# Blocks and circularity in labour requirements: An interplay between Clusters and Subsystems in the EU

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**Abstract**

Clusters and subsystems are two frequently used tools in inter-industry analysis, the former clarifying structure while the latter summarising circularity. Since industry blocks with crucial direct linkages will probably have strong indirect ties as well, localised intra-cluster feedback effects may play a prominent role in explaining total labour requirements. In this paper, we first quantify the labour redistribution taking place between industries and subsystems within and between clusters. Next, we extend the standard notion of vertically integrated labour to account for intra- and extra-cluster circularity, quantifying the extent to which overall productivity growth in every subsystem originates from intra-cluster industries. Both issues are illustrated for the consolidated European Union economy (EU27) between 2000 and 2007.

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Dependence and independence, hierarchy and circularity [...] are the four basic concepts of structural analysis. (Leontief, 1986[1963], p. 166)

## 1. Introduction

The extent and pace of economic growth crucially depends on the *structure* of the economic system. In particular, a detailed analysis of intersectoral linkages and *industry blocks*[3] — i.e. "the most important chains of sectors in the input-output table, as these denote the most important or *fundamental structure* of an economic system" (Hoen, 2002, p. 134, italics added) — on the one hand, and of the *circularity* of production — i.e. the extent and roundaboutness of linkages between industries within the comprehensive production process of each single item of final demand — on the other, are of utmost importance for the definition of effective industrial policies. While the former task may be achieved through cluster analysis, the latter relies on reduction procedures.

The *reduction* of an Input-Output (IO, hereinafter) model was formally introduced by Leontief (1967);[4] it consists in choosing a group of commodities in terms of which all others have to be expressed, their output thus being eliminated.[5] By doing so, the unit of analysis switches from the industry to the corresponding final de-

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[3]In what follows the terms 'cluster', 'industry block' and 'community' will be interchangeably used to indicate a subset of industries sharing strong connections.

[4]See also Guccione and Gillen (1995).

[5]A concept that can be traced back to the origins of economic analysis: "Adam Smith discussed at length the

mand subsystem (in the sense of Sraffa, 1960), and the reduced commodities participate into vertically integrated productive capacity and labour (Pasinetti, 1973).[6]

Instead, the focus of *cluster* analysis in an IO context has been mainly twofold: mapping the fundamental structure of an economic system via graphic techniques, and looking for consistent aggregation criteria (Ghosh, 1960). In both cases, the starting point has been that of scaling down the degree of complexity of the information provided by IO tables, through the identification of industry groups connected by above-average linkages.

Even though IO literature is rich of attempts in both directions, to our knowledge few attention has been devoted to the fact that clusters' identification provides a *partition* of the inter-industry network, which can be treated in a similar fashion (from a purely formal point of view) as the partition into regions of a multi-regional IO table.

In particular, by exploiting block partitioning of matrices, it is possible to quantify backward and forward linkages between clusters along the same lines as has been done for the case of different regions by e.g. Miyazawa (1966) and Miller (1969). Such extension was already envisaged by Miyazawa (1966), and further developed in Miyazawa (1971), who studied "the interdependence between service and goods-producing sectors", partitioning the IO matrix according to this criterion. A similar idea was also put forward by Milana (1985), who posed the question of how to build subsystems for gross, rather than net, output, exploiting Miyazawa's internal and external matrix multipliers.[7]

A key point of computing internal and external multipliers is that of measuring feedback and spillover effects. In particular, if we consider two industry clusters (A and B), final demand directed towards group A might induce demand for group B intermediates which, to be produced, require further inputs from group A, generating a feedback effect. Instead, demand for intermediates by group A, produced by industries in group B, to satisfy final demand of group A products, gives rise to spillovers.

The aim of the present paper is that of further developing the analysis of internal and external matrix multipliers, by rendering endogenous the partition of the IO table into industry blocks via cluster analysis. Moreover, by deriving vertically integrated sectors at the cluster level we identify *blocks of circularity*. In so doing, linkage measures between each *cluster* and the rest of the economy are introduced. A hierarchy of such clusters according to their closeness to final demand is further obtained on the basis of the difference between vertically integrated and direct labour.

Linkage indicators usually refer to the systemic effects induced by a single sector, or to the effect of all other activities on a given specific industry. The main original feature of the present contribution with respect to traditional analyses is that of singling out linkages between industries conforming structural paths, by grouping activities whose interactions have an above-average weight in determining systemic effects.

More specifically, the two research questions faced by the present contribution are the following:

- what is the labour redistribution between intra-cluster and extra-cluster industries and subsystems?
- what is the proportion of total (direct and indirect) labour requirements of each subsystem which is due to: (a) self-contained intra-cluster circularity, or (b) induced inter-cluster feedback and spillover effects?

The first question aims at identifying net backward linkages at the cluster level, which could be exploited by properly coordinated final demand expansions in order to stimulate employment. Additionally, it may render explicit the

question of whether corn should be measured in labor units required to grow it, or, on the contrary, labor measured in terms of corn that a worker needs to live" (Leontief, 1967, p. 419).

[6]In what follows, the terms 'subsystem' and 'vertically integrated sector' will be interchangeably used.

[7]Building on Milana (1985), Heimler (1991) computed what he called the industries' *degree of vertical integration*, i.e. the ratio of value added directly or indirectly due to each industry's *gross output* to total value added.

extent of phenomena such as tertiarisation. The answer to the second question provides a separate assessment of internal synergies within a cluster, on the one hand, and of feedback and spillover effects between clusters, on the other, which may be of importance in the design of performance indicators for industrial and innovation policies.

In fact, motivating the distinction between reduction and clustering, emphasized throughout the paper, within the context of industrial and innovation policy design is highly relevant. A 'wide' taxonomy of industrial policy includes (Pelkmans, 2006, p. 47): (i) framework aspects (e.g. competition policy, quality standards), (ii) horizontal interventions (e.g. labour force training, public procurement, R&D stimuli) and (iii) sectoral/specific interventions (e.g. *filières*, trade policy, specialised technology policy).

On the one hand, reduction procedures, focusing on direct and indirect content of a common element (labour, energy inputs, R&D expenditure) per unit of final demand, might be a valuable tool for assessing the systemic effects of *horizontal* industrial policy. For example, R&D stimuli or labour training programmes imply changes in industry-specific intensities, whose propagation through the IO network may be quantified. Subsystems are particularly useful for this task.

On the other hand, clustering leads precisely to the identification of linkage-based *filières*, providing a basic input to *sectoral* industrial policies. State aid targeted towards strategic industries might benefit from a 'graph' of strongly connected sectors, which renders clear, for example, to what extent there are synergies between health services, pharmaceutical and medical equipment industries, or if the construction industry depends more on its forward suppliers (real estate and financial services) rather than on its input providers (sand, cement, wood and metal products).

Therefore, by combining reduction and clustering procedures, we suggest a possible route to merge horizontal and sectoral dimensions of industrial policy design.

The rest of the paper is organised as follows. The next Section sets the present contribution in the broader context of structural analysis from an IO perspective. Methodology is discussed in Section 3; Section 4 reports the results of an empirical application to the consolidated IO tables of the European Union (EU27). The last section summarises the logic of the metrics proposed and discusses the significance and limitations of the empirical results obtained.

## 2. Structural Analysis: Reduction, Linkages, Clusters and Hierarchy

Simplifying the complexity of the intricate web of relations depicting the circular process of production has been a common task in economic analysis since its very beginning (see, for example, the 'zig-zag' of Quesnay, 1972 [1759]). The birth of Input-Output Tables, and especially of structural analysis, brought this issue to the fore with renewed centrality, for both theoretical and computational reasons. In fact, IO tables provide detailed data about inter-industry linkages; this richness of data, while valuable, needs to be organised and synthesised in order to become informative. The way in which this is done mostly depends on the task at hand.

When the task is that of performing intertemporal — or international — comparisons, complexity can be effectively simplified by computing synthetic indicators via a *reduction* procedure. An example of this kind of approach is the analysis of changes in labour productivity in terms of *(growing) subsystems* (see Sraffa, 1960; Pasinetti, 1973, 1988), where the economic system is partitioned into as many sectors as there are final commodities, and the set of means of production entering the circular process of production of each final product is summarised by the corresponding vertically (hyper-)integrated labour coefficient, obtained through the Leontief inverse.

Another task in which the Leontief inverse plays a crucial role has been that of identifying *key sectors*, i.e. "above average contributors to the economy from either an ex post or an ex ante perspective" (Sonis et al., 1995, p. 233). In particular, Rasmussen's (1956) normalised column and row sums of the Leontief inverse, conveying Hirschman's (1958) idea of backward and forward linkages, opened up a prolific stream of contributions.[8] Another recently reappraised strand of research is based on Dahmén's idea of *development blocks* (see for example Dahmén, 1988). To borrow Enflo's (2008, p. 57) words, "[t]he concept of 'development block' stresses the co-evolution of parts of the economy. At the core of a development block there is some central innovation(s) around which complementary activities are formed." The task, in both cases, is that of clarifying the relation between inter-industry linkages and development, or the more practical one of finding out priorities in investment strategies aimed at initiating economic development.

Key sectors, however, identify tipping points of a rather complex inter-industry configuration, whose structural paths still need to be uncovered. A further task has been precisely that of identifying, by applying graph theoretic tools, those intermediate transactions of major importance, and visualising the resulting *structure*. Many contributions in this area focused on the search for industry blocks. This approach was explicitly introduced by Ghosh (1960) in a seminal paper aimed at isolating "groups of industries [that] tend to form blocks with a great deal of buying and selling within blocks but relatively little between blocks" (Ghosh, 1960, p. 88). In the arc of decades, various community detection techniques were developed in IO literature.[9]

Network theory, and specifically cluster-based methodologies, have also been used as a way of finding convenient aggregation rules for industries displaying similar input structures.[10] In fact, as technical progress goes on, traditional industry classifications become progressively more inadequate and hide important structural aspects of an economic system. In this respect, it is worth mentioning a strand of literature that questioned these standard taxonomies and stressed the necessity of redefining them based on different characteristic dimensions.[11]

Finally, we can mention the task of "establish[ing] a *hierarchy* of sectors leading from primary products to final goods" (Kurz et al., 1998a, p. xxi; italics added), i.e. triangularisation of IO tables.[12] The aim is that of classifying industries according to their closeness to final demand. However, the problem is mathematically hard to solve and only leads to suboptimal solutions.[13] In fact, IO matrices could exhibit a perfectly triangular form only in presence of one-way linkages, i.e. of one-way *dependence* rather than interdependence. In other words, in the case of absence of *circularity*.

The present paper sets in at the juncture of these research areas, advancing a methodology to link some of their aspects. First of all, by means of a graph theoretic tool we shall identify industry blocks and visualise the resulting fundamental inter-industry structure. Secondly, we will rank clusters according to their relative closeness to final demand. Thirdly, we are going to apply the reduction procedure of vertical integration, in order to compute total labour productivity changes at the cluster level and identify the most dynamic individual subsystems of the economy.

[8] See for example Diamond (1976); Mcgilvray (1977); Hewings (1982); Guccione (1986); Clements and Rossi (1995).

[9] Without pretence of exhaustivity, see Czamanski's (1971) attempt at finding industrial complexes based on the eigenspace of a value-added-correlation matrix; Slater's (1977) agglomerative hierarchical clustering criterion based on what he calls Functionally Integrated Industries (FII); Aroche-Reyes's (1996) Important Coefficient (IC) analysys; Schnabl's (1995) Elasticity Coefficient (EC) Analysis; Minimal Flow Analysis (MFA) in Schnabl (1994, 2001); Oosterhaven et al. (2001), advancing a method for singling out 'which direct linkages are important enough to be considered as potentially cluster-building'; Hoen (2002), putting forward a procedure for block-diagonalisation of the adjacency matrix associated to an IO table.

[10] See for example Blin and Cohen (1977); Cabrer et al. (1991).

[11] For example, see Munir and Phillips (2002); Peneder (2003); Bhojraj et al. (2003); Dalziel (2007); Hicks (2011).

[12] A very interesting example is Lamel et al.'s (1972) application to European economies.

[13] For details and further references, see Kurz et al. (1998a, p. xxi). Moreover, see Leontief (1986[1963]) where various possible structures of IO tables are presented corresponding to different dependence/interdependence configurations.

## 3. Methodology

*3.1. Accounting framework, industry blocks and final demand subsystems*

Consider the basic IO accounting identity:[14]

$$\mathbf{x} = \mathbf{X}\mathbf{e} + \mathbf{y} \tag{1}$$

where $\mathbf{X}$ is the matrix of inter-industry transactions, $\mathbf{x}$ and $\mathbf{y}$ are the vectors of gross outputs and final uses by industry, respectively. From (1), it is common practice to derive the related matrices:

$$\mathbf{A} = \mathbf{X}\widehat{\mathbf{x}}^{-1} \tag{2}$$
$$\mathbf{D} = \widehat{\mathbf{x}}^{-1}\mathbf{X} \tag{3}$$
$$\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1} \tag{4}$$

where $\mathbf{A}$ is the matrix of direct input coefficients (which is read column-wise), $\mathbf{D}$ is the market-shares matrix (which is read row-wise) and $\mathbf{B}$ is the Leontief inverse, i.e. the matrix of total (direct and indirect) input requirements per (monetary) unit of final demand.

In the language of network (and graph) theory, $\mathbf{X}$ can be seen as the matrix representation of an IO network, i.e. its adjacency matrix. Clustering methodologies usually take this matrix as their point of departure.

Spectral graph theory has been extensively used to devise methods for the decomposition of networks into communities (see, for example, Fortunato, 2010). Given the great number of available methods, the choice crucially depends on the characteristics of the network under study.

IO networks have peculiarities restricting such a choice. To begin with, the level of disaggregation of normally available IO tables implies that almost all industries have intermediate buying/selling relations with every other industry: IO networks are *strongly connected.* This, in turn, implies that the *magnitude* of intermediate flows is fundamental in determining whether two industries are strongly or weakly connected: IO networks are *weighted.* The *direction* of the flows, i.e. whether we are looking at purchases or deliveries, is also essential in drawing a map of system structure: IO networks are *directed.*

Moreover, inter-industry flows are originated by a final sector (as final demand expenditure) and gradually return to this final sector (as value added), which thus represents both the 'entrance' and the 'exit': IO networks have *boundaries.* In fact, whenever an industry sells its output, it receives back a monetary counterpart. While a fraction of it is devoted to the purchase of the necessary intermediate inputs — giving rise to a whole set of further monetary and commodity flows — the remainder fraction becomes income (exiting this circular flow). The original flow becomes smaller and smaller with each iteration of this process and finally converges to zero: IO networks are *dissipative.*

These considerations led us to identify industry blocks through the so-called Spectral Bisection algorithm for modularity maximisation, a well established methodology developed by Newman (2006) and further generalised by Leicht and Newman (2008). While addressing the reader to the original papers for technicalities, the conceptual rationale of this method runs as follows.

Given each industry's intermediate purchases and deliveries in proportion to the economy-wide total, it is possible to compute average flows going in both directions between any couple of industries. The Spectral Bisection algorithm partitions the network by comparing such average flows to the actual ones and thus identifying higher than average purchases/deliveries.[15] Indirect as well as direct linkages are taken into account: if strong links exist between industry $i$ and $j$ and between $j$ and $k$, then $i$, $j$ and $k$ will be classified as belonging to the same cluster, even if the link between $i$

[14] All throughout the paper, vectors are indicated by lower case boldface characters (e.g. $\mathbf{v}$), and are column vectors unless explicitly transposed (e.g. $\mathbf{v}^T$), while matrices are indicated by upper case boldface characters (e.g. $\mathbf{X}$), except for lower case characters with a hat (e.g. $\widehat{\mathbf{z}}$), indicating diagonal matrices with the vector elements on the main diagonal. Moreover, $\mathbf{e} = [1 \dots 1]^T$ is the sum vector and $\mathbf{e}_i = [0 \dots 1 \dots 0]^T$, with 1 in the $i$-th. position, is a column selector vector.

[15] One of the crucial points of this method lies in iteratively bisecting the network on the basis of the dominant eigenvector of the adjacency matrix and sub-matrices.

and $k$ is weak.

Finally, it is necessary to choose the matrix to be partitioned: $\mathbf{X}$ or the direct coefficient matrix $\mathbf{A}$. Given that in this paper clustering is applied to draw a map of the fundamental structure of the system, the first choice seems more appropriate. In fact, matrix $\mathbf{A}$ keeps track of industries' input structure per unit of output, disregarding their relative importance. On the contrary, considering value flows allows us to take this important feature into consideration. In this way, bigger industries are attractors for smaller ones, and thus are the core of the resulting blocks. Applying Spectral Bisection to the $\mathbf{A}$ matrix would imply considering all industries on an equal footing, assessing similarity in industries' input-structure rather than the strength of bilateral connections in absolute terms. In fact, the task of uncovering the relative proportions — and thus the structure — of each activity is accomplished at the subsystem level via the procedure of vertical integration.

Building final demand subsystems consists in logically re-partitioning gross outputs, intermediate consumption, and employment by industry into as many parts as there are components in the final demand vector, each of these parts accounting for the total (direct and indirect) commodity input and labour requirements to reproduce a single component of final demand. Thus, each subsystem replicates the general interdependence (i.e. circular flow) of an hypothetical economy producing only one final product; all industries participate (directly or indirectly) in every subsystem through input provision.

Analytically, gross outputs by subsystem can be obtained by solving for $\mathbf{x}$ in (1) and noting that the resulting vector can be partitioned into as many parts $\mathbf{x}^{(i)}$ as there are elements in $\mathbf{y}$, by computing:

$$\mathbf{x}^{(i)} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y}^{(i)}, \quad i = 1, \ldots, n \tag{5}$$

with $\mathbf{y}^{(i)} = \mathbf{e}_i y_i$, such that: $\mathbf{y} = \sum_{i=1}^{N} \mathbf{y}^{(i)}$, obtaining: $\mathbf{x} = \sum_{i=1}^{N} \mathbf{x}^{(i)}$.

In what follows, we shall concentrate on industry employment and subsystem labour. Defining direct labour coefficients as $\mathbf{a}_l^T = \mathbf{l}^T \widehat{\mathbf{x}}^{-1}$, where $\mathbf{l}^T$ is the industry employment vector, and making use of (5), it is possible to compute the labour requirements of each industry $i$ to produce gross output $x_i$ ($L_i$), and those of its associated subsystem to produce final demand $y_i$ ($L^{(i)}$):

$$L_i = \mathbf{a}_l^T \widehat{\mathbf{x}} \mathbf{e}_i = a_{l_i} x_i \tag{6}$$

$$L^{(i)} = \mathbf{a}_l^T \widehat{\mathbf{x}}^{(i)} = \mathbf{a}_l^T (\mathbf{I} - \mathbf{A})^{-1} \mathbf{y}^{(i)} = v_i y_i \tag{7}$$

where $v_i = \mathbf{a}_l^T (\mathbf{I} - \mathbf{A})^{-1} \mathbf{e}_i$ is the 'vertically integrated labour coefficient' of subsystem $i$, quantifying the direct and indirect labour requirements from all its supporting industries per (monetary) unit of final demand (Pasinetti, 1973, p. 6).

In compact form, $L_i$ and $L^{(i)}$, for $i = 1, \ldots, n$ are given by $\mathbf{l}^T = \mathbf{a}_l^T \widehat{\mathbf{x}}$ and $\mathbf{l}_v^T = \mathbf{v}^T \widehat{\mathbf{y}}$. Note that the sum over either industries or subsystems equals total employment $L$, i.e. $\sum_{i=1}^{n} L_i = \sum_{i=1}^{n} L^{(i)} = L$. However, there is a fundamental difference. While $a_{l_i}$ in (6) only concerns labour inputs of a single industry per unit of gross output, subsystem labour per unit of final demand $v_i$ in (7) involves the whole network of intermediate inputs in a single coefficient. Hence, the reciprocal of this labour intensity coefficient for each subsystem provides a measure of total (direct and indirect) labour productivity ($\alpha_l^{(i)}$), and its associated rate of growth ($\varrho_i$) a measure of total labour productivity changes:[16]

$$\alpha_l^{(i)} = \frac{1}{v_i} = \frac{y_i}{L^{(i)}} \tag{8}$$

$$\varrho_i = \mathrm{d} \ln \alpha_l^{(i)} = -\mathrm{d} \ln v_i \tag{9}$$

By computing a weighted average of subsystem-specific growth rates we obtain an economy-wide rate of total labour productivity growth:[17]

$$\varrho^* = \frac{\sum_{i=1}^{n} \varrho_i L^{(i)}}{\sum_{i=1}^{n} L^{(i)}} \tag{10}$$

[16] Note that by using data in monetary units $\alpha_l^{(i)}$ is an index with respect to statistical basic prices, but if the interest of the analysis is on measuring *changes* in labour productivity, by using industry × industry matrices in constant prices, movements of $\alpha_l^{(i)}$ will — to a certain extent — reflect movements in labour content (i.e. volume changes).

[17] Growth rate $\varrho^*$ corresponds, in this context, to Pasinetti's(1981, pp. 101-4) 'standard rate of growth of productivity'.

Given that $\varrho_i$ reflects labour saving trends it is always important to consider the employment dimension associated to changes in productivity, as high values for $\varrho_i$ might be due to employment reduction, which would hinder effective demand. Therefore, $\varrho_i$ should always be jointly considered with its associated subsystem labour growth rate: $\mathrm{d}\ln L^{(i)}$. Moreover, it should be kept in mind that, by computing these growth rates, it is not our intention to enter into the realm of dynamic Input-Output models, which would surely require a more careful discussion (see, e.g. Kurz et al., 1998b). Instead, our aim is to perform a simple productivity accounting exercise based on period-by-period magnitudes, i.e. measuring productivity levels in two (historical) time periods and computing their proportional difference.

Combining Spectral Bisection as a clustering method and the reduction procedure of vertical integration[18] — to obtain a set of final demand subsystems — can shed light on, at least, two issues: the redistribution of labour between intra- and extra- cluster industries and subsystems, and the role of intra-cluster 'synergies' in subsystem labour and total labour productivity changes. These two issues are explored in each of the following subsections, respectively. An exposition of the main points that follow in the context of a simple 5×5 example are illustrated in Appendix A.

### *3.2. Hierarchy and Circularity: Labour redistribution between industries and subsystems*

Among several descriptive measures of the economic importance of a sector, the net backward linkage or net multiplier (Dietzenbacher, 2005, p. 423) captures the degree by which "economy-wide output generated by final demand in $j$ is larger than the amount of $j$'s output that is generated by all the other industries' final demands" (Miller and Blair, 2009, p. 559). In terms of employment, the net multiplier for sector $i$ is given by:[19]

$$\lambda_i = \frac{L^{(i)}}{L_i} = \frac{v_i y_i}{a_{l_i} x_i} = \frac{\sum_j a_{l_j} b_{ji} y_i}{\sum_j a_{l_i} b_{ij} y_j} \tag{11}$$

If $\lambda_i > 1$, sector $i$ induces more labour on others than the employment that other activities induce on industry $i$, and vice-versa for $\lambda_i < 1$.

However, by acknowledging the different unit of analysis implied by $L^{(i)}$ (the subsystem) and $L_i$ (the industry), an alternative interpretation can be given to the *difference* (rather than the ratio) $L^{(i)} - L_i$. This difference represents the labour redistribution between industries and subsystems. Hence, if $L^{(i)} - L_i > 0$, sector $i$ will absorb more labour from other industries than the employment it provides to other subsystems, and given that a subsystem produces *only* final goods, it means that sector $i$ will be relatively closer to final demand; vice-versa for $L^{(i)} - L_i < 0$.

Therefore, an indicator of the hierarchy (intended as the relative closeness to final demand) of a sector is given by:[20]

$$L^{(i)} - L_i = \sum_{j \neq i} a_{l_j} b_{ji} y_i - \sum_{j \neq i} a_{l_i} b_{ij} y_j \tag{12}$$

Two main additive components may be derived from (12):

$$\varphi_{-i,i} = \frac{\sum_{j \neq i} a_{l_j} b_{ji} y_i}{L^{(i)}}, \quad \psi_{i,-i} = \frac{\sum_{j \neq i} a_{l_i} b_{ij} y_j}{L_i}$$

where $\varphi_{-i,i}$ measures the proportion of subsystem $i$'s labour absorbed from other industries, while $\psi_{i,-i}$ measures the proportion of industry $i$'s employment provided to other subsystems.

[18] In fact, vector $\mathbf{v}^T = \mathbf{a}_l^T(\mathbf{I} - \mathbf{A})^{-1}$ is obtained by expressing some commodities in terms of others (in this case, labour): $\mathbf{v}^T = \mathbf{a}_l^T + \mathbf{a}_l^T\mathbf{A} + (\mathbf{a}_l^T\mathbf{A})\mathbf{A} + \ldots$, i.e. an infinite series reflecting the labour required to reproduce: final demand ($\mathbf{a}_l^T$), direct productive capacity required by final demand ($\mathbf{a}_l^T\mathbf{A}$), direct productive capacity required to reproduce direct productive capacity required by final demand ($\mathbf{a}_l^T\mathbf{A}^2$), and so on. However, it must be clear that vertically integrated labour refers always to current and co-existing employment, and should not be interpreted as running backwards in 'historical' time.

[19] Note that $x_i = \sum_j b_{ij} y_j$ and $v_i = \sum_j a_{l_j} b_{ji}$, where $\mathbf{B} = [b_{ij}]$ is the Leontief inverse defined in (4).

[20] Note that, differently from $\lambda_i$, self-loops cancel out: $L^{(i)} - L_i = \sum_j a_{l_j} b_{ji} y_i - \sum_j a_{l_i} b_{ij} y_j = \sum_{j \neq i} a_{l_j} b_{ji} y_i + a_{l_i} b_{ii} y_i - \sum_{j \neq i} a_{l_i} b_{ij} y_j - a_{l_i} b_{ii} y_i = \sum_{j \neq i} a_{l_j} b_{ji} y_i - \sum_{j \neq i} a_{l_i} b_{ij} y_j$.

It is straightforward to relate these additive components to the original net multiplier $\lambda_i$:[21]

$$\lambda_i = \frac{1-\psi_{i,-i}}{1-\varphi_{-i,i}} \tag{13}$$

However, note that $\varphi_{-i,i}$ and $\psi_{i,-i}$ concern the systemic effects of all industries on a given subsystem, and the importance of a given industry to all other subsystems, respectively. By introducing industry blocks in this context, we aim to separate those components of labour redistribution which are internal to a cluster from those that depend on extra-cluster linkages. To do so, it is necessary to further decompose $L^{(i)} - L_i, i = 1, \dots, n$ (or $\mathbf{l}_v^T - \mathbf{l}^T$, in compact notation). First, recall the definition of $\mathbf{B}$ in (4), and further define $\widehat{\boldsymbol{\beta}}$ and $\check{\mathbf{B}}$ as follows:[22]

$$\widehat{\boldsymbol{\beta}} = \sum_{i=1}^{n} b_{ii}\mathbf{e}_i\mathbf{e}_i^T, \quad \check{\mathbf{B}} = \mathbf{B} - \widehat{\boldsymbol{\beta}}$$

Introducing $\mathbf{B}$ in $\mathbf{l}^T$ and $\mathbf{l}_v^T$:

$$\mathbf{l}^T = \mathbf{a}_l^T\widehat{\mathbf{x}} = \mathbf{x}^T\widehat{\mathbf{a}}_l = \mathbf{y}^T\mathbf{B}^T\widehat{\mathbf{a}}_l \tag{14}$$

$$\mathbf{l}_v^T = \mathbf{v}^T\widehat{\mathbf{y}} = \mathbf{a}_l^T\mathbf{B}\widehat{\mathbf{y}} \tag{15}$$

and computing their difference gives:

$$\begin{aligned}\mathbf{l}_v^T - \mathbf{l}^T &= \mathbf{a}_l^T\mathbf{B}\mathbf{y} - \mathbf{y}^T\mathbf{B}^T\widehat{\mathbf{a}}_l \\ &= \mathbf{a}_l^T(\check{\mathbf{B}} + \widehat{\boldsymbol{\beta}})\widehat{\mathbf{y}} - \mathbf{y}^T(\check{\mathbf{B}} + \widehat{\boldsymbol{\beta}})^T\widehat{\mathbf{a}}_l \\ &= \mathbf{a}_l^T\check{\mathbf{B}}\widehat{\mathbf{y}} - \mathbf{y}^T\check{\mathbf{B}}^T\widehat{\mathbf{a}}_l\end{aligned} \tag{16}$$

where the terms $\mathbf{a}_l^T\widehat{\boldsymbol{\beta}}\widehat{\mathbf{y}}$ and $\mathbf{y}^T\widehat{\boldsymbol{\beta}}\widehat{\mathbf{a}}_l$ cancel out.[23]

By taking advantage of the fact that the Spectral Bisection algorithm produces a non-overlapping and mutually exclusive partition of the network, it is possible to accordingly re-order rows and columns, so that expression (16) can be decomposed *for each* cluster $C$ using partitioned matrices, distinguishing between $C$-type (intra-cluster) and $N$-type (extra-cluster) industries, as follows:

$$\begin{aligned}&\begin{bmatrix} \mathbf{l}_{vc}^T - \mathbf{l}_c^T & \mathbf{l}_{vn}^T - \mathbf{l}_n^T \end{bmatrix} = \\ &\begin{bmatrix} \mathbf{a}_{lc}^T & \mathbf{a}_{ln}^T \end{bmatrix} \begin{bmatrix} \check{\mathbf{B}}_{cc} & \mathbf{B}_{cn} \\ \mathbf{B}_{nc} & \check{\mathbf{B}}_{nn} \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{y}}_c & \mathbf{O} \\ \mathbf{O} & \widehat{\mathbf{y}}_n \end{bmatrix} \\ &- \begin{bmatrix} \mathbf{y}_c^T & \mathbf{y}_n^T \end{bmatrix} \begin{bmatrix} \check{\mathbf{B}}_{cc}^T & \mathbf{B}_{nc}^T \\ \mathbf{B}_{cn}^T & \check{\mathbf{B}}_{nn}^T \end{bmatrix} \begin{bmatrix} \widehat{\mathbf{a}}_{lc} & \mathbf{O} \\ \mathbf{O} & \widehat{\mathbf{a}}_{ln} \end{bmatrix}\end{aligned} \tag{17}$$

The first, i.e. intra-cluster, component of this partitioned vector (17) can be written as:

$$\begin{aligned}\mathbf{l}_{vc}^T - \mathbf{l}_c^T =& \mathbf{a}_{lc}^T\check{\mathbf{B}}_{cc}\widehat{\mathbf{y}}_c + \mathbf{a}_{ln}^T\mathbf{B}_{nc}\widehat{\mathbf{y}}_c \\ &- \mathbf{y}_c^T\check{\mathbf{B}}_{cc}^T\widehat{\mathbf{a}}_{lc} - \mathbf{y}_n^T\mathbf{B}_{cn}^T\widehat{\mathbf{a}}_{lc}\end{aligned} \tag{18}$$

where each addendum can be interpreted as:

i) $\mathbf{a}_{lc}^T\check{\mathbf{B}}_{cc}\widehat{\mathbf{y}}_c$: vertically integrated labour of each $C$-type subsystem coming from other industries in the cluster;

ii) $\mathbf{a}_{ln}^T\mathbf{B}_{nc}\widehat{\mathbf{y}}_c$: vertically integrated labour of each $C$-type subsystem coming from extra-cluster industries;

iii) $\mathbf{y}_c^T\check{\mathbf{B}}_{cc}^T\widehat{\mathbf{a}}_{lc}$: direct employment of each $C$-type industry contributing to vertically integrated labour of other subsystems of the cluster;

iv) $\mathbf{y}_n^T\mathbf{B}_{cn}^T\widehat{\mathbf{a}}_{lc}$: direct employment in each $C$-type industry contributing to vertically integrated labour of extra-cluster subsystems.

Therefore, by computing:

$$L^{(c)} - L_c = (\mathbf{l}_{vc}^T - \mathbf{l}_c^T)\mathbf{e}$$

we obtain the *cluster-level* labour redistribution between industries and subsystems, given by:[24]

$$\begin{aligned}L^{(c)} - L_c =& (\mathbf{a}_{lc}^T\check{\mathbf{B}}_{cc} + \mathbf{a}_{ln}^T\mathbf{B}_{nc})\mathbf{y}_c \\ &- (\mathbf{y}_c^T\check{\mathbf{B}}_{cc}^T + \mathbf{y}_n^T\mathbf{B}_{cn}^T)\mathbf{a}_{lc} \\ =& \mathbf{a}_{ln}^T\mathbf{B}_{nc}\mathbf{y}_c - \mathbf{a}_{lc}^T\mathbf{B}_{cn}\mathbf{y}_n\end{aligned} \tag{19}$$

i.e. the difference between two components:

[21] Writing (12) as: $L^{(i)} - L_i = \varphi_{-i,i}L^{(i)} - \psi_{i,-i}L_i$, dividing both sides by $L_i$, and solving for $\lambda_i$, gives the result reported in (13).

[22] Matrices with diagonal elements set to zero, will be indicated with a Czech hat, e.g. $\check{\mathbf{B}}$.

[23] These components correspond to self-loops, i.e. direct and indirect labour of industry $i$ *only*, required to produce final demand of industry $i$: $\mathbf{a}_l^T\widehat{\boldsymbol{\beta}}\widehat{\mathbf{y}}\mathbf{e}_i = \mathbf{y}^T\widehat{\boldsymbol{\beta}}\widehat{\mathbf{a}}_l\mathbf{e}_i = a_{l_i}b_{ii}y_i$.

[24] At the industry level: $\mathbf{a}_{lc}^T\check{\mathbf{B}}_{cc}\widehat{\mathbf{y}}_c \neq \mathbf{y}_c^T\check{\mathbf{B}}_{cc}^T\widehat{\mathbf{a}}_{lc}$, but for the cluster as a whole: $\mathbf{a}_{lc}^T\check{\mathbf{B}}_{cc}\mathbf{y}_c = \mathbf{y}_c^T\check{\mathbf{B}}_{cc}^T\mathbf{a}_{lc}$.

i) $\mathbf{a}_{ln}^T \mathbf{B}_{nc} \mathbf{y}_c$: *labour absorption* by intra-cluster subsystems from extra-cluster industries;

ii) $\mathbf{a}_{lc}^T \mathbf{B}_{cn} \mathbf{y}_n$: *employment provision* by intra-cluster industries to extra-cluster subsystems.

The sign of $L^{(c)} - L_c$ indicates whether the cluster is relatively closer to final demand (if positive) or to the rest of the inter-industry network (if negative). In fact, a positive value implies that the cluster absorbs more labour from the network than what it provides to it, and thus its production effort is mainly directed to final demand; vice-versa, a negative value indicates a relatively greater importance of the cluster as a producer of intermediates. Thus, in analogy to individual sector $i$'s measure, $L^{(c)} - L_c$ can be thought of as a 'hierarchy' indicator, ordering clusters according to their closeness to final demand.

Additionally, by considering not only components i)-ii) in (19) but also $\mathbf{a}_{lc}^T \check{\mathbf{B}}_{cc} \mathbf{y}_c = \mathbf{y}_c^T \check{\mathbf{B}}_{cc}^T \mathbf{a}_{lc}$, i.e. labour redistribution between $C$-type industries and subsystems — which cancel out at the level of the cluster aggregate — it is possible to advance four indicators (the cluster-level analogous to $\varphi_{-i,i}$ and $\psi_{i,-i}$ above) characterising the degree of circularity in labour redistribution for each cluster $C$.

The four indicators are summarised in Table 1: $\varphi_{cc}$ and $\varphi_{nc}$ measure the proportion of intra-cluster subsystems' labour coming from $C$-type industries (*in*-persistence of circularity) and $N$-type industries (labour absorption), respectively; $\psi_{cc}$ and $\psi_{cn}$ measure the proportion of intra-cluster industries' employment contributing to $C$-type subsystems (*out*-persistence of circularity) and $N$-type subsystems (employment provision), respectively.

### 3.3. *(In-)dependence and Dynamics: Vertically integrated labour within clusters*

Standard analyses of vertically integrated labour depart from vector $\mathbf{v}^T$ in (7), and distinguish between direct and indirect labour requirements.[25] Formally, we have:

$$\mathbf{v}^T = \mathbf{a}_l^T + \mathbf{a}_l^T \mathbf{H} \tag{20}$$

where $\mathbf{H} = \mathbf{A}(\mathbf{I} - \mathbf{A})^{-1}$ is the matrix of vertically integrated productive capacities (Pasinetti, 1973, p. 6).[26] For each sector $i$, $v_i = \mathbf{v}^T \mathbf{e}_i$ can be written as:

$$v_i = a_{l_i} + \sum_j a_{l_j} h_{ji}$$

being the sum of a direct (industry) labour coefficient ($a_{l_i}$) and of a composite indirect component, obtained as a linear combination of productive capacity coefficients for subsystem $i$, in terms of labour.

As with the net multiplier $\lambda_i$ in (11) above, $v_i$ is a system measure reflecting the comprehensive effect of all industries acting upon subsystem $i$.

By introducing industry blocks in this context, and deriving vertically integrated labour coefficients at the cluster-level, we may separate production loops by origin and destination, i.e. intra- and extra-cluster components, in view of assessing to which extent each *group* of subsystems is independent from the rest of the IO network.[27]

To do so, reconsider the vector of vertically integrated labour coefficients:

$$\mathbf{v}^T = \mathbf{a}_l^T \mathbf{B} = \mathbf{a}_l^T (\mathbf{I} - \mathbf{A})^{-1}$$

in which columns and rows have been rearranged so as to distinguish between intra- and extra-cluster industries:

$$\begin{bmatrix} \mathbf{v}_c^T & \mathbf{v}_n^T \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{lc}^T & \mathbf{a}_{ln}^T \end{bmatrix} \begin{bmatrix} \mathbf{B}_{cc} & \mathbf{B}_{cn} \\ \mathbf{B}_{nc} & \mathbf{B}_{nn} \end{bmatrix} = \\ = \begin{bmatrix} \mathbf{a}_{lc}^T & \mathbf{a}_{ln}^T \end{bmatrix} \begin{bmatrix} \mathbf{I} - \mathbf{A}_{cc} & -\mathbf{A}_{cn} \\ -\mathbf{A}_{nc} & \mathbf{I} - \mathbf{A}_{nn} \end{bmatrix}^{-1} \tag{21}$$

[25] See, e.g. Gupta and Steedman (1971) and Flaschel (2010).

[26] Expression (20) is obtained by noting that $(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A}(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{H}$.

[27] By definition, subsystems are obtained by a *virtual* partition of the economy, which renders each of them autonomous. So what is the rationale for introducing clusters? Because we are looking for *actual* rather than *virtual* relative independence of each industry block with its associated set of subsystems.

Table 1: Degree of Circularity in Labour Redistribution for cluster $C$

| Indicator | | Employment coming from (industries) | Labour going to (subsystems) | As a proportion of cluster $C$ total |
|---|---|---|---|---|
| labour absorption | $\varphi_{nc} = (\mathbf{a}_{ln}^T \mathbf{B}_{nc} \mathbf{y}_c)/L^{(c)}$ | extra-$C$ | intra-$C$ | subsystems' labour |
| employment provision | $\psi_{cn} = (\mathbf{y}_n^T \mathbf{B}_{cn}^T \mathbf{a}_{lc})/L_c$ | intra-$C$ | extra-$C$ | industries' employment |
| *in*-persistence | $\varphi_{cc} = (\mathbf{a}_{lc}^T \check{\mathbf{B}}_{cc} \mathbf{y}_c)/L^{(c)}$ | intra-$C$ | intra-$C$ | subsystems' labour |
| *out*-persistence | $\psi_{cc} = (\mathbf{y}_c^T \check{\mathbf{B}}_{cc}^T \mathbf{a}_{lc})/L_c$ | intra-$C$ | intra-$C$ | industries' employment |

As shown in Appendix B, the vector of vertically integrated labour coefficients of cluster $C$ subsystems ($\mathbf{v}_c^T$) may be decomposed into three addenda:

$$\mathbf{v}_c^T = \mathbf{v}_{cc}^T + \mathbf{v}_{cnc}^T + \mathbf{v}_{nc}^T \tag{22}$$

with:

$$\begin{aligned} \mathbf{v}_{cc}^T &= \mathbf{a}_{lc}^T (\mathbf{I} - \mathbf{A}_{cc})^{-1} \\ \mathbf{v}_{cnc}^T &= \mathbf{a}_{lc}^T (\mathbf{I} - \mathbf{A}_{cc})^{-1} \mathbf{H}_{cn} \mathbf{H}_{nc} (\mathbf{I} - \mathbf{H}_{cn} \mathbf{H}_{nc})^{-1} \\ \mathbf{v}_{nc}^T &= \mathbf{a}_{ln}^T (\mathbf{I} - \mathbf{A}_{nn})^{-1} \mathbf{H}_{nc} (\mathbf{I} - \mathbf{H}_{cn} \mathbf{H}_{nc})^{-1} \end{aligned}$$

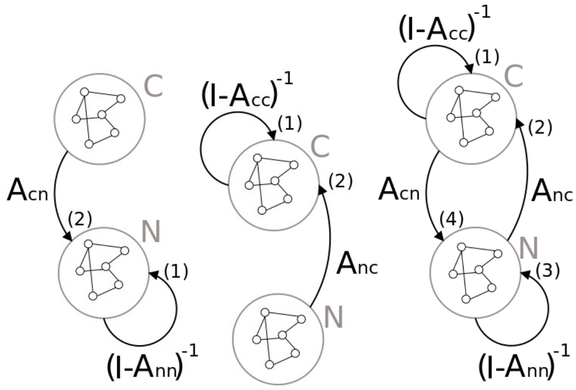
and:

$$\mathbf{H}_{cn} = \mathbf{A}_{cn} (\mathbf{I} - \mathbf{A}_{nn})^{-1} \tag{23}$$
$$\mathbf{H}_{nc} = \mathbf{A}_{nc} (\mathbf{I} - \mathbf{A}_{cc})^{-1} \tag{24}$$

Note that $\mathbf{H}_{cn}$ and $\mathbf{H}_{nc}$ are matrices of vertically integrated productive capacities, specifying total requirements of cluster $C$ inputs by cluster $N$ subsystems for $\mathbf{H}_{cn}$, and vice-versa for $\mathbf{H}_{nc}$.

To *see* self-loops, feedback and spillover effects in expression (22), we depict the circular relations of its main building blocks. Figures 1a, 1b and 1c display graphs for matrices $\mathbf{H}_{cn}$, $\mathbf{H}_{nc}$ and $\mathbf{H}_{cn}\mathbf{H}_{nc}$, respectively.

The number in parenthesis next to each edge indicates the 'roundaboutness' of the corresponding link. Consider, for example, panel 1b (numbers in parenthesis indicate the corresponding edge): the production of total input requirements per unit of final demand in cluster $C$ induces a self-loop (1), which itself requires cluster $N$ inputs to be produced (2), triggering a *spillover* effect. Instead, for panel 1c: total requirements of cluster $C$ products (1), absorb $N$ inputs (2), whose total



(a) $\mathbf{H}_{cn}$ (b) $\mathbf{H}_{nc}$ (c) $\mathbf{H}_{cn}\mathbf{H}_{nc}$

Figure 1: Circular relations between clusters

requirements to be produced (3) demand $C$ intermediates (4). Hence, $\mathbf{H}_{cn}\mathbf{H}_{nc}$ captures a *feedback* effect: own-cluster final demand comes back as demand for own-intermediates to satisfy other clusters' input requirements.[28]

By compounding the effects of $\mathbf{H}_{cn}$, $\mathbf{H}_{nc}$ and

[28]To a certain extent, our graphical representation in Figures 1a-1c resembles Leontief's (1991[1928]) description of the 'elementary scheme of the circular flow of an economy' by means of Figures 1-2 in his paper (Leontief, 1991[1928], pp. 185-6). For example, in our Figure 1c, each number in parenthesis corresponding to every edge specifies a *logical* direct or vertically integrated 'stage of the production process', where an increasing number indicates greater indirectness. In Leontief's (1991[1928], p. 185) Figure 1, the edge connecting $A^0$ with $A^1$ represents "all items whose distance in the production flow from point '$A$' is equal to one unit period, i.e. all those where element '$A$' is consumed as a cost element, may be labelled as group '$A^1$'. Points in the next group are then called '$A^2$', and so on" (Leontief, 1991[1928], p. 185). In fact, our graphical representation attempts precisely at conveying Leontief's original idea of the system as a "composite structure, in which the whole consists of several independently reproduceable groups, of which each (directly or indirectly) sat-

$\mathbf{H}_{cn}\mathbf{H}_{nc}$, we arrive at expression (22): vector $\mathbf{v}_{cc}^T$ contains only self-loops, $\mathbf{v}_{cnc}^T$ captures feedback effects (note the presence of $\mathbf{H}_{cn}\mathbf{H}_{nc}$) and $\mathbf{v}_{nc}^T$ captures the spillovers that final demand directed towards cluster $C$ products exerts on cluster $N$ employment.

Vector $\mathbf{v}_c^T$ may be aggregated over subsystems (using $\mathbf{y}_c$) and expressed in proportion to total labour requirements ($L^{(c)} = \mathbf{v}_c^T\mathbf{y}_c$), to give:

$$\frac{\mathbf{v}_{cc}^T\mathbf{y}_c}{L^{(c)}}+\frac{\mathbf{v}_{cnc}^T\mathbf{y}_c}{L^{(c)}}+\frac{\mathbf{v}_{nc}^T\mathbf{y}_c}{L^{(c)}} = \omega_{cc}+\omega_{cnc}+\omega_{nc} = 1 \quad (25)$$

These cluster-level scalar magnitudes represent:

- $\omega_{cc}$ intra-cluster employment generated by *self-contained* intra-cluster loops,
- $\omega_{cnc}$ intra-cluster employment induced by inter-cluster *feedbacks*,
- $\omega_{nc}$ extra-cluster employment induced by inter-cluster *spillovers*.

However, component $\omega_{cc}$ includes both synergies between industries in the cluster and self-consumption of each single industry. In order to quantify the former, we need to separate the latter. To do so, consider an alternative decomposition of matrix $\mathbf{B}$:

$$\mathbf{B} = (\mathbf{I} - \widehat{\boldsymbol{\alpha}})^{-1} + \check{\mathbf{A}}(\mathbf{I} - \check{\mathbf{A}})^{-1}$$

with:

$$\widehat{\boldsymbol{\alpha}} = \sum_i a_{ii}\mathbf{e}_i\mathbf{e}_i^T, \quad \check{\mathbf{A}} = \mathbf{A} - \widehat{\boldsymbol{\alpha}}$$

so that $\mathbf{v}^T$ may be written as:

$$\mathbf{v}^T = \mathbf{a}_l^T(\mathbf{I} - \widehat{\boldsymbol{\alpha}})^{-1} + \mathbf{a}_l^T\check{\mathbf{A}}(\mathbf{I} - \check{\mathbf{A}})^{-1} \quad (26)$$

The first addendum of (26) represents total (direct and indirect) labour required by self-consumption of industries. Given its diagonal character, it can easily be aggregated in terms of a set of mutually exclusive clusters. Hence, for each cluster $C$, the proportion of intra-cluster labour requirements originating from individual industries' self-consumption is given by:

$$\omega_{cc}^s = \frac{\mathbf{a}_{lc}^T(\mathbf{I} - \widehat{\boldsymbol{\alpha}}_c)^{-1}\mathbf{y}_c}{L^{(c)}} \quad (27)$$

Thus, synergies *between* industries within each cluster are given by $\omega_{cc} - \omega_{cc}^s$. In sum, these cluster-specific indicators $[\omega_{cc}^s, \omega_{cc}, \omega_{cnc}, \omega_{nc}]$ decompose vertically integrated labour into internal and external effects.

But one of the main advantages of vertically integrated magnitudes is that they reflect the overall effect of technical change, acting upon inter-industry relations, at a disaggregated (subsystem) level.[29] Hence, as a productivity accounting exercise, we may decompose sector $i$'s productivity growth rate — $\varrho_i$ in (9) — into the same three components of expression (22):

$$\varrho_i = -\mathrm{d}\ln v_i = \varrho_{cc_i}\omega_{cc_i}+\varrho_{cnc_i}\omega_{cnc_i}+\varrho_{nc_i}\omega_{nc_i} \quad (28)$$

with:

$$\begin{aligned}
\varrho_{cc_i} &= -\mathrm{d}\ln(\mathbf{v}_{cc}^T\mathbf{e}_i), \quad \omega_{cc_i} = (\mathbf{v}_{cc}^T\mathbf{y}^{(i)})/L^{(i)} \\
\varrho_{cnc_i} &= -\mathrm{d}\ln(\mathbf{v}_{cnc}^T\mathbf{e}_i), \quad \omega_{cnc_i} = (\mathbf{v}_{cnc}^T\mathbf{y}^{(i)})/L^{(i)} \\
\varrho_{nc_i} &= -\mathrm{d}\ln(\mathbf{v}_{nc}^T\mathbf{e}_i), \quad \omega_{nc_i} = (\mathbf{v}_{nc}^T\mathbf{y}^{(i)})/L^{(i)}
\end{aligned}$$

For each subsystem within cluster $C$, (28) distinguishes between productivity growth which is self-contained ($\varrho_{cc_i}$), induced by inter-cluster feedbacks ($\varrho_{cnc_i}$), and 'imported' from extra-cluster industries ($\varrho_{nc_i}$).

Accordingly, the standard rate of growth of cluster-labour productivity is a weighted average of the $\varrho_i$'s:

$$\begin{aligned}
\varrho_c^* =& \frac{\sum_{i\in C}\varrho_i L^{(i)}}{\sum_{i\in C}L^{(i)}} = \frac{\sum_{i\in C}\varrho_{cc_i}\omega_{cc_i}L^{(i)}}{\sum_{i\in C}L^{(i)}} + \\
& \frac{\sum_{i\in C}\varrho_{cnc_i}\omega_{cnc_i}L^{(i)}}{\sum_{i\in C}L^{(i)}} + \frac{\sum_{i\in C}\varrho_{nc_i}\omega_{nc_i}L^{(i)}}{\sum_{i\in C}L^{(i)}} = \\
=& \varrho_{cc}^* + \varrho_{cnc}^* + \varrho_{nc}^*
\end{aligned} \quad (29)$$

isfies the conditions for producing the others, and thus satisfies the conditions for its own reproduction" (Leontief, 1991[1928], p. 186). We sincerely thank one of the Editors for hinting us at this insightful connection.

[29] In Pasinetti's (1981) words: "Inter-industry relations, referring to any particular point of time, represent a cross-section of the vertically integrated magnitudes, whose movements through time express the structural dynamics of the economic system", p. 117.

being itself composed of three addenda: $\varrho^*_{cc}$, $\varrho^*_{cnc}$, and $\varrho^*_{nc}$. Expression (29) provides a cluster-level decomposition by origin and destination of labour-saving trends. As for individual subsystems, $\varrho^*_c$ should be considered together with the evolution of total labour ($\mathrm{d}\ln L^{(c)}$).

## 4. Empirical Results

This section reports empirical answers addressing the research questions posed in the Introduction. To this end, we analysed the case of the European Union (EU27) between 2000 and 2007, making use of the consolidated EU27 Supply-Use Tables (SUIOT Database) together with National Accounts data on employment by Industry, published by EUROSTAT.[30] Square industry × industry Input-Output tables at basic prices for domestic output have been obtained from the original Supply-Use tables by applying the fixed product sales structure technology assumption.[31] All Tables have been expressed in constant prices (base year=2000), deflating rows with price indexes for gross output by activity.

Table 2 displays the industry clusters resulting from the application of the Spectral Bisection algorithm; we obtained 9 industry blocks for the year 2007.[32] Notably, the Services (C08) block includes 16 out of 58 industries — among them, Public and Financial Services as well as Education. Moreover, Bio-tech, R&D and Hi-tech industries group into a specific cluster: Pharma-Hi Tech (C09) which also includes Health. Note how the blocks we found break up traditional industry classifications; e.g. the Construction cluster follows the complete physical transformation process of inputs into output (from Forestry and Stone-sand-clay-minerals to the Construction industry). Clearly, rigid classification infrastructures are challenged by dynamic clustering schemes (Hicks, 2011).

We are now in a position to provide an example of how the proposed methodology could be used to identify an industry block suitable for being the focus of, e.g., a public investment plan.

First of all, the target block must be responsive to stimuli coming from final demand. A hierarchical ordering of clusters according to their closeness to final demand is provided by column (4) of Table 3. We can see that the highest value is associated to cluster Heavy Machinery (C05).

Having a closer look at the figures associated to this cluster, Table 4, displaying the dynamics of labour and productivity growth at the cluster level, shows that the corresponding subsystems experienced a yearly average increase in employment (+0.32 p.p.), associated to a yearly average increase in labour productivity: +2.87 p.p., the highest of all industry blocks.

Table 5, identifying the most dynamic individual subsystems,[33] shows that Heavy Machinery is the cluster including the highest number of them, with 4 out of its 9 sectors displaying high rates of growth of both productivity (in the range going from 1.98 p.p. for DM35 Ships-railway-aircrafts to 4.02 p.p. for DM34 Motor-vehicles) and employment (from 0.31 p.p. for DM34 Motor-vehicles to 1.65 p.p. for DM35 Ships-railway-aircrafts). These figures further support the choice of Heavy Machinery as our target block, since a policy addressed to it would contribute to boost system-wide productivity, thus avoiding the channelling of resources towards backward sectors with low productivity dynamics.

Moreover, and most importantly, an hypothetical public investment policy should target a clus-

[30] See EUROSTAT (2011) for details. We have considered 58 out of 59 industries of the 2 digit NACE Rev. 1 Industry classification (results for CA12 Uranium, which represents only 0.00013% of gross output, have not been analysed).

[31] See ten Raa and Rueda-Cantuche (2009, p. 364) for a description of industry coefficient transformation models.

[32] Paper (C07) is actually an isolated industry which did not 'cluster' with other activities.

[33] The criterion we followed to identify dynamic subsystems has been a higher than average rate of growth of productivity coupled with increasing employment. In fact, increases in productivity which are associated to a decrease in employment might simply stress that we are in front of a declining sector; on the contrary, productivity increases coupled with employment enhancement cannot but be associated with dynamic subsystems, which are expanding their deliveries to final users while experiencing technological progress.

Table 2: Clustering results for the European Union (EU27)

(Year: 2007, Method: Spectral Bisection algortihm for modularity maximisation)

| | |
|---|---|
| **(C01) Agri-Food** | **(C09) Pharma-Hi Tech** |
| A01 Agriculture | DG24 Chemicals-pharma |
| B05 Fishing | DH25 Rubber-plastics |
| DA15 Food-beverages | DL30 Office-machinery-PC |
| DA16 Tobacco | DL32 ICT-equipment |
| H55 Hotel-restaurant | DL33 Medical-precision-equip. |
| | K73 R&D |
| **(C02) Construction** | N85 Health |
| A02 Forestry | |
| CB14 Stone-sand-clay-minerals | **(C05) Heavy Machinery** |
| DD20 Wood | CB13 Metal-mining |
| DI26 Glass-clay-cement-ceramic | DJ27 Iron-steel-aluminium-tub. |
| F45 Construction | DJ28 Structural-metal-products |
| | DK29 Mechanical-machinery |
| **(C03) Energy** | DL31 Electrical-machinery |
| CA10 Coal Mining | DM34 Motor-vehicles |
| E40 Electricity-gas | DM35 Ships-railway-aircrafts |
| E41 Water | DN36 Furniture-Sports-Toys |
| | DN37 Recycling |
| **(C04) Transport-Trade** | |
| CA11 Petroleum-gas-extraction | **(C08) Services** |
| DF23 Petroleum-refinery | DE22 Publishing-printing |
| I60 Transport-land | G52 Retail-trade |
| I61 Transport-water | I64 Post-telecomm. |
| I62 Transport-air | J65 Finance |
| I63 Storage-travel-agencies | J66 Insurance |
| G50 Sale-repair-vehicles | J67 Brokerage-credit-cards |
| G51 Wholesale-trade | K70 Real-estate |
| K71 Renting-equipment | K72 Computer-services |
| | K74 Business-services |
| **(C06) Dressing** | L75 Public-admin. |
| DB17 Textiles | M80 Education |
| DB18 Clothing | O90 Refuse-disposal |
| DC19 Leather | O91 Membership-organisations |
| | O92 Arts-entertainment |
| **(C07) Paper** | O93 Personal-services |
| DE21 Paper | P95 Household-services |

Notes: Industry codes refer to the 2 digit NACE Rev. 1 classification

Source: Own computation based on EUROSTAT SUIOT Database

Table 3: Hierarchy, Circularity and (In-)Dependence of Clusters in the European Union (Year: 2007)

| Clusters | | | Hierarchy | | | Circularity | | | | (In-)Dependence | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | $L_c/L$ (in %) | $L^{(c)}/L$ (in %) | (3)-(2) (in p.p.) | $\varphi_{nc}$ (in %) | $\psi_{cn}$ (in %) | $\varphi_{cc}$ (in %) | $\psi_{cc}$ (in %) | $\omega^s_{cc}$ (in %) | $\omega_{cc}$ (in %) | $\omega_{cnc}$ (in %) | $\omega_{cn}$ (in %) |
| Code | Description | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) |
| C01 | Agri-Food | 5 | 12.26 | 13.90 | 1.64 | 24.89 | 14.84 | 19.93 | 22.60 | 54.30 | 74.48 | 0.63 | 24.89 |
| C02 | Construction | 5 | 9.28 | 11.08 | 1.79 | 33.94 | 21.18 | 5.87 | 7.00 | 59.61 | 65.17 | 0.88 | 33.94 |
| C03 | Energy | 3 | 0.93 | 0.90 | -0.04 | 55.94 | 57.65 | 4.55 | 4.38 | 39.20 | 43.62 | 0.44 | 55.94 |
| C04 | Transport-Trade | 9 | 11.56 | 9.27 | -2.29 | 32.25 | 45.67 | 11.85 | 9.50 | 55.05 | 65.51 | 2.24 | 32.25 |
| C05 | Heavy Machinery | 9 | 7.47 | 9.85 | 2.38 | 45.34 | 27.92 | 11.87 | 15.66 | 42.38 | 53.36 | 1.30 | 45.34 |
| C06 | Dressing | 3 | 1.50 | 1.72 | 0.22 | 30.81 | 20.48 | 5.76 | 6.62 | 63.31 | 69.08 | 0.11 | 30.81 |
| C07 | Paper | 1 | 0.34 | 0.25 | -0.08 | 60.53 | 70.41 | 0.00 | 0.00 | 39.24 | 39.24 | 0.23 | 60.53 |
| C08 | Services | 16 | 44.46 | 38.54 | -5.91 | 10.91 | 22.76 | 14.98 | 12.99 | 73.73 | 86.52 | 2.57 | 10.91 |
| C09 | Pharma-Hi Tech | 7 | 12.20 | 14.49 | 2.29 | 26.03 | 12.15 | 2.25 | 2.67 | 71.63 | 73.52 | 0.46 | 26.03 |
| | Economy-wide | 58 | 100.00 | 100.00 | 0.00 | 23.83 | 23.83 | 11.93 | 11.93 | 63.77 | 74.58 | 1.58 | 23.83 |

Source: Own computation based on EUROSTAT SUIOT and National Accounts Databases

Notes: number of industries per cluster in col. (1); components of Table 1 in cols. (5)-(8); components of equations (25) and (27) in cols. (9)-(12)

Table 4: Dynamics of Subsystem Labour and Total Labour Productivity in the European Union (EU27)

(Average yearly growth rates over period 2000-2007)

| Clusters | | | $dlnL^{(c)}$ (in p.p.) | $\varrho^*_c$ (in p.p.) | $\varrho^*_{cc}$ (in p.p.) | $\varrho^*_{cnc}$ (in p.p.) | $\varrho^*_{nc}$ (in p.p.) |
|---|---|---|---|---|---|---|---|
| Code | Description | (1) | (2) | (3) | (4) | (5) | (6) |
| C01 | Agri-Food | 5 | -0.11 | 1.759 | 1.531 | 0.018 | 0.218 |
| C02 | Construction | 5 | 2.21 | -0.563 | -0.335 | 0.000 | -0.226 |
| C03 | Energy | 3 | 0.20 | 1.853 | 2.192 | 0.007 | -0.288 |
| C04 | Transport-Trade | 9 | 1.54 | 1.530 | 0.917 | 0.051 | 0.577 |
| C05 | Heavy Machinery | 9 | 0.32 | 2.827 | 1.743 | 0.045 | 1.045 |
| C06 | Dressing | 3 | -4.18 | 2.157 | 1.497 | 0.006 | 0.660 |
| C07 | Paper | 1 | -2.25 | 4.631 | 1.592 | 0.016 | 3.034 |
| C08 | Services | 16 | 1.21 | 0.818 | 0.552 | 0.015 | 0.258 |
| C09 | Pharma-Hi Tech | 7 | 1.35 | 1.748 | 0.906 | 0.024 | 0.824 |
| | Economy-wide | 58 | 0.95 | 1.235 | 0.822 | 0.021 | 0.399 |

Source: Own computation based on EUROSTAT SUIOT and National Accounts Databases

Notes: number of industries per cluster in col. (1); all components of equation (29) in cols. (3)-(6)

Table 5: Dynamic Subsystems in the European Union (EU27)

(Period: 2000-2007, Columns $\varrho_i$ and $\Delta\%L^{(i)}$ are average yearly growth rates)

| Subsystems with $\varrho_i \geq \varrho^* = 1.235$ and $\Delta\%L^{(i)} > 0$ | | | | | |
|---|---|---|---|---|---|
| Cluster | NACE | Subsystem | $\varrho_i$ (in p.p.) | $\Delta\%L^{(i)}$ (in p.p.) | $L^{(i)}/L$ (in %) |
| C03 Energy | E40 | Electricity-gas | 2.39 | 0.54 | 0.70 |
| C04 Transport-Trade | G51 | Wholesale-trade | 2.40 | 1.62 | 3.66 |
| | I60 | Transport-land | 1.89 | 0.32 | 1.49 |
| | I61 | Transport-water | 6.74 | 3.92 | 0.29 |
| C05 Heavy Machinery | DJ28 | Structural-metal-products | 2.51 | 0.59 | 0.98 |
| | DK29 | Mechanical-machinery | 2.90 | 0.53 | 2.69 |
| | DM34 | Motor-vehicles | 4.02 | 0.31 | 2.93 |
| | DM35 | Ships-railway-aircrafts | 1.98 | 1.68 | 0.69 |
| C08 Services | G52 | Retail-trade | 1.71 | 0.85 | 8.09 |
| | I64 | Post-telecomm. | 5.28 | 1.49 | 0.99 |
| | J65 | Finance | 3.70 | 3.41 | 0.95 |
| C09 Pharma-Hi Tech | DG24 | Chemicals-pharma | 3.85 | 0.83 | 1.51 |
| | DL33 | Medical-precision-equip. | 2.56 | 1.30 | 0.60 |

Source: Own computation based on EUROSTAT SUIOT and National Accounts Databases ter displaying strong synergies, both within its internal activities and towards the rest of the IO network. In order to see whether Heavy Machinery complies with these requirements, we can look at columns (5)-(12) of Table 3. Columns (7) and (8) display *in*- and *out*-persistence of circularity, respectively. Heavy Machinery has the second (following Agri-Food) highest value of the former, and a relatively high value of the latter, i.e. its cluster industries share a high degree of circularity.

In fact, high *in*- and *out*-persistence indicate that an important proportion of cluster-industries' employment participates into the corresponding cluster-subsystems' labour, so a coordinated stimulus towards internal final demand would effectively trigger employment within the industry block. In addition, demand spillovers generated within a (group of) subsystem(s) spread over internal industries according to the *in*-persistence of circularity. Therefore, the fact that a relatively high proportion of cluster-subsystems' labour comes from cluster industries implies a strong persistence of spillovers within the block.

Computing the difference between columns (10) and (9) of Table 3 reinforces this conclusion, since the synergies between the industries in the cluster account for $\omega_{cc} - \omega_{cc}^{s} = 10.98\%$ — the economy-wide average being 10.81% and the highest value being 20.18% (for the Agri-Food cluster).

Finally, columns (11) and (12) of Table 3 display feedback and spillover effects, respectively. For cluster Heavy Machinery, feedback effects are the highest among labour-absorbing clusters (those with a positive value in column (4) of Table 3) and the third highest value for the whole economy. Spillover effects are even more relevant: 45.34% of the cluster's vertically integrated labour — again, the highest value among labour-absorbing blocks without considering Paper — versus 23.83% for the economic system as a whole.

Spillovers are particularly important for labour absorbing clusters: boosting their demand produces virtuous effects on the rest of the economy, especially for those clusters which are mainly input providers — in our case, Energy, Transport-Trade and Services as shown by column (4) of Table 3. This would produce further multiplicative effects, whose extent might be assessed by replicating the present analysis for these three industry blocks. We can very succinctly do so, as a way of example, for Energy.

Energy is a very 'open' cluster: as shown by column (6) of Table 3, 57.65% of employment in its member industries feeds labour requirements of extra-cluster subsystems. At the same time, a high proportion of cluster-subsystems' labour comes from extra-cluster industries — almost 56% of total labour, the highest value disregarding the Paper industry, as shown by columns (5) and (12). This means that the positive systemic effects coming from Heavy Machinery are further multiplied by the Energy block.

To conclude, we provide a birds' eye view of the results coming from Tables 3, 4 and 5 for each cluster. Cluster Agri-Food (C01) experienced a higher than average productivity increase at the expense of total cluster labour reduction. Due to its high self-contained circularity, its spillover effects are quite weak, making it unsuitable for triggering employment in other industry blocks. Construction (C02) shows signs of backwardness, being the only cluster displaying a decrease in productivity (though coupled with an increase in employment).

On the contrary, the Energy (C03) cluster is a dynamic one, its productivity performance being reduced by the extra-cluster component. As it is basically oriented to the production of intermediates, its strong spillover effects can be activated by any source of final demand expansion. The same holds for cluster Transport-Trade (C04), with the only difference that it displays a higher degree of self-contained circularity.

As detailed above, Heavy Machinery (C05) is also a dynamic cluster, with both intra-cluster synergies and inter-cluster spillover effects that could trigger employment expansion in the whole economic system if active industrial policies to stimulate investments were undertaken. Cluster Dressing (C06) is persistently losing importance, with sharp total labour reduction and relatively poor feedback and spillover effects.

Even though Services (C08) cannot be listed among the dynamic clusters according to Table 5, its productivity, only slightly lower than average, couples with a good performance in terms of employment. Moreover, given the great importance of intra-cluster synergies with respect to inter-cluster effects, its share in total employment is bound to increase more than proportionally to any increase in final demand — unfortunately, the opposite holds for final demand reductions. Finally, cluster Pharma-Hi Tech (C09) is highly likely to be influenced by industrial and income policies, its output being mainly directed to final demand.

## 5. Summary and concluding remarks

Clustering and reduction procedures in an Input-Output context have been usually rendered operational through industry blocks and vertically integrated sectors, respectively. However, seldom had they been connected to one another within the field of structural analysis.

The aim of the present paper has been precisely to establish a bridge between them, providing an explicit formulation of redistribution patterns of labour among (endogenously found) clusters of industries, through block partitioned internal and external matrix multipliers.

To do so, we started from the separate consideration of industry blocks, identified by means of a Spectral Bisection algorithm for modularity maximisation, and the logical re-partitioning of inputs, outputs and employment into subsystems. By keeping in mind the map provided by mutually exclusive blocks of industries, the use of partitioned inverse matrices allowed us to isolate internal and external effects in the specification of subsystem labour requirements.

With this combination of clustering and reduction procedures, we obtained computable notions of hierarchy, circularity, (in-)dependence and productivity growth at the cluster level.

Hierarchy was established according to the relative distance of a cluster to final demand, as captured by the difference between subsystem labour and industry employment ($L^{(c)} - L_c$). Circularity has been quantified by the proportion of employment going from extra-cluster industries to intra-cluster subsystems ($\varphi_{nc}$), and vice-versa ($\psi_{cn}$); as well as by the persistence of employment remaining within its industry block ($\varphi_{cc}$, $\psi_{cc}$). The degree of (in-)dependence of each cluster has been assessed through a decomposition of vertically integrated labour into purely intra-cluster self-loops ($\omega^s_{cc}$) and synergies ($\omega_{cc} - \omega^s_{cc}$), on the one hand, and induced inter-cluster feedback ($\omega_{cnc}$) and spillover ($\omega_{nc}$) effects, on the other. Finally, productivity growth has been depicted by means of total labour productivity changes of cluster-subsystems ($\varrho^*_c$).

Clustering applied to the consolidated EU27 economy resulted in 9 industry blocks, providing a reading key of the main agglomerative forces in the region: biotechnology (cluster Pharma-Hi Tech), energy and logistics (clusters Energy and Transport-Trade), and electrical-mechanical machinery (cluster Heavy Machinery). Moreover, traditional *filières* (clusters Agri-Food, Construction, Dressing), and a 'mega' Services block have been identified.

The presence of a 'mega' cluster of Services (including 16 out of 58 industries) represents a limitation of the community detection method adopted here. Proceeding to fine-tune the algorithm on the basis of particular properties of inter-industry tables is an avenue for further research.[34]

The joint consideration of the set of metrics introduced in Section 3, pointed to cluster 'Heavy Machinery' as a potential candidate for a targeted public intervention. However, it should be clear that our methodology is only a first approximation to any concrete policy action. In order to proceed, a careful study of intra-cluster hierarchies, and an even more detailed research into the firm demography within each industry participating in a cluster, are two magnifying lens playing a complementary and relevant role.

[34] In particular, a modified modularity matrix which incorporates the principle of hypothetical extraction to quantify bilateral linkages, rather than computing the difference between observed transactions and bi-proportional averages, could be an interesting starting point.

Having focused on the consolidated EU27 economy is of particular interest, because a key aspect of EU 'industrial innovation policy' has been to set-up a European Cluster Observatory,[35] which has undertaken the crucial methodological task of arriving at an operational definition, identification and measurement of clusters in European *regions* (see European Commission, 2008). Rather than relying on IO linkages, this has been done by computing 'locational employment correlation coefficients' between couple of industries across regions and then grouping together those activities nearly always geographically associated. The "cluster template" (in the sense of Porter, 2003) so obtained has been applied to conduct sectoral studies.[36] The fact that the spatial dimension plays a major role in this method points to a limitation of our exercise: we have identified *aspatial* industry blocks, even though clusters are commonly intended as "*spatially* defined sets of networks of firms" (Bailey et al., 2011, p. 303, italics added). However, this may be overcome if our methodology were applied to regional IO data.[37] In any case, to the end of conducting cluster-based industrial policy, the spatial unit of analysis should ideally correspond to the geographical domain of the policy authority.

In this respect, a unified EU27 analysis seems quite appropriate, given that sectoral industrial policies at a national level seem to be increasingly restricted within the institutional setting of European treaties (see, e.g. Pelkmans, 2006, p. 47). And even at the EU level it is becoming increasingly complex to develop specific, rather than horizontal, industrial policy interventions. This sounds paradoxical indeed, given that "[s]ectoral and specific industrial policy lies at the origin of the Community. European economic integration began in 1952 with a 'deep' free trade area in coal and steel, called the European Coal and Steel Community." (Pelkmans, 2006, p. 50).

Going beyond the empirical application here presented, the metrics introduced may be used to address a variety of issues; for example, the shift of advanced economies to service industries. While the traditional conception of the relative weight of services as a growing percentage of value added has been questioned by possible price index effects, the shift of employment shares to services is a sustained empirical fact (see, e.g. Montresor and Vittucci Marzetti, 2011).

Sometimes, however, this is over-simplified or misinterpreted as the transition to an economy producing mostly intangibles, where the role of manufacturing is of minor importance. Evidence in Section 4 suggests that a group of activities conforming the core of the Services block is mainly a provider of inputs to *other* (mostly manufacturing) clusters. Given the composition of this industry group, such a result is in line with the view that "KIBS [Knowledge Intensive Business Services] represent the most important case of structural change driven by intermediate demand", (Savona and Lorentz, 2005, p. 15). Moreover, the productivity gap between some service subsystems (like Business-services) and those dynamic sectors reported in Table 5, suggests that a trend verified in the US economy has also occured in Europe, i.e. "high productivity performers in manufacturing have been relatively successful at outsourcing sluggish services" (ten Raa and Wolff, 2001, p. 161).

Moreover, within the 'tertiarisation' discussion, at least for the case of the EU27, simplified dichotomies like manufacturing vs. services disregard that clustering has clearly broken-up standard activity classifications (e.g. the Pharma-Hi Tech cluster in Table 2), confirming the relevance of studies which call for the need of dynamic classification schemes (see, e.g. Dalziel, 2007; Hicks, 2011).

[35] `http://www.clusterobservatory.eu`

[36] Unfortunately, clustering has been based on the NACE Rev. 2 classification at a 4-digit disaggregation level, while EUROSTAT IO tables are disaggregated using the 2-digit NACE Rev. 1 classification, preventing detailed comparisons. However, for example, our cluster Heavy Machinery (C05) roughly corresponds to the merging of clusters 'Aerospace', 'Automotive', 'Heavy Machinery', 'Metal manufacturing' and 'Production Technology' identified by the Cluster Observatory.

[37] A new EU-FP7 project called 'SmartSmec' (to start in 2013) will derive an integrated multi-regional IO scheme for more than 200 EU27 NUTS2 regions (see Tukker and Dietzenbacher, 2013, p. 15).

A further key final consideration is in place. The emphasis of this paper has been on the method advanced, the empirical exploration has had an auxiliary role, which has been to illustrate the application potential of combining the indicators proposed. In fact, the results that can be obtained acquire more relevance the more disaggregated are the IO tables utilised. In this paper, however, we have avoided the use of highly detailed empirical interindustry networks to the benefit of simplicity and synthesis. If the method is considered relevant, then suitable applications may follow.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## Appendix A. A 5 × 5 example

In order to clarify the measures we derived in Section 3, we provide here a simple $5 \times 5$ example of an economy with five industries $(1, \ldots, 5)$ and two clusters $(C,\ N)$. Such a simple economy is characterised by:

$$\mathbf{X} = \begin{bmatrix} 170 & 150 & 58 & 45 & 34 \\ 200 & 300 & 227 & 18 & 32 \\ 41 & 36 & 200 & 100 & 96 \\ 12 & 27 & 85 & 180 & 74 \\ 26 & 42 & 169 & 89 & 250 \end{bmatrix} \mathbf{y} = \begin{bmatrix} 195 \\ 223 \\ 200 \\ 183 \\ 218 \end{bmatrix}$$

By applying the Spectral Bisection algorithm to matrix $\mathbf{X}$, two clusters emerge: $C = \{1, 2\}$ and $N = \{3, 4, 5\}$.

Technique, in this simple example, is given by:

$$\mathbf{A} = \begin{bmatrix} 0.26 & 0.15 & 0.09 & 0.08 & 0.04 \\ 0.31 & 0.30 & 0.34 & 0.03 & 0.04 \\ 0.06 & 0.04 & 0.30 & 0.18 & 0.12 \\ 0.02 & 0.03 & 0.13 & 0.32 & 0.09 \\ 0.04 & 0.04 & 0.25 & 0.16 & 0.31 \end{bmatrix} \mathbf{a}_l = \begin{bmatrix} 0.08 \\ 0.08 \\ 0.09 \\ 0.07 \\ 0.09 \end{bmatrix}$$

and, in vertically integrated terms:

$$\mathbf{B} = \begin{bmatrix} 1.59 & 0.40 & 0.57 & 0.42 & 0.28 \\ 0.86 & 1.72 & 1.20 & 0.60 & 0.45 \\ 0.28 & 0.20 & 1.82 & 0.62 & 0.44 \\ 0.17 & 0.15 & 0.52 & 1.72 & 0.35 \\ 0.29 & 0.24 & 0.90 & 0.69 & 1.74 \end{bmatrix} \mathbf{v} = \begin{bmatrix} 0.26 \\ 0.23 \\ 0.43 \\ 0.33 \\ 0.28 \end{bmatrix}$$

Subsystems and industry labour are given by:

$$\mathbf{l} = \begin{bmatrix} 53 \\ 84 \\ 60 \\ 40 \\ 73 \end{bmatrix} \mathbf{l}_v = \begin{bmatrix} 51.61 \\ 50.94 \\ 85.82 \\ 59.67 \\ 61.97 \end{bmatrix} (\mathbf{l}_v - \mathbf{l}) = \begin{bmatrix} -1.39 \\ -33.06 \\ 25.86 \\ 19.67 \\ -11.03 \end{bmatrix}$$

Focusing on cluster $C$, the four components i)-iv) listed on page 8, and diagrammatically illustrated in Figure A.2, for this simple case, are:

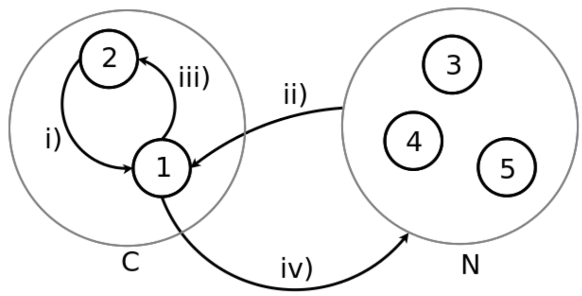


Figure A.2: Labour redistribution between $C$-type (intra-cluster) and $N$-type (extra-cluster) industries/subsystems $(i = 1)$

i) $\mathbf{a}_{lc}^T \check{\mathbf{B}}_{cc} \widehat{\mathbf{y}}_c = \begin{bmatrix} 14.03 & 7.32 \end{bmatrix}$

ii) $\mathbf{a}_{ln}^T \mathbf{B}_{nc} \widehat{\mathbf{y}}_c = \begin{bmatrix} 12.31 & 11.32 \end{bmatrix}$

iii) $\mathbf{y}_c^T \check{\mathbf{B}}_{cc}^T \widehat{\mathbf{a}}_{lc} = \begin{bmatrix} 7.32 & 14.03 \end{bmatrix}$

iv) $\mathbf{y}_n^T \mathbf{B}_{cn}^T \widehat{\mathbf{a}}_{lc} = \begin{bmatrix} 20.42 & 37.67 \end{bmatrix}$

or, in relative terms:

i) $\varphi_{cc} = \begin{bmatrix} 0.14 & 0.07 \end{bmatrix}$: excluding self-loops, $21\%(= 0.14 + 0.07 \times 100)$ of total labour requirements of subsystems 1 and 2 comes from industries 1 and 2;

ii) $\varphi_{nc} = \begin{bmatrix} 0.12 & 0.11 \end{bmatrix}$: 23% of total labour requirements of subsystems 1 and 2 comes from industries 3, 4 and 5;

iii) $\psi_{cc} = \begin{bmatrix} 0.05 & 0.10 \end{bmatrix}$: 15% of direct employment of industries 1 and 2 goes to subsystems 1 and 2;

iv) $\psi_{cn} = \begin{bmatrix} 0.15 & 0.27 \end{bmatrix}$: 42% of direct employment of industries 1 and 2 goes to subsystems 3, 4 and 5;

Hence, cluster $C$ is mainly important for its employment provision towards external subsystems (given its relatively high value of $\psi_{cn}$). This is also reflected in the negative entries for sectors 1 and 2 in $(\mathbf{l}_v - \mathbf{l})$, which imply a cluster hierarchy indicator $(L^{(c)} - L_c)/L = -0.11(= -(1.39+33.06)/310)$ (i.e. the cluster is relatively far from final demand). Note that this occurs even if final demand for industry 2 is highest among all 5 sectors, as it is the *relative* output proportion between intermediate and final uses which is recursively applied when constructing logical subsystems. This exemplifies how the metrics proposed helps to avoid misleading conclusions based on direct observation.

According to expression (25), total intra-cluster vertically integrated labour can be decomposed into:

$\omega_{cc} = 0.68$: 68% of total labour requirements by subsystems 1 and 2 is generated by self-loops and synergies internal to industries 1 and 2;

$\omega_{cnc} = 0.09$: 9% of total labour requirements by subsystems 1 and 2 are due to feedback effects, i.e. employment from industries 1 and 2 indirectly enters subsystems 1 and 2, because it is required to produce own-intermediates, demanded by subsystems 3, 4 and 5, in order to produce inputs to satisfy final demand directed towards subsystems 1 and 2;

$\omega_{nc} = 0.23$: 23% of total labour requirements by subsystems 1 and 2 is generated by inter-cluster spillovers, i.e. employment from industries 3, 4 and 5 directly and indirectly participating in vertically integrated labour of subsystems 1 and 2.

By computing expression (27) we have that self-loops of industries 1 and 2 are $\omega_{cc}^s = 0.47$ (i.e. 47% of $L_c$), therefore, synergies amount to $\omega_{cc} - \omega_{cc}^s = 0.68 - 0.47 = 0.21$, i.e. 21% of vertically integrated labour in cluster $C$ circulates between internal industries.

Finally, by combining these measures we have that feedback effects represent around 43% of intra-cluster synergies $\omega_{cnc}/(\omega_{cc} - \omega_{cc}^s) = 0.09/(0.68 - 0.47) = 0.428$, which points to a relevant role of *inter*-cluster linkages in inducing *intra*-cluster demand, a result which does not follow by considering subsystem labour on its own.

## Appendix B. Vertically integrated labour of intra-cluster subsystems

To obtain a decomposition for $\mathbf{v}_c^T$, develop first the partitioned inverse $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$ in (21). Applying Banachiewicz-Schur matrix inversion formula,[38] each block $\mathbf{B}_{cc}$, $\mathbf{B}_{cn}$, $\mathbf{B}_{nc}$, $\mathbf{B}_{nn}$ is given by:

$$\begin{aligned}
\mathbf{B}_{cc} =& (\mathbf{I} - \mathbf{A}_{cc} - \mathbf{A}_{cn}(\mathbf{I} - \mathbf{A}_{nn})^{-1}\mathbf{A}_{nc})^{-1} \\
\mathbf{B}_{cn} =& (\mathbf{I} - \mathbf{A}_{cc})^{-1}\mathbf{A}_{cn} \\
& \times (\mathbf{I} - \mathbf{A}_{nn} - \mathbf{A}_{nc}(\mathbf{I} - \mathbf{A}_{cc})^{-1}\mathbf{A}_{cn})^{-1} \\
\mathbf{B}_{nc} =& (\mathbf{I} - \mathbf{A}_{nn})^{-1}\mathbf{A}_{nc} \\
& \times (\mathbf{I} - \mathbf{A}_{cc} - \mathbf{A}_{cn}(\mathbf{I} - \mathbf{A}_{nn})^{-1}\mathbf{A}_{nc})^{-1} \\
\mathbf{B}_{nn} =& (\mathbf{I} - \mathbf{A}_{nn} - \mathbf{A}_{nc}(\mathbf{I} - \mathbf{A}_{cc})^{-1}\mathbf{A}_{cn})^{-1}
\end{aligned}$$

Substituting for $\mathbf{H}_{cn}$ and $\mathbf{H}_{nc}$ — expressions (23)-(24) in the main text — in $\mathbf{B}$ and after some algebraic operations, the partitioned inverse may be written as:

$$\mathbf{B} = \begin{bmatrix} (\mathbf{I} - \mathbf{A}_{cc})^{-1} & \mathbf{0} \\ \mathbf{0} & (\mathbf{I} - \mathbf{A}_{nn})^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{H}_{cn} \\ \mathbf{H}_{nc} & \mathbf{I} \end{bmatrix} \times \begin{bmatrix} (\mathbf{I} - \mathbf{H}_{cn}\mathbf{H}_{nc})^{-1} & \mathbf{0} \\ \mathbf{0} & (\mathbf{I} - \mathbf{H}_{nc}\mathbf{H}_{cn})^{-1} \end{bmatrix}$$

Pre-multiplying $\mathbf{B}$ by the partitioned vector of direct labour coefficients $\mathbf{a}_l^T = \begin{bmatrix} \mathbf{a}_{lc}^T & \mathbf{a}_{ln}^T \end{bmatrix}$ we obtain $\mathbf{v}^T = \begin{bmatrix} \mathbf{v}_c^T & \mathbf{v}_n^T \end{bmatrix}$:

$$\begin{bmatrix} \mathbf{v}_c^T & \mathbf{v}_n^T \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{lc}^T(\mathbf{I} - \mathbf{A}_{cc})^{-1} & \mathbf{a}_{ln}^T(\mathbf{I} - \mathbf{A}_{nn})^{-1} \end{bmatrix} \times \begin{bmatrix} \mathbf{I} & \mathbf{H}_{cn} \\ \mathbf{H}_{nc} & \mathbf{I} \end{bmatrix} \begin{bmatrix} (\mathbf{I} - \mathbf{H}_{cn}\mathbf{H}_{nc})^{-1} & \mathbf{0} \\ \mathbf{0} & (\mathbf{I} - \mathbf{H}_{nc}\mathbf{H}_{cn})^{-1} \end{bmatrix}$$

Focusing only on vertically integrated labour of subsystems in cluster $C$, we have:

$$\begin{aligned}
\mathbf{v}_c^T =& \mathbf{a}_{lc}^T(\mathbf{I} - \mathbf{A}_{cc})^{-1}(\mathbf{I} - \mathbf{H}_{cn}\mathbf{H}_{nc})^{-1} \\
& + \mathbf{a}_{ln}^T(\mathbf{I} - \mathbf{A}_{nn})^{-1}\mathbf{H}_{nc}(\mathbf{I} - \mathbf{H}_{cn}\mathbf{H}_{nc})^{-1}
\end{aligned}$$

[38] See, e.g. Abadir and Magnus (2005, pp. 106-7).

but given that:

$$(\mathbf{I}-\mathbf{H}_{cn}\mathbf{H}_{nc})^{-1}=\mathbf{I}+\mathbf{H}_{cn}\mathbf{H}_{nc}(\mathbf{I}-\mathbf{H}_{cn}\mathbf{H}_{nc})^{-1}$$

we finally obtain:

$$\begin{aligned}\mathbf{v}_c^T =&\mathbf{a}_{lc}^T(\mathbf{I}-\mathbf{A}_{cc})^{-1}\\ &+\mathbf{a}_{lc}^T(\mathbf{I}-\mathbf{A}_{cc})^{-1}\mathbf{H}_{cn}\mathbf{H}_{nc}(\mathbf{I}-\mathbf{H}_{cn}\mathbf{H}_{nc})^{-1}\\ &+\mathbf{a}_{ln}^T(\mathbf{I}-\mathbf{A}_{nn})^{-1}\mathbf{H}_{nc}(\mathbf{I}-\mathbf{H}_{cn}\mathbf{H}_{nc})^{-1}\end{aligned}$$

which coincides with expression (22) in the main text.


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