Pasinetti's 'Structural Change and Economic Growth': A conceptual excursus‡

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ABSTRACT A clear and organic exposition of Pasinetti's theoretical framework of *Structural Change and Economic Growth* has been prevented by misunderstandings and ambiguities concerning basic categories and terminology. The pre-institutional character of the approach, the nature of its equilibrium paths and the significance — and normative character — of the 'natural' economic system are some of the most controversial issues. The aim of this article is to present a conceptual excursus of the model to establish a solid foundation for fruitful discussions to be held with other Classical approaches.

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1. Introduction

Pasinetti's *Structural Change and Economic Growth* has been, since its publication in 1981, the object of many reviews and comments, and it is one of the most cited works regarding the topic of structural change (For example, see: Silva & Teixeira 2008). However, many aspects of the book, both conceptual and analytical, have not been grasped, or only partially grasped, which has prevented a complete understanding of the implications and potentialities of Pasinetti's approach.

The first stumbling block has usually been the pre-institutional — sometimes misinterpreted as a pre-industrial — and normative rather than positive — character of the model.[1] A second problem, related to the previous one, is the often missed distinction between the general dynamic analysis of the price and quantity systems, the dynamic equilibrium paths — one for each possible *exogenous* combination of distributive variables — and the 'natural' economic system, resulting from a particular closure of the price system.[2] A third issue of importance is the vertically hyper-integrated (VHI hereinafter) character of the framework, on which we particularly focus in this article, in order for the model — and some of its most far reaching insights — to be more fully understood.[3]

This article comprises a conceptual *excursus* of Pasinetti's model and is organised as follows. Section 2 deals with the pre-institutional character of Pasinetti's (1981) model. Section 3 presents a synthetic exposition of the model *before* the introduction of the 'natural' economic system. Section 4 presents the

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[1] For example, see: Asimakopulos (1982, p. 1566), Harris (1982, p. 29) and Taylor (1995, p. 699).

[2] For example, see Parrinello (2004).

[3] On this point, see the criticisms put forward by Schefold (1982) and Taylor (1995).

rationale for the particular closure of the price system adopted by Pasinetti (1981) highlighting some of the main insights. Section 5 is a methodological note on the notion of equilibrium and its role throughout the analysis. Finally, section 6 presents some concluding remarks.

2. The pre-institutional analysis of an industrial system

Before going into the details of Pasinetti's (1981) analytical formulation, we present a brief methodological introduction in order for 'this theoretical framework . . . [to be] appropriately understood and correctly used. It is a basic framework, a skeleton, so to speak, which is meant to remain at a *pre-institutional level of investigation'* (Pasinetti 1985, p. 274, emphasis added). This quote is from the reply Pasinetti gave to a review, by Nina Shapiro (1984), of *Structural Change and Economic Growth*. The point he raised is crucial: the analytical framework he developed can be understood and correctly used only if its pre-institutional character is constantly and clearly kept in mind. Therefore, although we will draw on Pasinetti's (1981) methodology in sections 4 and 5 below, we provide a general hint here, before going into the analytical description of the model, in order for the latter to be properly understood.

The first thing that should be made clear is the meaning Pasinetti attaches to the word 'capitalistic' as opposed to 'capitalist'. While the latter refers to the set of *social relations* of production typical of a system with private ownership of the social means of production, in contrast to those of, for example, a centrally planned one, the former term describes the very physical-technological nature of the production process in any *industrial* society, i.e. a society in which the accumulation of purchasing power in the form of capital goods has *social*

implications, for example, regarding the level and composition of employment (Pasinetti 1983). The notion of 'capitalistic' production processes concerns the *production of commodities by means of commodities*, i.e. the productive consumption of circulating and fixed capital as the *means* of production, to be used together with labour, and which should be *necessarily* replaced and accumulated, *if* this circular flow is to be reproduced at an expanding scale.[4]

Pasinetti's (2005, p.247) focus has *always* been on 'industrial societies, with their tendency towards change and towards an evolving structure, as against the more static conditions of pre-industrial societies'. Nonetheless, the pre-institutional analysis he puts forward has sometimes been (mis)interpreted as a pre-capitalist, or pre-capitalistic, one, in spite of the fact that he has never used such expressions, and he has always made explicit, and repeated, reference to 'pure production systems' within the context of an 'industrial society'.

Pasinetti's aim is that of analysing the working of a *capitalistic*, and not of a capitalist, economic system. As discussed in sections 4 and 5 below, the framework he develops is by no means an attempt at describing the functioning of an actual capitalist system. Nor it is an attempt at describing the functioning of a centrally planned economy, as someone could be induced to conclude.

What does it mean, therefore, that Pasinetti's (1981) analysis has been carried out at the *pre-institutional* level? The issue is not a trivial one. It is apparent that many commentators did not grasp the meaning of such a statement

[4] Given that Pasinetti does *not* conceive capital as time, the term 'capitalistic' in this context does not stand for 'roundaboutness' (as it has been intended by, for example, Wicksell). In Pasinetti's (1981) analysis, it is co-existing and concurrent labour that is applied to any reduction process of heterogeneous capital goods in terms of homogeneous labour for the purpose of summarising the extent and degree of the division of labour.

and thereby failing to grasp the very nature of Pasinetti's framework.[5] It is our contention that many — actual or pretended — ambiguities in Pasinetti's (1981) exposition are due to this misunderstanding.

As Pasinetti states in the Introduction to *Structural Change and Economic Growth*, his approach to economic theory starts from a very precise standpoint:

> 'It is my purpose...to develop first of all a theory which remains neutral with respect to the institutional organisation of society. My preoccupation will be that of singling out, to resume Ricardo's terminology, the 'primary and natural' features of a pure production system'
> (Pasinetti 1981, p. 25).

A 'separation' — as Pasinetti (2007) subsequently called it — is therefore needed between two stages of analysis, each concerning a very specific kind of economic investigation. [6] The rationale of this separation emerges very clearly from the Preface to the 1981 book (p. xiii, emphasis added):

> '*There is ... a sharp discrimination between those economic problems that have to be solved on the ground of logic alone — for which economic theory is entirely autonomous — and those economic problems that arise in connection with particular institutions, or with particular groups' or individuals' behaviour* — for which economic theory is no longer autonomous and needs to be integrated with further hypoth*eses, which may well come from other social sciences. It is with the first type of problems that the present work is basically concerned.*'

Of course, Pasinetti's claim for the *logical* priority of the first stage — the *pre-institutional* one — with respect to the second stage — the *institutional* one — by

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[5] See the discussion in, for example, Delorme & Dopfer (1994) as well as the summary and comments in Reati (2000).

[6] In this more recent book, Pasinetti (2007) stresses such a distinction in a much sharper way.

no means implies that he is disregarding the role of institutions. On the contrary, their role is of primary importance as they are the *means* to shape the real world:

> 'All these considerations only come to confirm how important is to keep the logical problems concerning the 'natural' economic system quite separate from those concerning the institutions, and to consider the institutions for what they really are — means, and not ends in themselves. Once their instrumental role is properly understood and recognised, it becomes much easier also to operate on them in as detached a way as is possible; to treat them as instruments susceptible to be continually improved and changed, in relation to their suitability (or unsuitability) to ensure tendencies, or near-tendencies, towards agreed ends.'
>
> (Pasinetti 1981, p. 155).

Institutions are means, not ends, but in order for them to be used to drive society 'towards agreed ends' it is first of all necessary to know the fundamental mechanisms they are called upon to counteract, or to favour, or simply to take advantage of. Without this knowledge, institutions cannot pursue any instrumental role.

Pasinetti's *vision* is that the 'primary and natural' features of an economic system have to be studied independently of a particular institutional set-up. Nonetheless, the task of describing an economic system without reference to a particular institutional set-up is not a trivial one. It might prove difficult to realise how an economic system can be thought of without strong reference to the institutions which shape it, since no actual economic system could have been brought into existence without them. This task can be accomplished by looking for those *physical* requirements necessary for an industrial system to carry out its production process, and *grow*, i.e. without the 'need to go beyond the purely

physical characteristics of capitalistic production' (Pasinetti 1983, p. 411). The way in which Pasinetti puts this idea into practice is clarified in sections 3 and 4 below.

3. General dynamic analysis and equilibrium dynamics

Pasinetti's (1981) *Structural Change and Economic Growth* provides us with a model of economic growth starting from a complete description of an economic system in a single-period *equilibrium*, defined as 'a situation in which there is full employment of the labour force and full utilisation of the existing productive capacity' (Pasinetti 1981, pp. 48-49). This situation can be thought of as the *initial condition* of a general multi-sector dynamic model, which 'has been developed for the purpose of detecting the 'permanent' causes moving an economic system, irrespective of any accidental or transitory deviation which may temporarily occur' (Pasinetti 1981, p. 127).

We will now introduce a synthetic exposition of the model; more precisely, the specification of the model considering capital goods produced by means of labour alone, for this is the main case with which Pasinetti (1981) deals.[7]

3.1 The quantity and price systems

This single-period description starts from the physical quantity and commodity price systems, composed of $2m+1$ equations each, m being the number of final consumption commodities produced; the first set of m equations concern consumption commodities, the second set capital goods, and the last equation is an

7. The basic notation can be found in the Appendix.

equilibrium condition. The production of each consumption commodity i requires a specific capital good k_i.[8]

As to the physical quantity system, the equations concerning consumption commodity i and the corresponding capital good k_i together describe total quantities produced by VHI sector i: $x_i(t)$ and $x_{k_i}(t)$.[9] The last equation establishes the condition for the full employment of total labour available in the system. Analytically, the system is given by:

$$\begin{cases} x_i(t) = a_{in}(t)x_n(t) \\ x_{k_i}(t) = T_i^{-1}x_i(t) + a_{k_i n}(t)x_n(t) \\ x_n(t) = \sum_i a_{ni}(t)x_i(t) + a_{nk_i}(t)x_{k_i}(t) \end{cases} \quad (3.1)$$

More precisely, x_i is determined by the average per capita demand for final consumption commodity i multiplied by total population, while x_{k_i} is the sum of two components: the number of units of productive capacity for consumption good i necessary for the replacement of worn out productive capacity, and the number of units of productive capacity demanded as new investment:[10] the capital-producing industry has to provide not only those units of productive capacity necessary for keeping the initial stock intact, but also those units required to expand it.[11]

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[8] Hereinafter, unless differently stated, subscript i is intended as $\forall i = 1,2,\cdots,m$.

[9] The concept of vertical hyper-integration is already present in Pasinetti (1981) even though not always explicitly. For a rigorous statement and development of this concept, and of its analytical properties, see Pasinetti (1988).

[10] $T_i^{-1}a_{in}(t)x_n(t) = x'_{k_i}(t)$ and $a_{k_i n}(t)x_n(t) = x''_{k_i}(t)$, respectively.

[11] This is a crucial difference between the notion of vertically integrated sectors and that of vertically hyper-integrated sectors (see Pasinetti 1973, Pasinetti 1988).

As for the price system, there will be a price for each final consumption commodity i, $p_i(t)$, and a price for each capital good k_i associated with it, $p_{k_i}(t)$. A further equation establishes the condition for the full expenditure of (full employment) national income. Analytically, the price system can be written as:

$$\begin{cases} p_i(t) = \left(\pi_i(t)\dfrac{k_i(t)}{x_i(t)} + \dfrac{1}{T_i}\right)p_{k_i}(t) + a_{ni}(t)w(t) \\ p_{k_i}(t) = a_{nk_i}(t)w(t) \\ w(t) + \pi_i(t)\dfrac{k_i(t)}{x_i(t)} = \sum_i a_{in}(t)p_i(t) + a_{k_in}(t)p_{k_i}(t) \end{cases} \tag{3.2}$$

Since we are assuming that capital goods are produced by labour alone, each price p_{k_i} is determined by direct labour requirements multiplied by the wage rate, while prices p_i also include an additional component corresponding to the direct costs of replacing worn out productive capacity and a profit margin on the existing stock of productive capacity.

3.2 Vertically hyper-integrated productive capacity

In general, the means of production required to obtain one unit of a final consumption good is a sector-specific composite commodity in which the same intermediate inputs enter in *technically given* proportions. This motivates the definition of a particular unit of measurement for this physical composite commodity, one for each sector: the *units of VHI productive capacity*. Each of these units is the sum of three components: direct requirements for the production of one unit of final consumption commodity i; direct requirements for the replacement of worn-out direct and indirect capital goods needed for the production of one unit of final consumption commodity i; and, direct requirements for the

production of all intermediate commodities directly and indirectly needed for the expansion of productive capacity in line with the growth of final demand for consumption commodity *i*.

In Pasinetti's (1981) theoretical scheme, the units of productive capacity used for the measurement of capital goods are actually units of *direct* productive capacity for final consumption commodities. This becomes clear when looking at the most complex case, in which capital goods are produced by means of labour *and* capital goods (see Pasinetti 1981, pp. 43-45). However, it is our contention not only that units of *vertically hyper-integrated* productive capacity are the most appropriate units of measurement for capital goods, but also that Pasinetti himself, in 1981, had already begun to argue in terms of vertical hyper-integration, even if the complete analytical implications were still to be drawn (many of them finally reached a rigorous formulation with the publication of Pasinetti 1988).[12] In the present, simpler case, however, no analytical difference can be found between direct, vertically integrated and vertically hyper-integrated productive capacity, since final consumption commodities are the only ones produced by means of capital goods; we can thus interpret units of productive capacity as being VHI without the necessity of reformulating Pasinetti's (1981) analytical framework.[13]

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[12] This is particularly clear when matching the chapters of the book which 'have been almost entirely re-written' (Pasinetti 1981, p. xiv) since the time of his PhD Thesis with the entries in the index concerning vertical *hyper*-integration.

[13] In fact, direct productive capacity includes capital goods employed for the production of final consumption commodities, while VHI productive capacity also includes capital goods required for the production of other capital goods (i.e. worn out and additional productive capacity). In the present simplified context, however, the last two components are nil by definition given that capital goods are assumed to be produced by means of labour alone, and thus direct and VHI productive

For the sake of simplicity, we will use 'sectors' for 'vertically hyper-integrated sectors', and 'productive capacity' for 'vertically hyper-integrated productive capacity', except where the complete expressions are considered more appropriate. A unit of productive capacity will refer to the specific final commodity that requires it, and therefore to the specific sector in which it is produced. In this way, the analysis opens up the possibility of separating the pace of accumulation of the means of production (the number of units of productive capacity) from its physical composition.

In order to simplify the exposition, Pasinetti (1981) regards these composite commodities as particular capital goods, specific to each consumption good. Therefore, in the present context, a sector is made up by two industries: one producing the final consumption good, and the other one producing the corresponding capital good. These two industries play an asymmetric role, since

> 'the physical quantities of the means of production appear as playing a sort of ancillary role with respect to the physical quantities of final demand [for consumption goods]; the former being, so to speak, 'at the service' of the latter' (Pasinetti 1988, pp. 125-126).

3.3 Single-period equilibrium conditions for flows and stocks

In order for both quantity and price systems to have non-trivial (i.e. non-zero) solutions, a specific *macroeconomic condition* has to be satisfied, which states that, in equilibrium, producing the whole set of commodities consumed by each worker requires exactly one unit of VHI labour.[14]

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capacity are *de facto* the same.

[14] Given that both the physical quantity and commodity price systems are formulated as sets of

If this condition is not satisfied, the systems are contradictory, i.e. equations in the two systems cannot *simultaneously* hold. However, because of the particular mathematical structure of the problem, we can still get meaningful solutions for quantities and prices, but the last equation in each system will not be satisfied, i.e. we shall not be in a situation of full employment of the labour force and full expenditure of total income. [15] On the contrary, if this single condition is satisfied, the solutions will correspond to a situation of full employment and full expenditure of income. To this situation we shall refer as a *flow-equilibrium* situation.

When the macroeconomic condition holds, and thus the system is in a situation of flow-equilibrium, we get two indeterminate linear homogeneous systems, which means we have solutions for *relative* quantities and *relative* prices: we need to choose a scale factor for each system. For the quantity system this factor is total available labour, i.e. an exogenous variable; for the price system, the choice is arbitrary. Following Pasinetti (1981, pp. 92-93), we choose the wage rate, and therefore we take it as given both at a specific point in time and through time.[16]

2m+ 1 linear and homogeneous equations, they have non-trivial solutions if the coefficient matrix built from any of the systems (3.1) or (3.2) is singular, i.e. if its determinant is zero. The condition for this to happen is the same for both systems:

$$\sum_i a_{in}(t)a_{ni}(t) + \sum_i T_i^{-1}a_{in}(t)a_{nk_i}(t) + \sum_i a_{k_in}(t)a_{nk_i}(t) = 1 \quad (N.1)$$

The three addenda under summation are direct labour, indirect labour (i.e. direct labour necessary for replacing worn out productive capacity) and hyper-indirect labour (i.e. direct labour necessary for the expansion of productive capacity), respectively summing up to VHI labour for each growing subsystem *i*.

[15] For details, see Pasinetti (1981, pp. 33-34).

[16] This means setting $x_n(t) = \bar{x}_n(t)$ and $w(t) = \bar{w}$, respectively, in expressions (3.1) and (3.2). The

In principle, there is no difference between production prices obtained when the price system is formulated in terms of industries and when it is formulated in terms of sectors. The technique in use and the distributive variables do not change as a consequence of adopting the procedure of vertical hyper-integration, which is simply a way of re-classifying and partitioning activities in order to explicitly acknowledge for the relationship between each activity producing a final consumption commodity and those activities producing the means of production for self-replacement and expansion of the corresponding productive capacity.[17] The difference emerges as a consequence of measuring capital goods in terms of units of productive capacity; in this way, each price p_{k_i} does not stand for the price of one 'ordinary unit' of commodity k_i, but for the price of one unit of productive capacity for consumption good *i*.

As stated above, an equilibrium position entails full employment of total available labour — which implies a *single* condition concerning *flows* — and full utilisation of the existing productive capacity in each sector i — a *series* of sectoral conditions concerning *stocks*. The condition concerning flows has already emerged as the condition for non-trivial solutions to the quantity and price systems, which

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solutions are then given by:

$$\begin{cases} x_i(t) = a_{in}(t)\bar{x}_n(t) \\ x_{k_i}(t) = T_i^{-1}a_{in}(t)\bar{x}_n(t) + a_{k_in}(t)\bar{x}_n(t) \end{cases} \qquad (N.2)$$

$$\begin{cases} p_i(t) = a_{ni}(t) + a_{nk_i}(t)\left(\pi_i(t)\dfrac{k_i(t)}{x_i(t)} + \dfrac{1}{T_i}\right)\bar{w} \\ p_{k_i}(t) = a_{nk_i}(t)\bar{w} \end{cases} \qquad (N.3)$$

Note that, in the solution for $p_i(t)$, Pasinetti (1981, p. 41) implicitly assumes that $x_i(t) = k_i(t)$. This amounts to assuming that productive capacity available at the beginning of time period t is totally used up. In order to make the formulation as general as possible, we decided not to make such assumption at this stage.

[17] See Pasinetti (1973, p. 7, section 5) and Pasinetti (1988, p. 130, section 4).

is a *macroeconomic* condition since it refers to the economic system as a whole, irrespective of the number of sectors. Moreover, 'it emerges from a model which has been developed on a *multi-sector* basis, thereby revealing its truly macro-economic nature' (Pasinetti 1981, p. 35). As to stocks, we have a *series* of sectoral conditions which mean that in each sector the number of units of productive capacity available at the beginning of the period must be exactly equal to the number of units of final consumption good to be produced during the same time period.[18]

With the statement of these conditions, the description of single-period equilibrium is complete. For the analysis we will perform, an initial situation of both flow and stock equilibrium is assumed, i.e. we assume that, at time $t = 0$, both the macroeconomic condition for flow-equilibrium and the series of stock-equilibrium conditions hold true.

3.4 Dynamic method and equilibrium conditions through time

The dynamic method adopted by Pasinetti is that of specifying exponential growth of total available labour, average per capita demand, and labour input requirements;[19] (single-period) equilibrium prices and quantities being therefore

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[18] Analytically, this means that $k_i(t) = x_i(t)$.

[19] According to:

$$x_n(t) = x_n(0)e^{gt}, a_{in}(t) = a_{in}(0)e^{r_i t}, a_{ni}(t) = a_{ni}(0)e^{-\rho_i t}, a_{nk_i}(t) = a_{nk_i}(0)e^{-\rho_{k_i} t} \quad (N.4)$$

for $i = 1,2,\cdots,m; k_1,\cdots,k_m$. For the sake of simplicity, we are here assuming steady rates of change of the relevant variables, though this is not the procedure adopted by Pasinetti (1981), at least for the rate of change of final demand for consumption commodities (See Pasinetti 1981, p. 82). This is a crude simplification, though it is not possible — according to the authors — to take full advantage of the increasing realism of working with non-steady rates of change if the model is

linear structures whose components follow exponential dynamics which, however, do not imply full employment and full utilisation of productive capacity after time period $t = 0$.[20] In particular, full utilisation of productive capacity depends on a series of *stock* conditions. In the present model, the stocks of the economic system change according to the flow of demand for new investment, linking one period to the following one.[21]

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specified in continuous time. For the scope of the present work, moreover, the simplification adopted does not compromise the conclusions to be reached.

[20] Taking expressions (N.2) and (N.3) evaluated at time period $t = 0$, and inserting the dynamics described in (N.4) we obtain the following solutions for physical quantities and commodity prices, respectively:

$$\begin{cases} x_i(t) = a_{in}(0)\bar{x}_n(0)e^{(g+r_i)t} \\ x_{k_i}(t) = T_i^{-1}a_{in}(0)\bar{x}_n(0)e^{(g+r_i)t} + a_{k_in}(t)\bar{x}_n(0)e^{gt} \end{cases}$$

$$\begin{cases} p_i(t) = \left(a_{ni}(0)e^{-\rho_i t} + a_{nk_i}(0)\left(\pi_i(t)\dfrac{k_i(t)}{x_i(t)} + \dfrac{1}{T_i}\right)e^{-\rho_{k_i}t}\right)\bar{w} \\ p_{k_i}(t) = a_{nk_i}(0)e^{-\rho_{k_i}t}\bar{w} \end{cases}$$

[21] The set of accounting identities describing capital accumulation is $\dot{k}_i(t) \equiv x''_{k_i}(t)$ (where, for any variable $y(t)$ in the system, $\dot{y}(t) \equiv dy(t)/dt$. Given that $x''_{k_i}(t) = a_{k_in}(t)x_n(t)$, we obtain $\dot{k}_i(t) = a_{k_in}(t)x_n(t)$. Therefore, the series of coefficients $a_{k_in}(t)$ 'is the only one that affects the stocks of the economic system, i.e. productive capacity in each sector; hence it cannot be taken as given from outside' (Pasinetti 1981, p. 85). This opens up for the possibility to perform a general dynamic analysis by specifying a law of movement for the level of per capita new investment demand (a_{k_in}), allowing for the discrepancy between productive capacity available at the beginning of period t (k_i) and the units of productive capacity actually used up during period t (x_i). The specification of the dynamics of investment is a degree of freedom that, once closed, allows to perform an *institutional* analysis of different theories of capital accumulation. Another degree of freedom can be opened by changing the last equation of both the physical quantity and the commodity price systems, in order to explicitly allow for the possibility of flow-disequilibrium, for example, by writing:

$$\sum\nolimits_i a_{in}(t)a_{ni}(t) + \sum\nolimits_i T_i^{-1}a_{in}(t)a_{nk_i}(t) + \sum\nolimits_i a_{k_in}(t)a_{nk_i}(t) = \alpha \gtreqless 1$$

By explicitly considering the laws of motions of the exogenous variables, it is possible to derive the 'necessary requirements for equilibrium growth' (Pasinetti 1981, p. 25), i.e. to keep *flow-* and *stock-equilibrium* through time.[22] As regards the stock-equilibrium, the laws of motion of average per capita sectoral demands for new investment 'must be such as to be compatible with the process of economic growth and will therefore themselves be determined as part of the equilibrium conditions' (Pasinetti 1981, p. 85). Therefore, the growth requirements of productive capacity in each sector, i.e. demand for new investment, must exactly satisfy the growth of demand for each final consumption commodity in all periods beyond $t = 0$. We can thus obtain a set of *capital accumulation conditions*, which are the dynamic counterpart of stock-equilibrium ones, determining the sectoral equilibrium *rates of new investment* $(g + r_i)$, defined as the number of units of productive capacity, per unit of final demand for each consumption commodity i, necessary as new investment for the expansion of the corresponding productive capacity.[23]

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meaning that macroeconomic condition (N.1) is not satisfied if $\alpha \neq 1$. For a hint at different cases that can occur as a consequence of flow and stock disequilibria, see Pasinetti (1981, pp. 47-48).

[22] In particular, we can get the condition for keeping flow-equilibrium by inserting (N.4) into (N.1):

$$\sum_i a_{in}(0)a_{ni}(0)e^{(r_i-\rho_i)t} + \sum_i T_i^{-1}a_{in}(0)a_{nk_i}(0)e^{(r_i-\rho_{k_i})t} + \sum_i a_{k_in}(t)a_{nk_i}(0)e^{-\rho_{k_i}t} = 1 \quad (N.5)$$

It can be noted that the demand coefficients for new investment $a_{k_in}(t)$ are still taken as exogenously given, their specification being the subject of the following few paragraphs.

[23] Mathematically, since $\dot{k}_i(t) = a_{k_in}(t)x_n(t)$ and, in stock equilibrium, $k_i(t) = x_i(t)$, we have that $a_{k_in}(t)x_n(t) = \dot{x}_i(t)$. As the growth of final demand is given by $\dot{x}_i(t) = (g + r_i)a_{in}(t)x_n(t)$, the following set of sectoral *capital accumulation conditions* must be satisfied:

$$a_{k_in}(t) = (g + r_i)a_{in}(t), \;\; t > 0 \quad (N.6)$$

Crucially, therefore, equilibrium new investments are determined according to an 'accelerator mechanism' (Pasinetti 1960) operating at the level of *each* VHI sector. This *induced* character of *gross* investments is an essential building block of equilibrium situations in Pasinetti's (1981) book. '*Demand for capital goods ... is not autonomous. It is (logically) determined after demand for consumption goods has been decided already, and as a consequence of it, as a condition for equilibrium growth*' (Pasinetti 1981, p. 176, emphasis added).

The set of capital accumulation conditions can alternatively be expressed as ratios of sectoral new investment to production at current prices,[24] showing that, in order to keep stock-equilibrium,

> 'the ratio of new investments to the level of production must be equal, in each sector, to the technologically determined capital/output ratio multiplied by [the sum of] the rate of population growth [and the rate of growth of per capita demand]'
> (Pasinetti 1981, pp. 54-55).

In order to fully acknowledge the importance of capital accumulation conditions stated in this alternative way, we first must note the *vertically hyper-integrated character* of the capital/output ratio. In a traditional inter-industry scheme, the net output of the economy is the set of commodities produced for final consumption and new investment. However, in a VHI framework, the net output of the system is made up only of the set of commodities for final consumption, as new investment demand is part of the means of production required to expand productive capacity. Therefore, when thinking of the capital intensity of a sector i,

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[24] Analytically:

$$\frac{p_{k_i}(t)x''_{k_i}(t)}{p_i(t)x_i(t)} = (g + r_i)\frac{p_{k_i}(t)k_i(t)}{p_i(t)x_i(t)} = (g + r_i)\chi_i(t), \;\; t > 0 \quad (N.7)$$

where the $\chi_i(t)$'s are sectoral capital/output ratios.

its denominator (the net output) will be the value of final consumption commodity i produced in the system, while its numerator (the value of capital) will be the value of the units of productive capacity specific to each final consumption commodity i required to self-replace and *expand* productive capacity during period t.

In the light of this, the specification of an equilibrium schedule of capital accumulation in VHI terms reflects, on the one side, the interdependent nature of the production process as, in the most general case, a single industry producing a basic commodity (in the sense of Sraffa 1960) utilised as a capital good would participate in different sectors with a different capital intensity in each of them; and on the other side, it highlights the potential of working with VHI sectors, as

> 'the notion of a physical unit of productive capacity, by being defined with reference to the commodity that is produced, continues to make sense, as a physical unit, whatever complications technical change may cause to its composition in terms of ordinary commodities' (Pasinetti 1973, p. 24).[25]

This is the most remarkable property of the chosen unit of measurement: whatever the time period, whatever the stage of technical progress, whatever the technique actually in use, capital goods can always be measured in units of productive capacity, and the accumulation of capital can always be studied by evaluating the *number* of units of productive capacity that have to be produced during time period t to maintain stock-equilibrium at the beginning of time period $t + 1$. In this way, we can link the stocks of different time periods through relatively simple capital accumulation (equilibrium) conditions. Complementarily, the problem of

[25] In Pasinetti (1981), as each capital goods-producing industry is specific to each consumption goods-producing one, it is the *second* aspect that is emphasised, though the framework allows for further generalisation to reflect also the first one. See Pasinetti (1988).

the change in the *physical composition* of these units can be studied separately by exploiting the one-to-one correspondence between VHI and inter-industry relations as 'the production coefficients of a vertically [hyper]integrated model turn out to be a linear combination of the production coefficients of the corresponding input-output model'(Pasinetti 1981, p. 111).

The macroeconomic condition can now be further reformulated to incorporate the capital accumulation conditions.[26] For any specific *composition* of final demand *for consumption*, the equilibrium amount of gross investment is in this way univocally determined by the technique in use and by the dynamics of population and of final consumption demand itself.[27] In this way, therefore, the macroeconomic condition 'may be called the *effective demand condition* for keeping full employment' (Pasinetti 1981, p. 54, original emphasis), since it establishes whether a given composition of final demand for consumption is compatible with flow-equilibrium, i.e. with full-employment of the labour force. It therefore follows that 'the difficulty of increasing total effective demand is one of finding out, and achieving, at a sufficient speed, its appropriate structural composition, and not one of reaching any absolute level' (Pasinetti 1981, p. 242), highlighting the multi-sectoral foundation of an effective demand theory of output.

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[26] By substituting capital accumulation conditions (N.6) into the macroeconomic condition (N.5) and writing it as follows:

$$\sum_i a_{in}(0)a_{ni}(0)e^{(r_i-\rho_i)t} + \sum_i (g + r_i + T_i^{-1})a_{in}(0)a_{nk_i}(0)e^{(r_i-\rho_{k_i})t} = 1 \quad (N.8)$$

Note that the two addenda distribute total labour of the system between the labour requirements of final consumption commodities and the labour requirements of equilibrium *gross* investments.

[27] The left-hand side of (N.8) stands for the size of per-capita total effective demand in time period *t*.

3.5 Vertically hyper-integrated labour

The units of productive capacity are one of the two constituent components of the technique of a VHI sector, the other being *vertically hyper-integrated labour coefficients* $\ell_i(t)$.[28] The VHI labour coefficient for each sector i, $\ell_i(t)$, is the sum of three components: direct labour for the production of one unit of final consumption commodity i (direct labour); direct labour for the replacement of worn-out units of productive capacity for sector i (indirect labour); and, direct labour required for the expansion of productive capacity of sector i according to the growth of final demand for consumption good i (hyper-indirect labour).[29]

We can take advantage of this definition to express prices in terms of VHI labour. When capital accumulation conditions hold, the prices of final consumption commodities can be written as:

$$p_i(t) = \ell_i(t)\overline{w} + [\pi_i(t) - (g + r_i)]p_{k_i}(t) \quad (3.3)$$

Expression (3.3) establishes the production price of each final consumption commodity i as the sum of two components: the cost of VHI labour embodied in it, $\ell_i(t)\overline{w}$, and a profit-differential with respect to the sectoral equilibrium rate of new investment, $[\pi_i(t) - (g + r_i)]$, computed on the value of equilibrium productive

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[28] In order to define them, we shall start from the full-employment macroeconomic condition for flow-equilibrium. By inserting (N.6) into (N.5) and rearranging, we get:

$$\sum_i a_{in}(0)e^{r_i t}\left(a_{ni}(0)e^{-\rho_i t} + \frac{1}{T_i}a_{nk_i}(0)e^{-\rho_{k_i} t} + (g + r_i)a_{nk_i}(0)e^{-\rho_{k_i} t}\right) = 1$$

which, defining $\ell_i(t) \equiv a_{ni}(t) + \frac{1}{T_i}a_{nk_i}(t) + (g + r_i)a_{nk_i}(t)$, can be written as:

$$\sum_i a_{in}(t)\ell_i(t) = 1 \quad (N.9)$$

[29] The three components are given by $a_{ni}(t)$, $T_i^{-1}a_{nk_i}(t)$, and $(g + r_i)a_{nk_i}(t)$, respectively. For details, see Pasinetti (1981, p. 102).

capacity at current production prices — $p_{k_i}(t)$. This second component is not the (dual) value counterpart of necessary physical quantity requirements of (re-)production and expansion, but emerges as an amount of purchasing power created in excess to these requirements, that goes to the owners of the means of production through the process of income distribution. Its extent is a direct consequence of the theory of income distribution that shall be adopted to close the price system, and it will influence the whole process of structural dynamics, via its effect on the pattern of real income expenditure.

Another important variable we introduce into the analysis is the level of equilibrium employment in each sector i, given by the product of the corresponding VHI labour coefficient times the physical quantity of final consumption commodity i produced: $L_i(t) = \ell_i(t)x_i(t)$. In this respect, it is relevant to stress the VHI character of $L_i(t)$: in the most general specification of technology, a fraction of the total labour employed by a single industry producing a *basic* commodity would enter into the employment of *all* VHI sectors, either directly and/or (hyper)indirectly.

The comparison with the VHI nature of the sectoral capital/output ratios is straightforward. The composition of sectoral employment reflects not only the change in labour requirements of the industry producing the final consumption commodity concerned, but also the changing physical composition of the corresponding unit of productive capacity, and therefore the change in labour requirements of *all* the industries in the sector. It is for this reason that evaluating only the change in direct labour requirements cannot account for the interdependent and systemic nature of productivity changes. This opens up the

possibility of performing empirical investigations on the dynamics of productivity taking VHI sectors as the unit of analysis.[30]

3.6 Dynamic equilibrium paths

Assuming that the set of capital accumulation conditions and the effective demand condition hold, we can specify the equilibrium path of relative quantities and prices, as well as the evolution of sectoral employment.[31] The equilibrium solutions

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[30] The rate of growth of $\ell_i(t)$, which we may denote by $\rho_i'(t)$, is the rate of growth of VHI labour productivity of sector i, given by the weighted average of the rates of growth of direct, indirect and hyper-indirect labour productivity, the weights being the proportions of the three kinds of labour to total labour employed in VHI sector i, respectively:

$$-\frac{\dot{\ell}_i(t)}{\ell_i(t)} \equiv \rho_i'(t) = \rho_i \frac{a_{ni}(t)}{\ell_i(t)} + \rho_{k_i} \frac{T_i^{-1} a_{nk_i}(t)}{\ell_i(t)} + \rho_{k_i} \frac{(g + r_i) a_{nk_i}(t)}{\ell_i(t)}$$

Note that, within the Classical tradition, system measures of labour productivity have always relied on vertical integration rather hyper-integration to assess productivity changes. See, for example, Gupta & Steedman (1971) and De Juan & Febrero (2000).

[31] If 'we choose to reckon prices in terms of Classical 'labour commanded' ' (Pasinetti 1981, p. 99), the wage rate still being the basis for the price system, we set $\overline{w} = 1$. Hence, the equilibrium dynamic path of relative physical quantities, sectoral employment and commodity prices is given by, respectively:

$$\begin{cases} x_i(t) = a_{in}(0)\bar{x}_n(0)e^{(g+r_i)t} \\ x_{k_i}(t) = (T_i^{-1} + g + r_i)a_{in}(0)\bar{x}_n(0)e^{(g+r_i)t} \end{cases} \qquad (N.10)$$

$$\{L_i(t) = \ell_i(t)a_{in}(0)\bar{x}_n(0)e^{(g+r_i)t} \qquad (N.11)$$

$$\begin{cases} p_i^{(w)}(t) = \ell_i(t) + [\pi_i(t) - (g + r_i)]a_{nk_i}(0)e^{-\rho_{k_i}t} \\ p_{k_i}^{(w)}(t) = a_{nk_i}(0)e^{-\rho_{k_i}t} \end{cases} \qquad (N.12)$$

In what follows, whenever a *nominal* magnitude has a letter in brackets as a superscript, that letter will indicate the *numéraire* commodity adopted. Therefore, $p_i^{(w)}(t)$ indicates the price of commodity

for physical quantities, together with equilibrium sectoral employment, represent a set of growing subsystems, one for each final consumption commodity *i*. Each growing subsystem or, equivalently, hyper-subsystem, consists of three components: $x_i(t), x_{k_i}(t)$and $L_i(t)$. The first one represents the 'production of one single consumption good *i*, expanding through time at its particular rate of growth $(g + r_i)$' (Pasinetti 1988, p. 127). The second one represents the physical quantities for

> 'the maintenance of a circular production process that *both* reproduces all the means of production which are absorbed by the production process for [each] consumption good ... *and* also produces those means of production that are strictly necessary to expand such a circular process at a rate of growth $(g + r_i)$.'
>
> (Pasinetti 1988, p. 127, added emphasis)

Finally, the third one represents the 'absorption of a physical quantity of labour $L_i(t)$' (Pasinetti 1988, p. 127) required to produce physical quantities $x_i(t)$ and $x_{k_i}(t)$.

The rates of change of relative quantities, sectoral employment and relative prices describe the structural dynamics implied by dynamic *stock-* and *flow-equilibrium*. As regards sectoral physical quantities, their equilibrium evolution is completely determined by that of effective demand for the corresponding final consumption commodity on which each growing subsystem is built. This holds true for both $x_i(t)$ and $x_{k_i}(t)$, due to the adoption of the units of productive capacity as the particular units of measurement for capital goods.[32] Hence, the rate of

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i when the *numéraire* of the price system is the wage rate. For a complete analysis of the equilibrium structural dynamics of a growing economic system, see Pasinetti (1981, pp. 91-99).

[32] In fact, we have:

$$\frac{\dot{x}_i(t)}{x_i(t)} = \frac{\dot{x}_{k_i}(t)}{x_{k_i}(t)} = g + r_i$$

change of physical quantities is given by the sum of two components: the rate of growth of population, g, common to all sectors; and the rate of change of sectoral per-capita demands for final consumption commodities, r_i, specific to each sector. Since these are different from sector to sector, the whole structure of relative quantities is changing through time.

The equilibrium evolution of sectoral employment is determined both by the equilibrium dynamics of relative quantities and by that of VHI labour productivities.[33] Since r_i is different from $\rho_i'(t)$ (the rate of change of VHI labour productivity), and both are sector-specific, the whole structure of employment, i.e. the division of labour within the economic system, is continuously changing through time. This makes it clear how *sectoral reallocation* of employment is an essential requirement for the system to follow a full-employment path. It is worth noting that, in the most general case, a change in labour productivity in a single *industry* producing a basic commodity would change the *whole* structure of *sectoral* employment.

As regards relative prices, the equilibrium dynamics for the price of a unit of productive capacity for final consumption commodity i is particularly simple, due to the assumption that capital goods are produced by means of labour alone.[34] As a consequence, prices are completely determined by labour costs, and therefore their

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[33] In this case, we have:

$$\frac{\dot{L}_i(t)}{L_i(t)} = g + r_i - \rho_i'(t)$$

[34] It is given by:

$$\frac{\dot{p}_{k_i}^{(w)}(t)}{p_{k_i}^{(w)}(t)} \equiv \sigma_{k_i}^{(w)}(t) = -\rho_{k_i}$$

equilibrium evolution only depends on the changes of labour productivity in the industry producing the corresponding capital good.

On the contrary, the equilibrium path of consumption commodity prices reveals the consequences of the interaction between technical progress and changes in the distribution of income.[35] The rates of change of commodity prices are given by the weighted average of the rates of change of their two components, the first showing the univocally negative effect of increase in VHI labour productivity on production prices; the second quantifying the effect of a change in income distribution — through a variation in the sectoral profit rate — on the 'labour commanded' production prices.

As already stated, the first component reflects a necessary, physical self-replacement and expansion requirement; accordingly, its rate of change is completely determined by technology and equilibrium new investment, i.e. by the rate of change of VHI labour $\rho_i'(t)$. On the contrary, the second component of production prices also reflects income distribution. Accordingly, its rate of change depends not only on labour productivity in the capital goods producing industry, i.e. the rate of change of the price of the unit of productive capacity on which profits are computed, but also on the variation through time of sectoral rates of profit, i.e. on the rate of change of the profit differential with respect to the sectoral equilibrium rate of new investment.

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[35] In this case, we have:

$$\frac{\dot{p}_i^{(w)}(t)}{p_i^{(w)}(t)} \equiv \sigma_i^{(w)}(t) = -\rho_i'(t)\frac{\ell_i(t)}{p_i^{(w)}(t)} + \left(\frac{\dot{\pi}_i(t)}{\pi_i(t)-(g+r_i)} - \rho_{k_i}\right)\frac{(\pi_i(t)-(g+r_i))p_{k_i}^{(w)}(t)}{p_i^{(w)}(t)}$$

where $\sigma_i^{(w)}(t)$ is the rate of change of the relative price of commodity i when the numéraire is the wage rate.

Having described a full *set* of equilibrium paths, one for each possible realisation of the *sequence* of sectoral rates of profit — so far considered as exogenous magnitudes — this completes the description of the structural equilibrium dynamics of a growing economic system.

4. The 'natural' economic system

We are now in a position to introduce what Pasinetti calls the 'natural' economic system, i.e. that particular equilibrium path associated to one specific sequence of sectoral rates of profit, which, 'without recourse any longer to any exogenously given economic magnitude, now come to complete and close the whole relative price system of our theoretical scheme' (Pasinetti 1981, p. 131), due to the adoption of a particular *theory of the rate of profit*.

As explained earlier, the aim of Pasinetti's (1981) book is to develop a framework explaining the 'primary and natural' features of a growing economic system, independently of a particular institutional set-up. This, when coming to the issue of income distribution, would seem at first sight counter-intuitive, since the way in which income is distributed crucially depends on the character of the *social relations of production*, no less than on cultural, ethic, legal considerations; that is to say, precisely on the institutional set-up of society. In fact, all analyses taking income distribution as exogenous are clearly embedded in a specific institutional set-up. Then, how can a theory of the rate of profit be conceived that is independent of institutions?

As Pasinetti states, the 'natural' economic system deals with logical relations, based on magnitudes given from outside economic analysis (and therefore taken as exogenous), and emerging from the physical growth requirements of the

system. The problem must therefore be faced from this perspective: is there 'a natural rate of profit ... already *logically* implied in the previous theoretical framework *because the economic system* considered is a *growing one*' (Pasinetti 1981, p. 128, emphasis added)? The answer to this question is: yes.

The crucial point is that, at the pre-institutional stage, a theory of the rate of profit is *not* a theory of income distribution among income recipients, i.e. individuals or groups of individuals. This is because the very definition of the categories among which the purchasing power generated in the process of production is to be distributed essentially depends on the social relations of production of a particular institutional set-up. However, the very nature of an industrial system requires a separation between the means of production that enter a circular process and the set of commodities that are left out from the circular flow, once they are produced. Moreover, when the system is a growing one, the new investment requirements become a *necessary* expansion of the means of production.

Hence, prices of production must on the one hand be precisely those exchange ratios that satisfy the conditions of re-production and growth, i.e. *including* equilibrium accumulation of the means of production. But given that equilibrium requirements to expand productive capacity differ among sectors, the surplus factor in the price of production of each consumption commodity must reflect this difference.

On the other hand, prices of production provide for the purchasing power both to self-replace and expand productive capacity, and to consume those commodities not re-entering the circular flow. Consider that profits and wages just establish the amount of purchasing power that must be channeled to demand for means of production to expand productive capacity and to demand for final consumption commodities respectively. In this sense, profits and wages would

establish a truly *functional* distribution of income, as they stand for categories that channel purchasing power for different economic *functions*, arising from the conditions of production of physical quantities; specifically from the need to separate what enters the circular flow (being used as means of production) from what does not (being consumed).

As a consequence, from the above reasoning, it follows that profits must correspond to the purchasing power necessary for the equilibrium expansion of productive capacity in each VHI sector to take place: $\pi_i^*(t) = \pi_i^* = g + r_i$.[36] The equilibrium configuration corresponding to this structure of the rates of profit is the only one keeping the analysis at a strictly pre-institutional level.

4.1 A pure labour theory of value

When $\pi_i = \pi_i^*$, the equilibrium solutions for consumption commodity prices and their rate of change through time can be written, respectively, as:

$$p_i^{(w)*}(t) = \ell_i(t), \quad \frac{\dot{p}_i^{(w)*}(t)}{p_i^{(w)*}(t)} = -\rho_i'(t) \quad (4.1)$$

Expression (4.1) highlights the main result of the present formulation: when labour is the *numéraire* commodity for the price system, and the rates of profit are the natural ones, prices, i.e. 'labour commanded' prices, come to be exactly equal to 'labour embodied'. Therefore, this theoretical scheme implies a generalisation of a *pure labour theory of value*, where the equality of 'labour commanded' and 'labour embodied' is achieved thanks to a 're-definition of the concept of 'labour

[36] Since in equilibrium $x_i(t) = k_i(t)$, this follows from:

$$\pi_i^*(t)p_{k_i}(t)k_i(t) = (g + r_i)p_{k_i}(t)x_i(t), \quad t \geq 0 \quad (N.13)$$

embodied', which must be intended as the quantity of labour required directly, indirectly and hyper-indirectly to obtain the corresponding commodity as a consumption good' (Pasinetti, 1988, pp. 131-132).

With the introduction of the 'natural' rates of profit, both the value of productive capacity for self-replacement and the profits computed on the value of existing productive capacity perform the function of 'computing amounts of labour indirectly required elsewhere in the economic system for the equilibrium production of consumption good *i*' (Pasinetti 1981, p. 132). Not less importantly, this result holds *both* for the whole economic system *and* in each and every sector. Each growing subsystem following an equilibrium path of accumulation has a 'dual' value side that ascribes to natural prices a straightforward foundation, based on the 'basic principle of equal rewards for equal amounts of homogeneous labour' (Pasinetti 1981, p. 133).

Equally interesting, at the most general specification of technology, labour embodied in basic commodities produced by a single industry participate in the profits of all sectors. In this way, changes in the productivity of *labour* in one industry alter the value of *profits* of all sectors. Thus, it becomes clear that 'it is not the 'productivity of capital', or of any commodity, that turns out to be the *raison d'être* of the rate of profit. It is the growth, and the increasing productivity, of labour!' (Pasinetti 1981, p. 133).

4.2 Natural profits, wages, new investments and consumption

The relation between total natural profits and wages is a clearly asymmetrical one. Total national income produced in a specific time period, i.e. the *value* of total production at current prices, net of replacements, is distributed among total

(natural) profits and total wages. While the former emerge from the *physical* conditions for equilibrium growth as a necessity, if full employment and full capacity utilisation are to be maintained through time, the latter can be seen as a 'surplus', absorbing all the remaining national income. 'To produce, and to continually increase this 'surplus', through technical progress, is precisely the purpose of the whole production process' (Pasinetti 1981, p. 144).

In the same way, there is an asymmetric relation between total new investments and consumption. Total quantities produced in a specific time period, net of replacements, must be devoted in part to new investments and in part to final consumption. While the former are determined — by the structure of final demand for consumption and its evolution through time — as a *physical* requirement for equilibrium growth, the aggregate level of the latter can be seen as a 'surplus', absorbing all the remaining purchasing power.

As emerges by the very definition of natural profits, in the 'natural' economic system total profits will be equal to the value, at current prices, of total new investments; correspondingly, total wages will be equal to the value of total final consumption. But what is even more interesting is that this holds not as 'a mere over-all averaging-out result, but [as] the consequence of a whole series of equalities realised at each single sectoral stage' (Pasinetti 1981, p. 147).[37]

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[37] In fact, from condition (N.13) and from the expression for natural prices of consumption commodities (4.1), respectively, it follows that:

$$p^*_{k_i}(t)(g + r_i)a_{in}(t)x_n(t) = p^*_{k_i}(t)\pi^*_i x_i(t)$$
$$p^*_i(t)a_{in}(t)x_n(t) = w(t)L_i(t)$$

As a consequence, the value, at current prices, of total quantities, net of replacements, produced in each sector equals the total income it generates, i.e.:

$$p^*_i(t)a_{in}(t)x_n(t) + p^*_{k_i}(t)(g + r_i)a_{in}(t)x_n(t) = w(t)L_i(t) + p^*_{k_i}(t)\pi^*_i x_i(t)$$

A straightforward implication is that any price reduction due to increases in labour productivity immediately translates into a corresponding increase in the real purchasing power of wages, whatever the *numéraire* of the price system.[38] Hence, within the 'natural' economic system, the dynamics of the wage rate and the sectoral rates of profit have two different orders of magnitude (Pasinetti 1981, p. 143). The *level* of each π_i^* is given by two constant rates of change, while the *rate of change* of the *real* wage rate follows the dynamics of labour productivity in the

[38] In fact, any commodity or composite commodity can be chosen as the *numéraire* of the price system; analytically, this amounts to setting its price equal to unity, and keeping it constant through time. For example, if commodity h is chosen as the *numéraire*, we set $p_h^{(h)}(0) = 1$ and $\sigma_h^{(h)}(t) = \sigma_h^{(h)}(0) = 0$. Symmetrically to $\sigma_i^{(w)}$, which is the rate of change of the relative price of commodity i when the *numéraire* is the wage rate, $\sigma_h^{(h)}$ is the rate of change of the relative price of commodity h when the *numéraire* is the price of commodity h itself. Once the *numéraire* is specified, the wage rate has to be expressed in terms of it; this again means closing two degrees of freedom, i.e. we have to set both the wage rate at time zero and its rate of change in terms of the chosen *numéraire*. Within the 'natural' economic system, again taking commodity h as the *numéraire*, this means setting $p_h^{(h)*}(t) = w(t)\ell_h(t) = 1$, from where we obtain $w^{(h)}(t) = (\ell_h(t))^{-1}$ and therefore:

$$\begin{cases} w^{(h)}(0) = (\ell_h(0))^{-1} \\ \dfrac{\dot{w}^{(h)}(t)}{w^{(h)}(t)} \equiv \sigma_w^{(h)}(t) = \rho_h'(t) \end{cases}$$

the rate of change of the price of any consumption commodity i being given by:

$$\frac{\dot{p}_i^{(h)*}(t)}{p_i^{(h)*}(t)} = \rho_h'(t) - \rho_i'(t)$$

The rate of change of the wage rate in terms of the chosen *numéraire* — the *real* wage rate — is thus given by the rate of increase of labour productivity in the corresponding sector, whereas the rate of change of the price of commodity i, in terms of the chosen *numéraire*, is given by the difference of the rate of change of labour productivity in the corresponding sector with respect to the rate of change in VHI labour productivity in the sector producing the *numéraire* commodity.

VHI sector producing the commodity chosen as *numéraire*.[39] 'In the long run, therefore, while the real wage rate will persistently grow, the rate(s) of profit cannot but roughly remain at the same level' (Pasinetti 1981, p. 143).

4.3 Natural structural dynamics

By closing the relative price system with the 'natural' rates of profit, we are actually closing the last degree of freedom left open at the end of section 3. The resulting equilibrium path of sectoral relative physical quantities, employment, final consumption commodity prices and the prices of the units of VHI productive capacity, constitute the complete description of the 'natural' economic system. This description can be completed by noting that the only explicitly different analytical formulations, with respect to the general case, are given by expression (4.1) concerning final consumption commodity prices. The relative physical quantity system and sectoral employment would apparently be the same irrespective of the particular rates of profit chosen.

However, this result is a consequence of the fact that, in this framework, demand coefficients are taken as given. But the structure of demand is strongly dependent on consumers' real income, which in turn is determined by the structure of relative prices and therefore also by the ruling rate(s) of profit. To be more precise, therefore, we are not considering the $a_{in}(t)$'s as exogenous but we are considering as exogenous the mechanism by which changes in income distribution

[39] When the hypothesis of steady rate of change of per capita demand for consumption commodity i is removed, the natural rates of profit are no more exactly constant through time, but shall exhibit a roughly constant trend.

modify the structure of final consumption demand (and therefore of relative physical quantities). Such a mechanism is constantly at work.[40] Demand coefficients shall therefore change according to the particular configuration of the rate(s) of profit.

5. A methodological note

Before concluding, it is useful to reconsider Pasinetti's (1981) notion of equilibrium: 'A situation of equilibrium will simply be taken to mean a situation in which there is full employment of the labour force and full utilisation of the existing productive capacity' (Pasinetti 1981, pp. 48-49). The fact of referring to a '*situation* of equilibrium' is not trivial: the choice of the term highlights the transitory character of any equilibrium position eventually reached at a certain point in time. In fact, 'no connotation of automatism and no association with any particular adjustment mechanism is intended to be implied by such an expression' (Pasinetti 1981, p. 48).

The 'natural' economic system is by no means an attempt at describing the functioning of an actual capitalist system; it is an attempt at singling out the 'primary and natural' features of an *industrial* system, i.e. those 'necessary requirements for equilibrium growth' (Pasinetti 1981, p. 25). Equilibrium growth, however, entails neither the identification of a 'normal position' towards which the system tends in the long run — since the very structural dynamics of the economic system makes it impossible to identify a 'normal position' *persistent* enough to the continuous changes in the system's proportions — nor a logical *succession* of temporary equilibria spontaneously realised.

[40] See Pasinetti (1981, pp. 71-77).

The equilibrium dynamics defining the 'natural' economic system specifies the re-proportioning of productive capacity, relative quantities — and therefore sectoral employment — and relative production prices *necessary to comply with* the ever-changing structure of final demand for consumption goods and with the pace of technical progress. It is important to stress that this process of re-proportioning is not *spontaneous* but must be actively pursued if the new situation of equilibrium is to be reached period after period.

Furthermore, the empirical point of departure of the analysis must be explicitly mentioned: 'The coefficients that appear ... in the present (vertically [hyper]integrated) analysis must ... be interpreted as representing those physical quantities which can actually be observed' (Pasinetti 1981, p. 110), to which there corresponds — for *each* time period — a specific equilibrium situation. These equilibrium situations, together with the *necessary* dynamic conditions connecting them through time, establish a 'normative configuration'. In this sense, therefore, the 'natural' economic system is a 'norm; and the norm is always there — even if it is not so much apparent — in the *short* no less than in the long run' (Pasinetti 1981, p. 127n, added emphasis).

Since the whole structure of physical quantities and technical production requirements are continuously changing through time, the problem arises of how to perform a truly dynamic analysis, connecting equilibrium situations with completely different characteristics. Pasinetti solves the problem by developing the analytical device of vertical hyper-integration:

> 'By resolving all varieties of products into the same constituent elements — a flow of labour and a stock of capital goods both expressed in physical terms — the vertically [hyper]integrated approach leads to relations whose permanence over time is independent of specific technical possibilities' (Pasinetti 1981, p. 116).

It is worth stressing, however, that vertical hyper-integration is not a mere analytical device for making dynamic analysis possible; it also has a very important conceptual role within the development of the present theoretical framework — consider, for example, its role in the redefinition of the concept of 'labour embodied' in the theory of value implied by the 'natural' economic system.

6. Concluding remarks

The aim of this article has been to provide a reading key into controversial issues in the interpretation of Pasinetti's (1981) book. Tough reviews have been written[41] and some aspects remained unclear, due both to ambiguities in Pasinetti's exposition and to the fact that at the time of publication some concepts were still to be fully conceptualised by Pasinetti himself.[42]

One of the main tasks has been, first of all, the clarification of the pre-institutional character of Pasinetti's (1981) framework. Closely connected to this, the distinction between 'capitalist' and 'capitalistic' economic systems was stressed. Secondly, we separated Pasinetti's (1981) *general dynamic analysis* from his *equilibrium analysis*, a distinction which clearly emerges from the understanding of Pasinetti's particular notion of equilibrium situation. Third, we emphasised how

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41 Amongst these, see Asimakopulos (1982), Eltis (1982), Kregel (1982), Schefold (1982), Rymes (1984), Shapiro (1984), and Taylor (1995).

42 The development of the concept of vertical hyper-integration went through different stages. Pasinetti's (1981) book has been the final result of a process that began with his Doctoral dissertation at the University of Cambridge (Pasinetti 1962), partially published in Pasinetti (1965), and was itself an intermediate step towards the analytical elaboration of vertical hyper-integration, accomplished in Pasinetti (1988).

important is the *induced* character of new investment demand within such an equilibrium situation.

A further step, going beyond the identification of equilibrium trajectories, has been to discuss the definition of the 'natural system', which is the specific equilibrium path corresponding to a pre-institutional theory of income distribution. In this context, the 'natural rates of profit' are actually the *new investment rates* corresponding to equilibrium capital accumulation.

Finally, we have stressed the vertically *hyper*-integrated character of Pasinetti's (1981) book and in this way connecting it to Pasinetti's (1988) generalisation, an analytical step whose importance has not been fully grasped in the literature concerning structural change.

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Appendix

The basic notation in Pasinetti's *Structural Change and Economic Growth* is as follows:

$x_i(t)$	number of units of final consumption commodity i produced during time period t in the VHI sector i;
$x_{k_i}(t)$	gross investment in the VHI sector i, i.e. number of units of productive capacity for final consumption commodity i produced during time period t;
$x_n(t)$	total units of labour available at the beginning of time period t;
$p_i(t)$	price of a unit of final consumption commodity i during time period t;
$p_{k_i}(t)$	price of a unit of productive capacity for final consumption commodity i during time period t;
$w(t)$	wage rate during time period t;
$\pi_i(t)$	profit rate of the industry producing final consumption good i during time period t;
$a_{in}(t)$	average per capita demand for final consumption commodity i during time period t;
$a_{k_in}(t)$	average per capita demand for units of productive capacity for final consumption commodity i during time period t;
$a_{ni}(t)$	direct labour requirements for the production of one unit of final consumption commodity i during time period t
$a_{nk_i}(t)$	direct labour requirements for the production of one unit of productive capacity for final consumption commodity i during time period t;
T_i	reciprocal of the coefficient of wear and tear of one unit of productive capacity for final consumption commodity i;
$x'_{k_i}(t)$	demand for units of productive capacity for final consumption commodity i for replacement of worn out capacity during time period t;
$x''_{k_i}(t)$	net investment in the VHI sector i, i.e. new investment demand for units of productive capacity for final consumption commodity i during time period t;
$k_i(t)$	stock of units of productive capacity for the VHI sector i available at the beginning of time period t;
$\chi_i(t)$	capital/output ratio at current prices in time period t for VHI sector i;