investigate the behaviour of null signals travelling on the second brane when the inter-brane distance is large $(\mathcal{R} \gg 1/\mu)$. The fact that the second brane has to be closer to our brane than the horizon implies that we will be examining signals that are travelling at late times with respect to our Universe, as opposed to the previous Section where the only interesting situations occurred at early times.

In the $\mathcal{R} \gg 1/\mu$ limit, the function $A(\mathcal{R})$ becomes very large and can be approximated by

$$A(\mathcal{R}) \quad = \quad \frac{\tanh\mu\mathcal{R}}{1-\tanh\mu\mathcal{R}} \quad \simeq \quad \frac{1}{2}e^{2\mu\mathcal{R}}. \tag{6.42}$$

As before, we will require the ratios $u_2/A(\mathcal{R}) = (q+1)$ and $u_1/A(\mathcal{R})$ which is obtained from equation (6.34) and is approximately given by

$$\frac{u_1}{A(R)} \quad = \quad \frac{M\tanh\mu\mathcal{R} \;+\; 6}{(M \;-\; 6)\tanh\mu\mathcal{R}} \quad \simeq \quad 1 \;+\; \frac{12}{M}. \tag{6.43}$$

Therefore both u_1 and u_2 are exponentially large and so the expression for $\Delta D/D_1$ given by equation (6.30) can be simplified in the large $\mathcal{R}$ limit to

$$\frac{\Delta D}{D_1} \simeq qA(\mathcal{R})\left(\int_{(1+\frac{12}{M})A(\mathcal{R})}^{(q+1)A(\mathcal{R})} \frac{\mathrm{d}u}{u^{\frac{1}{q}+\frac{1}{2}}(u-A(\mathcal{R}))}\right)\left(\int_{(1+\frac{12}{M})A(\mathcal{R})}^{(q+1)A(\mathcal{R})} \frac{\mathrm{d}u}{u^{\frac{1}{q}+\frac{1}{2}}}\right)^{-1}. \qquad (6.44)$$

Assuming radiation dominance, the first integral on the right hand side of (6.44) can be solved to give

$$A^{\frac{3}{4}}\int_{u_1}^{u_2} \frac{\mathrm{d}u}{u^{3/4}(u-A(\mathcal{R}))} =$$
$$-2\arctan\left[\left(\frac{u_2}{A}\right)^{\frac{1}{4}}\right] + 2\arctan\left[\left(\frac{u_1}{A}\right)^{\frac{1}{4}}\right] + \ln\left[\frac{(\frac{u_2}{A})^{\frac{1}{4}}-1}{(\frac{u_1}{A})^{\frac{1}{4}}-1}\cdot\frac{(\frac{u_1}{A})^{\frac{1}{4}}+1}{(\frac{u_2}{A})^{\frac{1}{4}}+1}\right],$$

which when combined with (6.44) and (6.43), and after all subdominant terms are neglected, results in the following expression for the extra distance travelled on the second brane in the far brane limit

$$\frac{\Delta D}{D_1} \simeq 2\ln(M). \qquad (6.45)$$

Note that if instead of radiation domination we had chosen matter domination and set $q=3$, the leading term given in equation (6.45) would not have been altered. Replacing into (6.45) the maximum possible value of $M \simeq 10^{29}$ leads to the final result

$$\frac{\Delta D}{D_1} \simeq 130. \qquad (6.46)$$

Unlike when $\mathcal{R} \ll 1/\mu$, here ΔD is significantly greater than D_1 for an extended period of time. Solving the Friedman equation gives the late time relation between u and the cosmic time t as measured on our brane to be $u \simeq \mu^2 q^2 t^2/2$. This implies that $\Delta D > D_1$ during a time interval of $\Delta t \simeq (M_4^2/2M_5^3)\exp(\mu\mathcal{R}) = 10^{27}\exp(\mu\mathcal{R})\mathrm{TeV}^{-1}$. However, as can be seen from equation 6.44, if limits for the integral for $\Delta D/D_1$ are such that $u_2 \gg u_1$, then any acausal effect becomes negligible. Taking u_1 to be just before recombination and u_2 at the present day leads to $u_1 \sim 10^{12}$ and $u_2 \sim 10^{21}$, and therefore the horizon problem cannot be solved in this manner, though this effect may have important consequences in other areas of cosmology such as structure formation. One should note that the effectiveness of the acausality would also be lessened by the time taken for signals to cross between the branes.

6.6 Acausal Signals: Moving Branes

Up till now we have been examining the possibility of acausal signals travelling along stationary branes. It is however interesting to consider the effect of allowing the branes to move. As will be shown, for a general cosmological scenario the equation of motion of the second brane is very difficult to solve; various qualitative features can, however, be investigated and will be discussed in this section. We start with the equation for the time dependent inter-brane spacing $\mathcal{R}(t)$, derived previously (6.21)

$$\frac{\dot{\mathcal{R}}}{n} = \left(-\frac{a'\dot{a}}{a^2 n} + \mu^2\eta_2\sqrt{\eta_2^2-1}\right)\left(\frac{\dot{a}^2}{n^2a^2}+\mu^2\eta_2^2\right)^{-1}. \qquad (6.47)$$

This can be rewritten using equations (6.17), (6.19), (6.6) and (6.7) as

$$\frac{\dot{\mathcal{R}}}{n} = \frac{H_0(\eta_0 \cosh \mu\mathcal{R} - \sinh \mu\mathcal{R}) + \mu\eta_2\mathcal{H}_2(\cosh \mu\mathcal{R} - \eta_0 \sinh \mu\mathcal{R})^2}{(\eta_0 \cosh \mu\mathcal{R} - \sinh \mu\mathcal{R})^2 + \mathcal{H}_2^2(\cosh \mu\mathcal{R} - \eta_0 \sinh \mu\mathcal{R})^2}. \tag{6.48}$$

For the trivial case where η_2 is constant and therefore $\mathcal{H}_2 = 0$, equation (6.48) can easily be solved numerically as was done in [26]. For a cosmologically realistic case where η_2 is time dependent, the solution is less easily obtained. If, for example, we assume a standard equation of state on the second brane: $P_2 = \omega_2\rho_2$ which provided $\omega_2 > -1$, implies that $\rho_2 \propto 1/a_2^{q_2}$, we can then solve the Friedman equation

$$\mathcal{H}_2^2 = \frac{1}{a_2^2}\left(\frac{\mathrm{d}a_2}{\mathrm{d}\tau}\right)^2 = \frac{1}{q_2^2\eta_2^2}\left(\frac{\mathrm{d}\eta_2^2}{\mathrm{d}\tau}\right)^2 = \mu^2(\eta_2^2 - 1), \tag{6.49}$$

to give $\eta_2 = -1/\sin(q_2\mu\tau)$ and $\mathcal{H}_2 = \mu^2/\tan(q_2\mu\tau)$. However, to solve equation (6.48) we need to know η_2 and $\mathcal{H}_2$ in terms of t: our cosmic time, which is related to the second branes cosmic time τ by

$$\mathrm{d}\tau = \sqrt{n^2 - \dot{\mathcal{R}}^2}\,\mathrm{d}t. \tag{6.50}$$

This makes it much more difficult to solve for $\dot{\mathcal{R}}$ and due to this added level of complication we leave a more general study of (6.48) to future work. We now examine the situation qualitatively and argue that acausality in the moving brane model is not appreciably greater than in the static case.

From equation (6.47) it can be seen immediately that since $a' < 0$ for $0 < R < y_h$, then if $\eta_2 > 0$ then $\dot{\mathcal{R}} > 0$ for all R and the second brane will move away from our brane and freeze out at the horizon, which has a time dependent position given by $y_h = \arctan(1/\eta_0)$. If $\eta_2 < 0$, then $\dot{\mathcal{R}} = 0$ only when

$$\mu\eta_2 = \frac{a'(t,\mathcal{R})}{a(t,\mathcal{R})}. \tag{6.51}$$

Requiring also that $\ddot{\mathcal{R}} = 0$ gives the conditions for stationary branes as discussed previously. The stability of a stationary second brane has been investigated in [26] where it was shown that if both branes are de Sitter the equilibrium position is unstable, while if both branes are anti-de Sitter then it is stable. For our purposes, we only need to examine the case where the second brane moves in the positive y direction and freezes out at the horizon, as the other possibilities are that the second brane approaches our brane and collides with it which is physically unacceptable, or that the second brane is stable: the case which has been examined previously.

If the second brane does freeze out, then $\mathcal{R} \to y_h$ and $\dot{\mathcal{R}} \to n(t,R)$ as can be checked from equation (6.47). The expression for the distance travelled on the second moving brane is

$$D_2 = \int_{t_1}^{t_2} \frac{\sqrt{n^2(t,\mathcal{R}(t)) - \dot{\mathcal{R}}^2}}{a^2(t,\mathcal{R}(t))}\mathrm{d}t, \tag{6.52}$$

which will therefore tend to zero as $\dot{\mathcal{R}} \to n(t,R)$. Another problem is the distance that signals have to travel to return. In order for there to be a significant acausal effect, the null signal has to be travelling on the second brane for a substantial amount of time and therefore would be a large distance away from our brane when starting to return. The combination of these two effects: the distance travelled by a null signal on the second brane frozen at the horizon tending to zero, and the large distance returning signals would have to travel, suggest that little or no acausal effects would be observed on our brane.

7. Discussion

We have investigated the behaviour of null geodesics in several five-dimensional brane world scenarios. In the single brane case it was shown that apparent causality violations caused by such signals taking shortcuts through the bulk were small. The ratio of the speed of five-dimensional gravity signals to the speed of four-dimensional light c_g/c_γ was found to be in general not more than two. The effects of introducing a non-zero Weyl tensor component and of relaxing the mirror (or Z_2) symmetry were examined and shown in general to decrease the observed acausality.

It was found that during early times in the Universe many geodesics would return to our brane, as opposed to late times when most of the geodesics leaving our brane freeze out at the horizon. This is understandable, as at late times ($\rho \to 0$) the cosmological metrics that were considered tend to the standard Randall-Sundrum metric which, as previously discussed, results in geodesics not returning. The behaviour of these geodesics could be used in conjunction with the temperature dependent production rate of gravitons to exactly determine the amount of energy lost to the bulk throughout the history of the Universe as is discussed in [32].

Analysis of the two brane scenario (with $w = f = k = 0$) shows that signals can in some situations travel along the second brane significantly faster than along our brane. This effect, however, would only last for a certain time depending on the position of the second brane. In the near brane limit ($\mu\mathcal{R} \ll 1$) this period is very brief and any acausal effects would become negligible when longer time intervals are considered. In the far brane limit ($\mu\mathcal{R} \gg 1$) however, the period of interest can last for $\Delta t \simeq 10^{27} \exp(\mu\mathcal{R})\mathrm{TeV}^{-1}$and it was found that the apparent speeds of null signals were approximately 130 times faster on the second brane than on the first. It should be noted that a non-zero w and f would lessen the above effect as the crucial ratio $n(t, \mathcal{R})/a(t, \mathcal{R})$ is largest only when $w = f = 0$. Another factor that would detract from the effectiveness of this mechanism in the far brane limit is the time taken for signals to travel between the two branes.

In neither one nor two brane scenarios are the possible acausal effects significant enough to enable us to solve the cosmological horizon problem. The increase in the gravitational particle horizon could, however, have important effects on other areas of cosmology such as the initial conditions of inflation, structure formation and the production of topological defects. It would also technically be possible to measure the time delay between the detection of gravitational waves and light waves from a particular cosmic event; however, we believe that this would require significant advances in gravitational wave detector technology.

There is also the possibility that a higher dimensional model could produce a large enough acausal effect to solve the horizon problem [33–40]. Unfortunately, we are currently unaware of any metrics in $(d+1)$ dimensions with $d > 4$ where acausal signals are possible. For example there have been several six-dimensional 'brane world' models proposed that have a metric Ansatz of the general form

$$\mathrm{d}s^2 \quad = \quad \phi^2(x^i)\eta_{\alpha\beta}(x^\nu)\mathrm{d}x^\alpha \mathrm{d}x^\beta \quad + \quad g_{jk}(x^i)\mathrm{d}x^j \mathrm{d}x^k, \tag{7.1}$$

where α, β, and ν run across time and normal space dimensions (0 to 3) and i and j run across the two extra spatial dimensions (5 to 6). It can be seen that no acausal effects are possible since both the time and the three-dimensional spatial components of the metric have the same dependence on the extra dimensions, unlike the five-dimensional case where $n(t,y) \neq a(t,y)$. A fully cosmological six-dimensional metric could however overcome this problem.

Acknowledgments

This work is supported in part by PPARC.

References

[1] V. A. Rubakov and M. E. Shaposnikov, *Do we live inside a domain wall?*, *Phys. Lett.* **B125** (1983) 136.

[2] K. Akama, *An Early Proposal of "Brane World"*, *Lect. Notes. Phys* **176** (1982) 267–271, [hep-th/0001113].

[3] I. Antoniadis, *A possible new dimension at a few TeV*, *Phys. Lett.* **B246** (1990) 377.

[4] M. Visser, *An exotic class of Kaluza-Klein models*, *Phys. Lett.* **B159** (1985) 22, [hep-th/9910093].

[5] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *The Heirarchy Problem and New Dimensions at a Millimeter*, *Phys. Lett.* **B429** (1998) 263–272, [hep-ph/9803315].

[6] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *Phenomenology, Astrophysics and Cosmology of Theories with Sub-Millimeter Dimensions and TeV Scale Quantum Gravity*, *Phys. Rev.* **D59** (1999) [hep-ph/9807344].

[7] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, *New Dimensions at a Millimeter to a Fermi and Superstrings at a TeV*, *Phys. Lett.* **B436** (1998) [hep-ph/9804398].

[8] R. Sundrum, *Effective Field Theory for a Three-Brane Universe*, *Phys. Rev.* **D59** (1999) 85009, [hep-ph/9805471].

[9] L. Randall and R. Sundrum, *An Alternative to Compactification*, *Phys. Rev. Lett.* **83** (1999) 4690–4693, [hep-th/9906064].

[10] L. Randall and R. Sundrum, *A Large Mass Hierarchy from a Small Extra Dimension*, *Phys. Rev. Lett.* **83** (1999) 3370–3373, [hep-th/9905221].

[11] P. Binétruy, C. Deffayet, and D. Langlois, *Non-conventional cosmology from a brane-universe*, *Nucl. Phys.* **B565** (2000) 269–287, [hep-th/9905012].

[12] C. Csaki, M. Graesser, C. Kolda, and J. Terning, *Cosmology of One Extra Dimension with Localized Gravity*, *Phys. Lett.* **B462** (1999) 34–40, [hep-ph/9906513].

[13] J. M. Cline, C. Grojean, and G. Servant, *Cosmological Expansion in the Presence of an Extra Dimension*, *Phys. Rev. Lett.* **83** (1999) 4245, [hep-ph/9906523].

[14] R. Maartens, D. Wands, B. A. Bassett, and I. P. C. Heard, *Chaotic inflation on the brane*, *Phys. Rev.* **D62** (2000) 041301, [hep-ph/9912464].

[15] S. C. Davis, W. B. Perkins, A. C. Davis, and I. Vernon, *Cosmological Phase Transitions in a Brane World*, *Phys. Rev.* **D63** (2001) 083518, [hep-ph/0012223].

[16] D. J. H. Chung and K. Freese, *Can Geodesics in Extra Dimensions Solve the Cosmological Horizon Problem?*, *Phys. Rev.* **D62** (2000) 063513, [hep-ph/9910235].

[17] H. Stoica, S.-H. H. Tye, and I. Wasserman, *Cosmology in the Randall-Sundrum Brane World Scenario*, *Phys. Lett.* **B482** (2000) 205–212, [hep-th/0004126].

[18] N. Deruelle and T. Dolezel, *Brane versus shell cosmologies in Einstein and Einstein-Gauss-Bonnet theories*, *Phys. Rev.* **D62** (2000) 103502, [gr-qc/0004021].

[19] A.-C. Davis, I. Vernon, S. C. Davis, and W. B. Perkins, *Brane World Cosmology Without the Z_2 Symmetry*, *Phys. Lett.* **B504** (2000) 254–261, [hep-th/0008132].

[20] B. Carter and J. Uzan, *Reflection symmetry breaking scenarios with minimal gauge form coupling in brane world cosmology*, gr-qc/0101010.

[21] R. A. Battye and B. Carter, *Generic junction conditions in brane-world scenarios*, *Phys. Lett.* **B509** (2001) 331–336, [hep-th/0101061].

[22] R. A. Battye, B. Carter, A. Mennim, and J.-P. Uzan, *Einstein equations for an asymmetric brane-world*, hep-th/0105091.

[23] W. Mück, K. S. Viswanathan, and I. V. Volovich, *Geodesics and Newton's Law in Brane Backgrounds*, hep-th/0002132.

[24] R. Caldwell and D. Langlois, *Shortcuts in the fifth dimension*, gr-qc/0103070.

[25] C. Csáki, J. Erlich, and C. Grojean, *Gravitational lorentz violations and adjustment of the cosmological constant in asymmetrically warped spacetimes*, *Nucl. Phys.* **B604** (2001) 312–342, [hep-th/0012143].

[26] P. Binétruy, C. Deffayet, and D. Langlois, *The Radion in Brane Cosmology*, hep-th/0101234.

[27] P. Binétruy, C. Deffayet, U. Ellwanger, and D. Langlois, *Brane cosmological evolution in a bulk with cosmological constant*, *Phys. Lett.* (2000) 285–291, [hep-th/9910219].

[28] C. Charmousis, R. Gregory, and V. A. Rubakov, *Wave function of the radion in a brane world*, *Phys. Rev.* **D62** (2000) 067505, [hep-th/9912160].

[29] T. Tanaka and X. Montes, *Gravity in the brane-world for two-branes model with stabilized modulus*, *Nucl. Phys.* **B582** (2000) 259–276, [hep-th/0001092].

[30] U. Gen and M. Sasaki, *Radion on the de Sitter brane*, gr-qc/0011078.

[31] A. Karch and L. Randall, *Locally Localized Gravity*, *JHEP* **0105** (2001) 008, [hep-th/0011156].

[32] A. Hebecker and J. March-Russell, *Randall-Sundrum II Cosmology, AdS/CFT, and the Bulk Black Hole*, hep-ph/0103214.

[33] T. Gherghetta and M. Shaposhnikov, *Localizing Gravity on a String-Like Defect in Six Dimensions*, *Phys. Rev. Lett.* **85** (2000) 240–243, [hep-th/0004014].

[34] M. Gogberashvili and P. Midodashvili, *Brane-Universe in Six Dimensions*, hep-ph/0005298.

[35] A. Chodos and E. Poppitz, *Warp Factors and Extended Sources in Two Transverse Dimensions*, *Phys. Lett.* **B471** (1999) 119–127, [hep-th/9909199].

[36] A. G. Cohen and D. B. Kaplan, *Solving the Hierarchy Problem with Noncompact Extra Dimensions*, *Phys. Lett.* **B470** (1999) 52–58, [hep-th/9910132].

[37] R. Gregory, *Nonsingular global string compactifications*, *Phys. Rev. Lett.* **84** (2000) 2564–2567, [hep-th/9911015].

[38] I. Olasagasti and A. Vilenkin, *Gravity of higher-dimensional global defects*, *Phys. Rev.* **D62** (2000) 044014, [hep-th/0003300].

[39] C. Csaki, J. Erlich, T. J. Hollowood, and Y. Shirman, *Universal Aspects of Gravity Localized on Thick Branes*, *Nucl. Phys.* **B581** (2000) 309–338, [hep-th/0001033].

[40] J.-W. Chen, M. A. Luty, and E. Ponton, *A Critical Cosmological Constant from Millimeter Extra Dimensions*, *JHEP* **0009** (2000) 012, [hep-th/0003067].