Availability-driven optimal design of shared path protection in WDM networks

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Abstract

Availability, defined as the fraction of time a network service is operative, is a key network service parameter. Dedicated protection increases availability but also the cost. Shared protection instead decreases the cost, but also the availability.

In this paper, we formulate and solve an integer linear programming (ILP) model for the problem of minimizing the backup resources required by a shared-protected static optical network whilst guaranteeing an availability target per connection. The main research challenge is dealing with the non-linear expression for the availability constraint. Taking the working/backup routes and the availability requirements as input data, the ILP model identifies the set of connections sharing backup resources in any given network link.

We also propose a greedy heuristic to solve large instances in much shorter time than the ILP model with low levels of relative error (2.49% average error in the instances studied) and modify the ILP model to evaluate the impact of wavelength conversion.

Results show that considering availability requirements can lead up to 56.4% higher backup resource requirements than not considering them at all, highlighting the importance of availability requirements in budget estimation.

Keywords: optical networks, shared path protection, prioritized shared protection, availability-guarantees, integer linear programming, heuristic, WDM networks.

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1 Introduction

The high data transmission capacity of current fiber optic communication systems (on the order of 50-100 Tbps per fiber, depending on the transmission distance, on a single fiber core [2]) has made them the de *facto* technology of transport networks, the core of the Internet. This high data transmission capacity is achieved by the simultaneous transmission of optical signals in the same fiber. To achieve simultaneous transmission, every optical signal in a fiber is transmitted using a different wavelength. This technique is known as Wavelength Division Multiplexing (WDM), and the networks that use it for transmission are known as WDM networks. Typically, 40-80 wavelengths are used in currently installed systems (using 100 GHz and 50 GHz spectral spacing in the C band, respectively, according to the ITU-T recommendation G.694.1 [7]), although experimental demonstrations with hundreds of wavelengths have also been carried out.

As a result of the high data transmission capacity of WDM networks, the growing access to the Internet by the world population and the continuous emergence of new networked multimedia applications, the traffic transported by the Internet increases significantly each year: annual traffic growth between 25 and 70% has been reported [9], with an accumulated growth of 887.9% in the period 2000-2016 [18] and more than 3566 million Internet users around the world on June 30, 2016.

The success of the Internet has led many activities to be highly dependent on the good quality of the service that the network can provide. This, in turn, has triggered much research activity aiming to ensure that the network is planned in such a way that the relevant performance metrics are optimized. Therefore, several optimization methods for solving resource assignment problems [3, 8] and survivability provisioning [3, 8, 17] have been used. Normally, the aim of these planning tasks is minimizing the network cost whilst considering constraints on one or more quality service metrics [11, 12].

Among the different quality metrics considered in the network planning and design process, availability is one of fundamental importance as it measures the capacity of the network to be operative in spite of the occurrence of failures. In fact, most SLAs (Service Level Agreements) established between network providers and their customers specify a minimum level of network availability, defined as the fraction of time the network service is required to be operative.

Sometimes, the required level of availability for a given connection (an optical communication channel between a source and destination node) can be provided with a single working connection (the connection normally used to transmit information). In a WDM network, a connection is established using a lightpath, defined by the route and the wavelength used along that route (if the network nodes are equipped with full wavelength conversion capability, different wavelengths can be used along the route). However, the usual situation is that the availability provided by a single lightpath is not enough and then one (or more) backup lightpaths must be established. Thus, when a failure renders the working lightpath inoperative, the information can still reach the destination node using a backup lightpath. As a result, in spite of the occurrence of a failure, the user perceives the network service as operative. This type of protection is known as dedicated path protection.

The backup lightpaths of connections using dedicated path protection are seldom utilized to transmit information. For example, according to the failure statistics reported in [22], a 1609.34[Km] working lightpath would fail, in average, 0.012 times every year. If the cut occurs in a terrestrial link, its reparation time would take 12 hours. This leads to an average downtime of 8.64 min per year. As a result, if a backup lightpath was necessary (due to the availability requirements established in the SLA), then it would be used just 8.66 min per year, being idle the rest of the time. From the network operator perspective, given that network resources (as backup lightpaths) are expensive, having them reserved but idle should be avoided as much as possible.

To increase the backup network resource usage, a technique called shared protection is used. Under shared protection, different connections share their backup network resources. Thus, backup resources are used a higher fraction of time as they act as a backup of several connections. However, this comes at the expense of a decreased availability with respect to dedicated protection, highlighting a trade-off between availability and backup resource usage. Thus, shared protection might not be an attractive solution for services for which network connectivity is a mission-critical factor (e.g., banks or stock markets) but it could be a very good option for customers not affected by lower levels of availability (residential customers, educational institutions, mining companies, etc).

The number of backup lightpaths and the type of protection provided to the connections (dedicated or shared) must be defined during the network planning stage. To do so, two different approaches are commonly used: a) providing every network connection with the same number of backup lightpaths or type of protection, as in [14, 15, 19] or b) providing every network connection with the number of backup lightpaths or type of protection that ensures the level of availability required by that connection. The latter approach is known as availability-guaranteed or availability-aware network planning process and the task of actually establishing the corresponding working and backup lightpaths is carried out by availability-aware provisioning algorithms.

Availability-aware provisioning algorithms can be classified as dynamic or static. Dynamic ones must establish the working/backup lightpaths on demand and thus, there is not much time to compute the lightpath allocation. For that reason, heuristics are the most used approach to solve the problem, as in [10, 13, 16, 20]. In a static scenario instead, the set of connections to establish are known *a priori* and there is enough time to run optimization techniques such as integer linear programming (ILP) models. In [23, 24] an ILP model was proposed to solve the problem of minimizing the number of backup wavelengths with availability guarantees. However, only the dedicated protection case is analyzed. The complexity that a shared protection scheme introduces in the ILP model is not addressed. The same case of dedicated protection with availability guarantees is also studied in [25, 26], but only heuristic approaches are applied. Finally, in [4] an ILP to evaluate the wavelength requirements of shared and dedicated protected connections is studied. To deal with the non-linear expression for the availability constraint, such an equation is dropped and the objective function is changed to maximize the availability of paths. In all the static cases, full wavelength conversion is assumed.

Table 1 summarises the references on availability-guaranteed provisioning.

Table 1: Comparison of previous works on availability-guaranteed provisioning.

REF	Shared	Dedicated	Method	Scenario	Types of failure	Full wavelength conversion ?
[20]	Yes	Yes	Heuristic	Dynamic	Double link failure	Yes
[10]	Yes	No	Heuristic	Dynamic	Double link failure	Yes
[13]	Yes	No	Heuristic	Dynamic	Multiple	Yes
[16]	Yes	No	Heuristic	Dynamic	Multiple (link & node) failures	No
[25]	No	Yes	Heuristic	Static	Multiple	Yes
[26]	No	Yes	Heuristic	Static	Multiple	Yes
[4]	Yes	Yes	• ILP • Heuristic	Static	Multiple	Yes
[23]	Yes	Yes	• ILP (only for unprotected and dedicated protected connections) • Heuristic (for unprotected, dedicated and shared protected connections)	Static	Multiple	Yes
[24]	No	Yes	ILP (only for unprotected and dedicated protection connections)	Static	Up to 10 link failures	Yes

In this paper we address the problem of determining the backup wavelength requirements of a static optical network operating under shared protection. Our contributions to this already studied problem are: a novel way of dealing with the non-linear expression for the availability constraint and derivation of new numerical results based on that constraint; studying the impact of a new availability-aware priority mechanism on availability equations and results; defining variables that allow identifying which wavelength is shared by which connections in every network link; and a new set of equations capturing the wavelength continuity constraint and its impact on the number of backup wavelengths. We address the problem of planning a static WDM network with availability guarantees under shared protection as follows:

- First, the set of connections that are to be allocated a backup lightpath to meet the availability requirement is determined.

- Next, for the set of connections determined in the first step, we formulate and solve an ILP that aims at minimizing the total number of backup wavelengths such that the availability of every connection meets a given target value. The input data are the working/backup routes and the availability requirements of the set of connections determined in the first step. The proposed ILP model considers an availability-based priority scheme for the access to the shared resources and also allows identifying which connections can share backup resources in every network link.

- Finally, a heuristic approach to solve the same problem in a shorter time is implemented and a modification of the original ILP to consider the wavelength continuity constraint is presented.

Unlike previous work in the area of shared protection in WDM networks, that only provide the number of backup wavelengths required by the whole network but not how to exactly configure the backup resources of each link, we provide essential information about the set of connections that share a backup wavelength in every network link. In this way, we produce results relevant to the network operators who need to know how to exactly configure their backup wavelengths to provide the availability requested by the customers (as just providing the number of backup resources does not allow one to configure the network). To diminish the amount of information that must be processed, instead of storing connection-by-connection information, we store link-by-link information.

The rest of the paper is organized as follows. In Section 2 the network and failure models are presented. In Section 3 dedicated and shared path protection schemes are described. Next, in Section 4, the expression for availability of connections operating under shared path protection scheme is derived. Then, the proposed ILP model and the greedy heuristic are given in Sections 5 and 6, respectively. Section 7 presents the numerical results obtained with the ILP model and the heuristic along with upper and lower bounds on the number of backup wavelengths required. Finally, Section 8 concludes this work.

2 Network and failure model

The network is represented by a graph $G = (N, L)$ where N is the set of nodes and L is the set of unidirectional links. The number of elements in the sets N and L are denoted by $|N|$ and $|L|$, respectively.

Every network link $l \in L$ can be either in the operative or non-operative state. The mean time link l is in the operative state is denoted by $MTTF_l$ (Mean Time To Failure). The mean time link l is under failure is denoted by $MTTR_l$ (Mean Time To Repair). The mean time between two consecutive failures on link l is denoted by $MTBF_l$ (Mean Time Between Failure) and is equal to the sum of $MTTF_l$ and $MTTR_l$. We assume that network links fail independently and that the network is in the steady state. Therefore, the probability that link l is in the operative state, denoted by p_l, is given by

$$p_l = \frac{MTTF_l}{MTTF_l + MTTR_l} \tag{1}$$

According to the statistics reported in [22], the unavailability due to link failures is three orders of magnitude higher than that due to equipment failures at the nodes. Therefore, in this paper it is assumed that a lightpath may not be available due to link failures only.

Let C be the set of all point-to-point connections that might be established in the network. The maximum number of elements in C is equal to $|N|(|N|-1)$, and each one is defined by its source and destination node. Once established, we assume that a connection remains in place for a long time (weeks to years), configuring a static scenario. To establish a connection at least one working lightpath must be implemented. Let LP_c^W be the working lightpath of connection c. LP_c^W is defined by: a) a route (given as input data), denoted as R_c^W, corresponding to the ordered set of links starting at the source node and ending at the destination node of connection c and b) a wavelength in each link in R_c^W (not necessarily the same, if full wavelength conversion is assumed).

If A_c^{Target}, the target availability required for the connection c, cannot be provided by the working lightpath alone, then a backup lightpath must be implemented. Let LP_c^B be the backup lightpath of connection c, defined by a route (given as input data) and a wavelength in each link. The set of links used by LP_c^B is denoted by R_c^B and we define C_l as the set of connections whose backup routes use the link l.

To ensure that a link failure does not affect simultaneously the working and backup lightpaths of a connection, R_c^W and R_c^B are assumed to be link-disjoint.

3 Dedicated and shared path protection

In a WDM network it must be ensured that two lightpaths in the same link do not use the same wavelength. Besides this requirement, additional constraints must be met depending on the type of protection used. They are described in the next subsections.

3.1 Dedicated path protection

In this case, resources (wavelengths in links) must be exclusively reserved in every link of the routes of working and backup lightpaths. Wavelengths reserved in the working routes are used under normal operation and wavelengths reserved in the backup routes are only used when the corresponding working route fails.

As a way of illustration, in Figure 1 a 9-node mesh network with two established connections (solid black and gray lines) is shown. The working lightpath of connection 1 (from node 0 to node 2) follows the route made of the nodes 0-1-2 whilst the working lightpath of connection 2 (from node 3 to node 5) follows the route made of the nodes 3-4-5. The backup lightpaths of connections 1 and 2 follow the routes made of nodes 0-3-6-7-8-5-2 and 3-6-7-8-5, respectively (dashed lines).

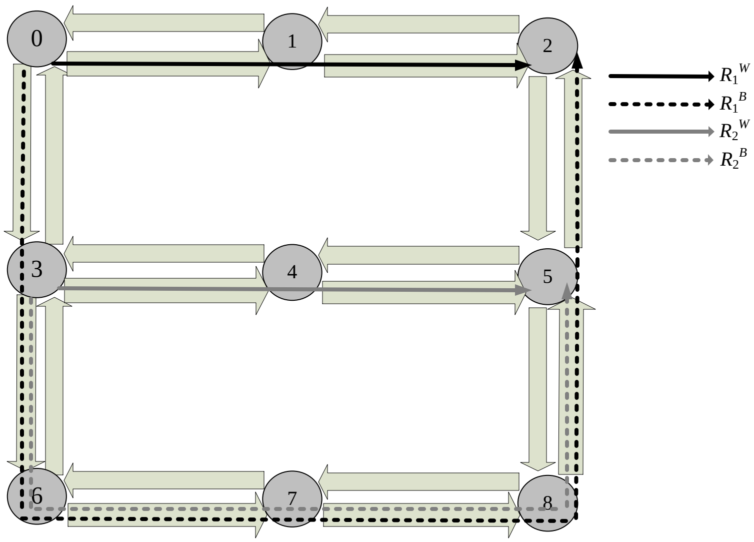


FIG. 1: Working and backup lightpaths for two connections.

Under this type of protection, it must be ensured that lightpaths with common links (e.g., both backup lightpaths in Figure 1 use links 3-6, 6-7, 7-8 and 8-5) use different wavelengths.

3.2 Shared path protection

In this case, the same backup wavelength can be shared among several backup lightpaths. In the example of Figure 1, the same wavelength in links 3-6, 6-7, 7-8 and 8-5 could be allocated as backup to both working lightpaths. As a result of sharing, wavelength contention might arise if more than one working lightpath sharing backup resource fails. In that case, a backup wavelength could be required to be used by more than one backup lightpath simultaneously.

In case of a single link failure, backup wavelength contention is eliminated if the corresponding working lightpaths of the connections sharing backup wavelengths are link-disjoint. In case of multiple link failures, backup wavelength contention cannot be efficiently eliminated and thus, a priority system for the access to the backup wavelengths must be defined.

As a way of illustration of a backup wavelength contention situation, consider Figure 1. Assume that only link 0-1 fails. In that case, because working lightpaths are link-disjoint, only one of them is affected by the single link failure and no backup wavelength contention arises. Instead, if link 3-4 fails while link 0-1 is still under repair, backup wavelength contention arises as both working lightpaths require using the shared backup wavelengths in links 3-6, 6-7, 7-8 and 8-5. In that case, if connection 1 has higher priority than connection 2, the shared backup wavelengths will be allocated to connection 1 and connection 2 becomes non-operative.

In this paper we define the following availability-based priority system to deal with the problem of backup wavelength contention: among all working connections sharing the same backup wavelength, the connection with the lowest value of availability under dedicated protection has the highest priority to access the shared backup resources. The connection with the second lowest availability level under dedicated protection has the second highest priority to access the shared resource and so on. In this way, as shared protection reduces the availability of all connections sharing backup resources, the connections most likely to have difficulties meeting the target availability under shared protection are less affected by the availability reduction. Because of mathematical convenience, in this paper we assume that if $i > j$, then connection i has lower priority than connection j.

4 Availability of connections under prioritized shared protection

The availability of connection c operating with a protection scheme is denoted by A_c and is given by

$$A_c = \Pr\{LP_c^W \text{ is available}\} + \Pr\{LP_c^W \text{ is unavailable and } LP_c^B \text{ is available}\} \tag{2}$$

Denoting by A_c^W and A_c^B the availability of the working and backup lightpath for connection c, respectively, and since paths LP_c^W and LP_c^B are link-disjoint, then

$$A_c = A_c^W + (1 - A_c^W)A_c^B \tag{3}$$

The availability of a lightpath depends not only on the availability of the links along its route but also on the availability of the wavelength to be used. In this paper, we assume that a particular wavelength is unavailable for a given connection only when it is used by another connection or when the corresponding link is under failure (in which case, all wavelengths in that link become unavailable). Wavelength unavailability due to hardware malfunctioning (e.g., laser failure) is not considered here. Thus, the availability of the working lightpath R_c^W (where the wavelengths are always available unless the link fails) reduces to

$$A_c^W = \Pr\{LP_c^W \text{ is available}\} = \prod_{l \in R_c^W} p_l \tag{4}$$

The expression for $\Pr\{LP_c^B$ is available$\}$ under shared protection can be evaluated as

$$A_c^B = \Pr\{R_c^B \text{ and } R_i^W \text{ are available } \forall i \in X_c\} \tag{5}$$

where $X_c \subseteq X_c^{All}$. Here X_c^{All} is the set of all connections i that might share resources with connection c such that:

- The backup lightpaths of connections c and i that have at least one common backup link.
- Connection i has higher priority than connection c to access the common wavelength(s).
- The working lightpath of connection i does not have links in common with the backup lightpath of connection c.

Mathematically, X_c^{All} is defined as

$$X_c^{All} = \{i \in C : c > i \wedge R_i^B \cap R_c^B \neq \emptyset \wedge R_i^W \cap R_c^B \neq \emptyset\}$$

On the other hand, X_c is the set of connections that actually share resources with connection c.

Equation (5) states that, for the backup lightpath of connection c to be available, not only all the links along its route must be operative but also all the working lightpaths of connections sharing backup wavelengths with connection c and that have higher priority than connection c must be operative.

Note that (5) is an approximation, as it does not consider the low-probability case where the wavelengths for R_c^B are available although at least one working path i ($\forall i \in X_c$) is unavailable. This occurs when the working path of connection i failed, but also did its backup connection due to a failure in a link that is not shared with connection c.

Note also that the set R_c^W is link-disjoint with sets R_c^B and R_i^W ($\forall i \in X_c$), and that R_i^W is not necessarily link-disjoint with R_c^B. Then

$$A_c^B = \prod_{l \in (\bigcup_{i \in X_c} R_i^W) \bigcup R_c^B} p_l \tag{6}$$

According to this, the availability of connection c under shared protection depends on the set of shared connections X_c, and can be expressed as

$$A_c(X_c) = A_c^W + (1 - A_c^W) \times \prod_{l \in (\bigcup_{i \in X_c} R_i^W) \bigcup R_c^B} p_l \tag{7}$$

It can be seen that the key element to decide is the set of shared connections X_c for all $c \in C$. Dedicated path protection is obtained when $X_c = \emptyset$ for all $c \in C$, so the maximum availability is obtained, but this requires a large number of wavelengths available at each link. On the contrary, if we group as many connections as possible, then $X_c = X_c^{All}$. In such a case, several wavelengths can be saved, but the resulting availability A_c can be too low.

So, the natural problem to study is how to minimize the total number of backup wavelengths subject to guaranteeing a target availability for every network connection in a WDM network using shared protection. That is

$$\min \sum_{l \in L} (\text{number of backup wavelengths used in link } l) \tag{8}$$

$$\text{s.t. } A_c(X_c) \geq A_c^{Target}, \forall c \in C \tag{9}$$

Note that even for a given set of allowed shared connections X_c for all $c \in C$, to evaluate the minimum number of backup wavelengths required in each link is an NP-hard problem. In fact, this problem is equivalent to the NP-hard problem MINIMUM CLIQUE PARTITION (see Appendix A). In the following section, an ILP model that aims at minimizing the number of backup wavelengths used under shared protection is presented.

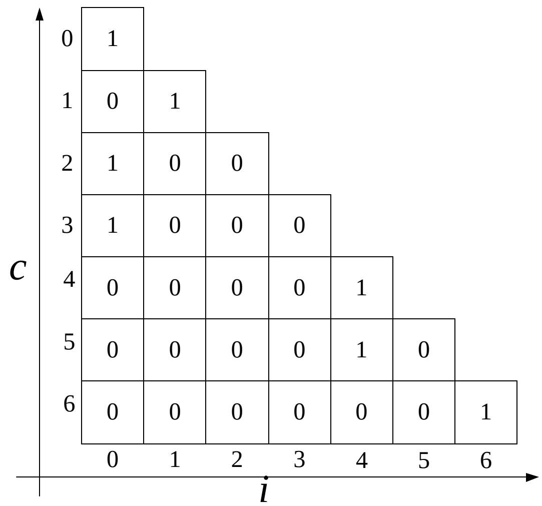


FIG. 2: Variable structure $y_{l,c,i}$ for a link l.

5 The integer linear programming model

Our ILP model uses binary decision variables $x_{c,i}$ for each $c \in C$ and $i \in C (i < c)$ such that $x_{c,i} = 1$ if $i \in X_c$ and $x_{c,i} = 0$ if $i \notin X_c$.

As the number of wavelengths required in each link cannot be obtained directly from the set X_c, it is necessary to decide, at each link, which connections share the same wavelength.

Let $y_{l,c,i}$ be a binary decision variable for $l \in L, c \in C_l, i \in C_l$ $(c > i)$, such that $y_{l,c,i} = 1$ when the backup lightpath of connection c shares a wavelength on link l with a higher-priority connection i. A unique representative connection is selected for each group of connections sharing a backup wavelength in a link. The representative connection is the one with the highest priority in the group. All remaining connections in a group only need to be associated to this representative connection to indicate they belong to the same group. Thus, if connections i, j and k (in descending order of priority) share a wavelength on link l, then connection i is the representative connection of this group. This is indicated by making $y_{l,i,i} = 1$. To indicate that connections j and k are part of this group we make $y_{l,i,j} = 1$ and $y_{l,i,k} = 1$. No more elements must be made equal to 1 to represent this group.

Figure 2 illustrates the behavior of variable $y_{l,c,i}$ for a determined link l. In the figure, the lower triangular section of matrix Y_l, of size $|C_l| \times |C_l|$, is shown, where $|C_l|$ is the number of connections whose backup routes use link l. The lower the number of the row/column, the higher the priority of the connection. In this particular example, the backup lightpaths of connections $0-6$ use the link under study, with connection 0 having the highest priority. Each element in the matrix represents the value of the binary decision $y_{l,c,i}$, with c having lower priority than connection i. The meaning of the elements of matrix Y_l is as follows:

- *Column*: the elements (c,i) in column i that are equal to 1 belong to a group of connections sharing a backup wavelength in link l whose representative connection (the connection with the highest priority) is connection i. In the example, we can observe 4 groups: *a*) the group represented by connection 0 that contains connections 0, 2 and 3, *b*) the group represented by connection 1 that only contains this connection (that is, connection 1 does not share backup wavelengths with any other connection in link l), *c*) the group represented by connection 4, containing connections 4 and 5 and *d*) the group represented by connection 6, containing only this connection.

- *Row*: each row can have only one element different from 0. For each row r, the column where a value equal to 1 is found corresponds to the identification of the connection representing the group to which connection r belongs. In the example, connections 0, 1, 2, 3, 4, 5 and 6 are represented by connections 0, 1, 0, 0, 4, 4 and 6, respectively.

The decision of selecting a representative for each group has been made in order to avoid symmetry problems in the final ILP model.

Note that the total number of backup wavelengths corresponds to the number of groups of connections sharing wavelengths in each link. Hence, we can count the number of wavelengths required at each link by the number of representative connections with $y_{l,c,c} = 1$. This allows formulating the objective function as

$$\min \sum_{l \in L} \sum_{c \in C_l} y_{l,c,c} \tag{10}$$

The selection of a representative connection at each link for each group is obtained by the following constraints:

$$\sum_{i \in C_l : c \geq i} y_{l,c,i} = 1 \qquad \forall l \in L, \forall c \in C_l \tag{11}$$

$$y_{l,c,i} \leq y_{l,c,c} \quad \forall l \in L, \forall c \in C_l, \forall i \in C_l : c > i \tag{12}$$

The first equation makes sure that, for a given link l and connection c, each connection i ($\forall i \in C_l$) can be either the representative connection of only one group of connections (that is, $y_{l,i,i} = 1$ and $y_{l,c,i} = 0$ for the rest of elements of row c such that $c > i$) or be part of at most one group represented by another connection c with higher priority (that is, $y_{l,c,i} = 1$).

The second expression ensures that, if the backup lightpaths of connections i and c share a wavelength at least in one link of their backup routes (that is, if $y_{l,c,i} = 1$), then c must be the representative connection of this group of backup lightpaths using this wavelength. That is, if $y_{l,c,i} = 1$, then $y_{l,c,c}$ must be also 1.

Note that although connections k and j can share a wavelength (then $x_{k,j} = 1$) $y_{l,j,k}$ can be equal to 0 if connection j is not the representative connection of this group.

To ensure that the decision variable $y_{l,c,i}$ allows modeling the grouping of connections per link in a correct way, we require to add constraints linking variables $y_{l,c,i}$ and $x_{c,i}$.

If $i \notin X_c$ ($x_{c,i} = 0$), then $y_{l,c,i} = 0$ for every link l. This is forced with the next constraint:

$$y_{l,c,i} \leq x_{c,i} \quad \forall l \in L, \forall c \in C_l, \forall i \in C_l : c > i \tag{13}$$

Note that sharing a wavelength is not a transitive operation. For example if $x_{i,j}=1$, $x_{i,k}=1$, and if backup routes R_j^B and R_k^B are disjoint, then $x_{j,k}=0$. However, if the backup lightpaths of three connections have in common at least one link (also that their primary routes R_i^W, R_j^W and R_k^W are disjoint), then in this case the transitivity applies: that is $x_{i,j}=1$, $x_{i,k}=1$ then $x_{j,k}=1$, if $j\in X_i$, $k\in X_i$ and $k\in X_j$. Hence, we require to decide the sharing of wavelengths at a link level. Hence, the following condition must be met:

$$y_{l,i,k}+y_{l,j,k}\leq 1+x_{i,j}\quad \forall l\in L,\forall k\in C_l,\forall i\in C_l,\forall j\in C_l : j<i\wedge k<i\wedge k<j$$

This constraint forces $x_{i,j}$ to 1 if the connections i and j are in a group represented by connection k. This inequality also implies that if $x_{i,j}=0$, then the connection i or connection j might be part of the group represented by connection k, but not represented by i and j. Note that in this case ($x_{i,j}=0$), connection i could belong to the group represented by connection k and not containing connection j at a given link, whilst in another link connection j could belong to the group represented by and not containing connection i. This is equivalent to changing the backup wavelength used along the backup routes.

Variables $x_{i,j}$ require additional bound constraints. The first, for variables $x_{c,i}$, is to set $x_{c,i}=0$ if the connections c and i can not share wavelengths (because their backup routes are disjoint or their working routes are not disjoint). This is formulated as

$$x_{c,i}=0\quad \forall c\in C,\forall i\in C : c>i\wedge i\notin X_c^{All} \tag{14}$$

Finally, to model the availability constraint in (9) -recall that p_l is the probability that link l is in operative state-, we can reorder the terms in (7) and apply the logarithmic function to each term, to obtain

$$\sum_{l\in(\bigcup_{i\in X_c} R_i^W)\bigcup R_c^B} \log(p_l)\geq \log\left(\frac{A_c^{Target}-A_c^W}{1-A_c^W}\right)\quad \forall c\in C \tag{15}$$

Considering that the expression (15) is only for connections operating with both working and backup lightpaths, the values for A_c^{Target} must satisfy

$$A_c^W<A_c^{Target}<A_c(\emptyset) \tag{16}$$

The condition $A_c^W<A_c^{Target}$ establishes that at least a backup lightpath is required to satisfy the target availability. The condition $A_c^{Target}<A_c(\emptyset)$ requires that the availability requirement must be achievable operating under dedicated protection scheme.

To write (15) as a linear constraint, we require to select the set of links $(\bigcup_{i\in X_c} R_i^W)\bigcup R_c^B$ for X_c. To do so, we define a binary variable $z_{c,l}$ such that $z_{c,l}=1$ if $l\in(\bigcup_{i\in X_c} R_i^W)\bigcup R_c^B$. In order to do that, we require two set of inequalities:

$$z_{c,l}=1\qquad \forall c\in C,\forall l\in R_c^B \tag{17}$$

$$z_{c,l}\geq x_{c,i}\qquad \forall c\in C,\forall l\in R_i^W,\forall i\in C : c>i \tag{18}$$

Using these new variables, we can formulate (15) as

$$\sum_{l\in L} z_{c,l}\times\log(p_l)\geq \log\left(\frac{A_c^{Target}-A_c^W}{1-A_c^W}\right)\quad \forall c\in C \tag{19}$$

In summary, the final integer linear programming model is given by

$$\min \sum_{l\in L}\sum_{c\in C_l} y_{l,c,c} \tag{20}$$

$$\text{s.t.} \sum_{l\in L} z_{c,l}\times\log(p_l) \geq \log\left(\frac{A_c^{Target}-A_c^W}{1-A_c^W}\right), \forall c\in C \tag{21}$$

$$z_{c,l} = 1 \qquad , \forall c\in C, \forall l\in R_c^B \tag{22}$$

$$z_{c,l} \geq x_{c,i} \qquad , \forall c\in C, \forall i\in X_c^{All} \tag{23}$$

$$, \forall l\in R_i^W : c > i$$

$$x_{c,i} = 0 \qquad , \forall c,i\in C : i\notin X_c^{All}, c>i \tag{24}$$

$$y_{l,c,i} \leq x_{c,i} \qquad , \forall l\in L, \forall c,i\in C_l : c>i \tag{25}$$

$$\sum_{i\in C_l: c\geq i} y_{l,c,i} = 1 \qquad , \forall l\in L, \forall c\in C_l \tag{26}$$

$$y_{l,c,i} \leq y_{l,i,i} \qquad , \forall l\in L, \forall c,i\in C_l : c>i \tag{27}$$

$$y_{l,i,k} + y_{l,j,k} \leq 1 + x_{i,j} \qquad , \forall l\in L, \forall k,i,j\in C_l : j<i\wedge k<i\wedge k<j \tag{28}$$

$$x_{c,i} \in \{0,1\} \qquad , \forall c,i\in C : c>i \tag{29}$$

$$y_{l,c,i} \in \{0,1\} \qquad , \forall l\in L, \forall c,i\in C_l : c>i \tag{30}$$

$$z_{c,l} \in \{0,1\} \qquad , \forall l\in L, c\in C_l \tag{31}$$

6 A greedy heuristic

The previous ILP model can be hard to solve, especially on large instances of the problem. To be able to solve large instances, in this section we present a heuristic solution. Such solution is obtained using a greedy algorithm that, starting from $X_c = \emptyset$ for all $c\in C$, iteratively groups two connections (i.e., $x_{c,i} = 1$), reducing the number of required wavelengths, until no more grouping is possible without violating the required availability for each connection A_c^{Target}.

Remember that given a set of allowed shared connections X_c, the availability of each connection $A_c(X_c)$ can be obtained directly from (7), but the minimum number of required wavelengths at each link is NP-hard to compute. Hence, we use a different objective to select the "best" pair of connections to be grouped in each iteration: for each pair of connections (c,i), the benefit obtained by sharing a backup wavelength is defined by

$$B_{c,i} = \left(A_i(X_i) - A_i^{Target}\right)\times\left|R_i^B \bigcap R_c^B\right| \tag{32}$$

where the first term of the right side corresponds to the difference between the target availability of connection i (the one with the lowest priority between c and i) and its resulting availability under shared protection (resulting from making connections c and i sharing a backup wavelength in all their common backup links). The higher the positive value of this difference, the higher the potential of backup wavelength sharing and thus, the higher the probability of including another connection in this group. The second term in (32) is the number of common links in the backup routes of connections c and i. Thus, the higher the value of this term, the higher the backup resource savings obtained by allowing these two connections to share backup resources.

Hence, the heuristic finds, at each iteration, the pair of connections (c,i) with $i \in X_c^{All}$ that attains the maximum benefit $B_{c,i}$ as a candidate to share backup wavelengths. Next, to ensure that all connections meet the availability constraint formulated in (9), for each iteration of the greedy algorithm the fulfillment of the following expression is evaluated:

$$A_i(X_i) \geq A_i^{Target} \tag{33}$$

where i is the connection with lower priority of the pair (c,i) (the availability of connection c does not change). If the condition established by (33) is not met, then the candidate is not considered part of the solution (i.e., $x_{c,i} = 0$) and it is not further considered in the following iterations of the greedy heuristic.

When there are no more candidates that meet the expression (33), the greedy algorithm stops and the solution for each variable $x_{i,j}$ is stored in matrix X.

Next, to correctly find the groups of backup routes that share the same wavelength in each link, i.e., to evaluate the minimum number of wavelengths required in each link, we require solving an ILP. But, as the values of X_c are fixed, the general problem becomes separable and thus, less complex. For each link $l \in L$, we solve

$$L_l(X) := \min \sum_{c \in C_l} y_{c,c} \tag{34}$$

$$\text{s.t. } y_{c,i} = 0 \quad , \forall c \in C_l, \forall i \in C_l : x_{c,i} = 0 \tag{35}$$

$$\sum_{i \in C_l : c \geq i} y_{c,i} = 1 \text{ , } \forall c \in C_l \tag{36}$$

$$y_{c,i} \leq y_{i,i} \quad , \forall c \in C_l, \forall i \in C_l : c > i \tag{37}$$

$$y_{i,k} + y_{j,k} \leq 1 \quad , \forall i,j,k \in C_l : j < i \wedge k < i \wedge k < j : x_{i,j} = 0 \tag{38}$$

$$y_{c,i} \in \{0,1\} \quad , \forall c \in C_l, \forall i \in C_l \tag{39}$$

which is equivalent to the ILP problem in the previous section for a fixed value of X_c for all $c \in C$.

The pseudocode of the heuristic is described in Algorithm 1.

7 Numerical results and discussion

The ILP model and the heuristic solution were solved using the IBM ILOG OPL Optimization Suite v6.3 in an Intel Xeon processor E5-2670 with 128 Gb RAM, with the total memory assigned for each instance.

The topologies studied are shown in Figure 3. The working route R_c^W for each connection $c \in C$ corresponds to the most reliable route between the source and destination node of c, assuming link failure independence. Similarly,

Algorithm 1 Greedy heuristic for backup wavelengths configuration for networks operating under shared protection scheme with availability requirement

Input: sets L, N and C.
Input: R_c^W, R_c^B for every $c \in C$.
Input: p_l, for every $l \in L$.
Input: A_c^{Target}, for every $c \in C$.
Input: X_c^{All}, for every $c \in C$.

Define $sw = 1$ (auxiliary variable)
Define X, such that $x_{c,i} = 1$, for every $i \in X_c^{All}$ and $x_{c,i} = 0$, if $i \notin X_c^{All}$.
Define $Candidates\_set$ as the set of pairs of connections (c,i) such that $x_{c,i} = 1$ (where connection c has lower priority than connection i)
while $sw = 1$ **do**
    (loop to modify matrix X to find the solution)
    $sw = 0$
    $B^{MAX} = 0$ (B^{MAX} is used to store the maximum benefit value, according to (32))
    $c^{MAX} = 0$, $i^{MAX} = 0$ (c^{MAX} and i^{MAX} are used to store the identification of the connections c and i corresponding to B^{MAX})
    for all $(c,i) \in Candidates\_set$ **do**
        (in this loop, the benefit of all candidates is evaluated and the next candidate to be incorporated into the solution is selected)
        $x_{c,i} = 1$
        Compute A_c, $B_{c,i}$
        (in the next, the availability requirement is evaluated for connection c, the connection with the lowest priority of the pair)
        if $A_c \geq A_c^{Target}$ **then**
            if ($B_{c,i} > B^{MAX}$) **then**
                (if the availability requirement is met for connection c and the benefit of candidate evaluated is higher than the previous candidates in $Candidates\_set$, then the variables B^{MAX}, c^{MAX}, i^{MAX} and $sw = 1$ are updated)
                $B^{MAX} = B_{c,i}$, $c^{MAX} = c$, $i^{MAX} = i$, $sw = 1$
            end if
        else
            (if the availability requirement is not met, then the candidate (c,i) is dropped from the list of possible candidates)
            Delete (c,i) from $Candidates\_set$
        end if
        $x_{c,i} = 0$
        Compute A_c
    end for
    if $sw == 1$ **then**
        (if any candidate meets the availability requirement, then the position associated with candidate (c^{MAX}, i^{MAX}) is changed to 1 in matrix X)
        $X[c^{MAX}, i^{MAX}] = 1$
    end if
end while
(the next loop is used to compute the total number of backup wavelengths)
$w_{total} = 0$
for all $l \in L$ **do**
    execute the ILP formulated in (34)-(39) for $l \in L$
    $w_{total} = \sum_{c \in C_l} y_{c,c}^l + w_{total}$
end for

Output: X (matrix whose element (i,j) is equal to 1 if connection i shares at least one backup wavelength with connection j, 0 otherwise)
Output: Y_l for each link (matrix with information about the groupings of connections sharing backup wavelengths in each link)
Output: w_{total} (the total number of backup wavelengths required)

each backup route R_c^B corresponds to the most reliable route that is arc-disjoint with R_c^W. However, we remark that the proposed methodology can be applied to any pair of working/backup routes, and other criteria can be applied to select these pairs of routes.

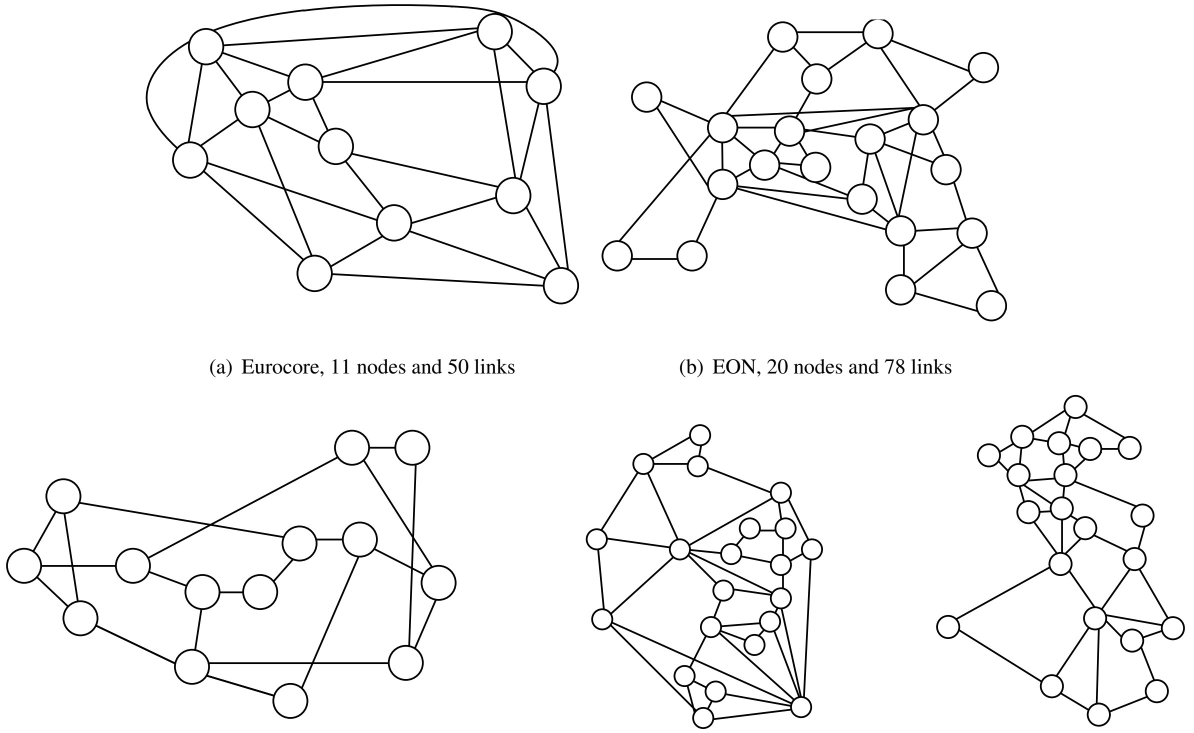
For each topology two scenarios were evaluated. In the first scenario (homogeneous scenario), all connections require the same value of availability with three possible values: 0.999, 0.9999, 0.99999. That is, $A_c^{Target} = A^{Target}, \forall c \in C$. In this scenario it might happen that some connections achieve the required availability using only the working lightpath and thus, they are excluded from further consideration (as they do not required backup resources). On the other hand, some connections could not achieve the required availability even with dedicated protection. They are also excluded from consideration. Therefore, the set of connections C (problem size) might have a number of elements lower than $|N| \times (|N|-1)$ as it is made of the connections that need to operate with exactly one backup lightpath to reach the target availability. Given the target availability, this set is unique.

In the second scenario (heterogeneous scenario), we use the concept of differentiated reliability (introduced in [5]), where every connection requires a different target availability, corresponding to a fraction of the availability obtained under dedicated protection (heterogeneous scenario). That is, $A_c^{Target} = \alpha \cdot A_c(\emptyset)$, $\forall c \in C$. In this case, no connection requires more than one backup lightpath to achieve the target availability. Thus, only those connections that achieve their required availability using just the working lightpath are excluded from the set C.

Tables 2 and 3 show the results for the homogeneous and heterogeneous scenarios, respectively. The first column of each table corresponds to the name of the network analyzed. Under the name, in brackets, the average number of hops of the working and backup lightpaths and the average availability of their links are given. The availability of each link, used to obtain the average availability of the network links, is evaluated according to (1). The MTTF of each link is calculated by multiplying the length of the link by the failure rate per length unit, equal to $2.73 \times 10^{-3}[\text{yr}^{-1} \times \text{km}^{-1}]$ for terrestrial links, as reported in [22], and equal to $1 \times 10^{-4}[\text{yr}^{-1} \times \text{km}^{-1}]$ for submarine links, as reported in [21]. The MTTR is assumed equal to 12[hr] and 336[hr] for terrestrial and submarine links, respectively.

The second column shows the target availability value (homogeneous scenario) or the value of α (heterogeneous scenario). The third column shows PS (problem size), equal to the number of connections considered in the problem (out of a total of $|N| \times (|N|-1)$). These are the connections that need to operate with exactly one backup lightpath to reach the target availability. The next four columns present the results in terms of number of backup wavelengths required in 4 cases (from left to right):

- LB: a scenario where the availability constraint in the ILP model is relaxed. That is, $X_c = X_c^{All}$ for all $c \in C$. Thus, this is a lower bound for the number of backup wavelengths.
- ILP: the results for the complete ILP model.
- Greedy: the results obtained by applying the greedy heuristic.
- UB: a scenario where full dedicated protection is used. That is, $X_c = \emptyset$ for all $c \in C$. Thus, this is an upper bound



(a) Eurocore, 11 nodes and 50 links

(b) EON, 20 nodes and 78 links

(c) NSFNet, 14 nodes and 42 links

(d) UKNet, 21 nodes and 78 links

(e) Italnet, 21 nodes and 72 links

FIG. 3: Studied topologies.

for the number of backup wavelengths.

In the ILP column, the percentage of additional resources required by the ILP model with respect to the LB, denoted by δ_{ILP}^{LB}, and the percentage of additional resources defined by the UB with respect to the ILP model, denoted by δ_{ILP}^{UB}, are given (first and second numbers between brackets, respectively). In the Greedy column, the percentage of additional resources required by the Greedy heuristic with respect to the ILP, denoted by δ_{Greedy}^{ILP}, and the percentage of additional resources required by the UB with respect to the Greedy heuristic δ_{Greedy}^{UB}, are also given.

From Table 2 it can be seen that in most cases, the problem size increases with the target availability value because connections that required only a working lightpath to achieve a low availability value need to include a backup lightpath to meet higher availability levels. However, it can also be seen that in some cases the problem size decreases with the target availability value because when such value becomes too high it is not possible to meet condition (16). Thus, those connections whose target availability cannot be achieved even with dedicated protection are removed from analysis. The only network where the problem size always increases with the target availability value is the UKNet topology, due to its high link reliability (because of short length of links). As a result, even high values of availability

Table 2: Number of backup wavelengths for the homogeneous scenario.

Topology $[\|\bar{R}\|; \bar{p_l}]$	A^{Target}	PS	Number of backup wavelengths			
			LB	ILP $[\delta_{ILP}^{LB};\delta_{ILP}^{UB}]$	Greedy $[\delta_{Greedy}^{ILP};\delta_{Greedy}^{UB}]$	UB
Eurocore [2.8; 0.9984]	0.999	98	124	124 [0; 44.1]	124 [0; 44.1]	222
	0.9999	110	126	126 [0; 48.7]	126 [0; 48.7]	246
	0.99999	76	84	98 [16.7; 38]	102 [4.1; 35.4]	158
EON [3.02; 0.997]	0.999	372	768	768 [0; 40.3]	768 [0; 40.3]	1286
	0.9999	298	502	555* [10.6; 38.2]	585 [5.4; 34.9]	898
	0.99999	52	58	74 [27.6; 30.2]	76 [2.7; 28.3]	106
NSFNet [3.25; 0.9981]	0.999	172	295	295 [0; 53.3]	295 [0; 53.3]	632
	0.9999	182	303	303 [0; 54.6]	311 [2.6; 53.4]	668
	0.99999	34	52	62 [19.2; 46.6]	66 [6.5; 43.1]	116
UKNet [3.25; 0.9995]	0.999	230	603	603 [0; 36.6]	603 [0; 36.6]	951
	0.9999	416	920	920 [0; 39.1]	920 [0; 39.1]	1511
	0.99999	420	922	922 [1; 38.8]	951 [3.1; 37.5]	1521
ItalNet [3.64; 0.9994]	0.999	294	988	988 [0; 32]	988 [0; 32]	1453
	0.9999	420	1115	1115 [0; 38.2]	1115 [0; 38.2]	1803
	0.99999	342	720	776* [7.8; 39.4]	791 [1.9; 38.3]	1281

(*) Best solution after memory limit.

can be achieved with a single backup lightpath.

It can also be seen that in 66% of the cases studied, the values in the LB and ILP columns are exactly the same, highlighting the fact that for low levels of availability requirements (e.g., 0.999) or for topologies with highly available links (e.g., UKNet) all backup wavelengths in a link can be shared without violating the availability requirements. In the remaining cases, the target availability is so high (0.99999) that to guarantee such value, the level of sharing of backup resources must decrease, leading to a higher requirement in the number of backup wavelengths. In these cases, the approach of simply sharing as much as possible, as usually done (e.g., [17]) is not good enough to ensure the required availability levels.

In Table 2 two cases where the optimal solution was not achieved, due to memory exhaustion, have been marked (with *). For these cases, the resulting GAP was of 1.2% for EON with $A^{Target} = 0.9999$ and 0.35% for ItalNet with $A^{Target} = 0.99999$. For the heterogeneous scenario, the optimal solution was reached in all cases.

Regarding the performance of the greedy heuristic, it can be seen that in the cases where it requires more backup wavelengths than the ILP solution, the maximum difference was just 15.7%. In Table 4 the running time (in seconds) for the ILP (RTI column) and the Greedy heuristic (RTG) is given. It can be seen that in most cases, the ILP solver takes much shorter times than the heuristic. However, there are a few cases where the ILP solver takes very long times (between 12 and 48 hours) and in some of them it could not find the optimum as it ran out of memory. Such different

Table 3: Number of backup wavelengths for the heterogeneous scenario.

Topology $[\lvert\bar{R}\rvert; \bar{p_l}]$	α	PS	Number of backup wavelengths			
			LB	ILP $[\delta^{LB}_{ILP};\delta^{UB}_{ILP}]$	Greedy $[\delta^{ILP}_{Greedy};\delta^{UB}_{Greedy}]$	UB
Eurocore [2.8; 0.9984]	0.999	98	124	124 [0; 44.1]	124 [0; 44.1]	222
	0.9999	110	126	126 [0; 48.8]	126 [0; 48.8]	246
	0.99999	110	126	140 [11.1; 43.1]	162 [15.7; 34.1]	246
EON [3.02; 0.997]	0.999	372	768	768 [0; 40.9]	768 [0; 40.9]	1286
	0.9999	380	772	796* [3.1; 40]	885 [11.2; 32.1]	1304
	0.99999	380	772	1206 [56; 7.5]	1224 [1.5; 6.1]	1304
NSFNet [3.25; 0.9981]	0.999	172	295	295 [0; 53.3]	295 [0; 53.3]	632
	0.9999	182	303	303 [0; 54.6]	303 [0; 54.6]	668
	0.99999	182	303	474 [56.4; 29]	536 [13.1; 19.8]	668
UKNet [3.25; 0.9995]	0.999	230	603	603 [0; 26.6]	603 [0; 26.6]	951
	0.9999	416	920	920 [0; 39.1]	920 [0; 39.1]	1511
	0.99999	420	922	922 [0; 39.4]	931 [1; 38.8]	1521
ItalNet [3.64; 0.9994]	0.999	294	988	988 [0; 32]	988 [0; 32]	1453
	0.9999	420	1115	1115 [0; 38.2]	1115 [0; 38.2]	1803
	0.99999	420	1115	1127* [1.1; 37.5]	1193 [5.9; 33.8]	1803

(*) Best objective after memory limit.

running times exhibited by the ILP solver for the same network is caused by the nature of the solution provided by the linear relaxation of the problem, which is the first step of the Branch&Bound algorithm. If such solution is integer (or almost integer), usually the solver ends its execution very quickly. Instead, if most of the solution is fractional, then the Branch&Bound algorithm could exhibit a very bad performance. The heuristic instead took less than 2 minutes for all the studied cases.

In Table 3 the same information shown for the homogeneous case is given for the heterogeneous case. As the target values for availability are now a fraction of the dedicated protection availability values, it is always possible to guarantee the constraint (16). For this reason, PS increases with the value of α.

It can be seen that in the 73% of the cases studied the ILP model and the lower bound require the same number of backup wavelengths. These cases exhibit low availability requirements or the network links have high availability values (UKNet and ItalNet, both with short links). In the remaining cases, it can be seen a small difference with the lower bound, except in two cases: EON and NSFNet for $\alpha = 0.99999$. This is because, compared to other topologies, these two exhibit the lowest values of link availability and longest routes.

For the 4 cases where the ILP computation time was on the order of days, we modified the optimality gap to 2% to verify whether a good solution could be found more quickly. Even so, there were still 2 cases where the time limit was exceeded, highlighting the usefulness of devising a heuristic to deal with those cases.

Table 4: Running time for cases of ILP and Greedy heuristic.

Topology	α/A^T	Homogeneous case			Heterogeneous case		
		PS	RTI[s]	RTG[s]	PS	RTI[s]	RTG[s]
Eurocore	0.999	98	0.06	0.452	98	0	0.252
	0.9999	110	0.06	0.38	110	0	0.332
	0.99999	76	0.05	0.12	110	0.85	0.231
EON	0.999	372	0.76	22.088	372	0.09	19.765
	0.9999	298	44268.39*	4.344	380	172804.48*	12.375
	0.99999	52	0.02	0.146	380	0.05	4.356
NSFNet	0.999	172	0.19	4.094	172	0.07	3.949
	0.9999	182	175.62	4.462	182	1.02	5.478
	0.99999	34	0.13	0.15	182	48.26	1.197
UKNet	0.999	230	0.25	6.094	230	0.01	6.415
	0.9999	416	0.64	44.37	416	0.03	42.806
	0.99999	420	54.04	40.339	420	6.74	42.733
ItalNet	0.999	294	0.57	31.331	294	0.01	31.38
	0.9999	420	1.29	104.224	420	0.06	95.076
	0.99999	342	65240.54*	22.185	420	172802.46*	61.19

(*) Running time after memory limit.

In Appendix B we briefly explore the case without wavelength conversion.

8 Conclusions

An ILP model to solve the problem of configuration of shared backup routes with an availability target in optical WDM networks was proposed and solved. To do so, unlike previous work, the ILP model included modeling the availability constraint as a linear expression and the solution allowed identifying the connections sharing resources in different links. The results obtained by solving the ILP model were equal to the lower bound (ILP without availability guarantees) for networks with high link availability and low number of hops in the routes. However, in networks with low levels of link availability or longer routes, the results in a scenario with availability requirements were different from the lower bound, highlighting the importance of considering the availability requirements when designing the network.

To solve instances where the ILP takes significant time to get to a solution (on the order of days), we proposed a greedy heuristic approach that showed a very good performance: an average and maximum relative error of 2.49% and 15.7%, with execution times lower than 2 minutes.

We also explored the modifications necessary for the ILP model to include the wavelength continuity constraint and obtained results for that case. Further analysis of this area as well as the impact of modifying assumptions on the input routes and the link failure independence is part of future research.

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Appendix A Proof of NP-completeness for the minimum number of wavelengths required for a link.

Lemma 1. *Let Ψ_c be a set of allowed shared connections for each $c \in C$ and let Ω_l be the set of connections using a given link $l \in L$. Then, to determine the minimum number of required wavelengths in a link l is an NP-hard problem.*

Prof. We use a reduction of MINIMUM CLIQUE PARTITION problem [6]. Given a graph $G = (V,E)$ we can construct an instance of our problem where $\Omega_l = V$ (i.e., each vertex of G is a connection in Ω_l) and two connections $u, v \in V$ can be grouped if and only if $\{u, v\} \in E$. Recall that two connections u and v can be grouped in a same wavelength if and only if $u \in \Psi_v$ and $v \in \Psi_u$. Hence, if it is possible to assign a subset S of connections into one wavelength, then the corresponding subset of nodes S forms an induced clique in G. Also, since each connection requires an assigned wavelength, then an assignment of wavelengths to Ω_l is equivalent to a clique partition of the graph G. Hence, a minimum assignment of wavelength to Ω_l is equivalent to a minimum partition of G into cliques.

Appendix B Extension to the case without wavelength conversion

For the case of lack of wavelength conversion, in the ILP model we can replace (28) by the following constraints:

$$
\begin{aligned}
x_{i,j} \geq x_{i,c} + x_{j,c} - 1 & \text{ , } \forall c \in C, \forall i \in C \\
& \text{ , } \forall j \in C | c < i \wedge c < j \wedge j < i \\
& \text{ , } R_i^B \cap R_j^B \neq \emptyset, R_i^B \cap R_c^B \neq \emptyset \\
& \text{ , } R_j^B \cap R_c^B \neq \emptyset
\end{aligned} \tag{40}
$$

$$
\begin{aligned}
x_{i,j} \leq 1 + (x_{j,c} - x_{i,c}) & \text{ , } \forall c \in C, \forall i \in C \\
& \text{ , } \forall j \in C | c < i \wedge c < j \wedge j < i \\
& \text{ , } R_i^B \cap R_j^B \neq \emptyset, R_i^B \cap R_c^B \neq \emptyset \\
& \text{ , } R_j^B \cap R_c^B \neq \emptyset
\end{aligned} \tag{41}
$$

$$
\begin{aligned}
x_{i,j} \leq 1 + (x_{i,c} - x_{j,c}) & \text{ , } \forall c \in C, \forall i \in C \\
& \text{ , } \forall j \in C | c < i \wedge c < j \wedge j < i \\
& \text{ , } R_i^B \cap R_j^B \neq \emptyset, R_i^B \cap R_c^B \neq \emptyset \\
& \text{ , } R_j^B \cap R_c^B \neq \emptyset
\end{aligned} \tag{42}
$$

Constraints (40), (41) and (42) guarantee a coherent behavior among different links. For example, if connections i and j share the same wavelength with connection c at some different links, then i and j also would share backup resources, because i, j and c are using the same wavelength.

The results for homogeneous and heterogeneous scenarios are presented in Tables 5 and 6, respectively. As in the previous case, the lower bound (LB) is obtained by removing the availability constraint.

Table 5: The homogeneous scenario for networks without wavelength conversion.

Topology $[\lvert\bar{R}\rvert; \bar{p_l}]$	A^{Target}	PS	GAP%	RT[s]	Number of backup wavelengths		
					LB	ILP$[\delta^{LB}_{ILP};\delta^{UB}_{ILP}]$	UB
Eurocore [2.8; 0.9984]	0.999	98	0	0.28	124	124 [0; 44.1]	222
	0.9999	110	0	0.45	126	126 [0; 48.7]	246
	0.99999	76	0	0.1	84	98 [16.7; 38]	158
EON [3.02; 0.997]	0.999	372	0.77	108011.9	768*	768 [0; 40.3]	1286
	0.9999	298	0.26	108000.06	504*	518 [2.8; 42.3]	898
	0.99999	52	0	0.03	58	74 [27.6; 30.2]	106
NSFNet [3.25; 0.9981]	0.999	172	0	7194.62	295	295 [0; 53.3]	632
	0.9999	182	0.65	108006.36	303	303 [0; 54.6]	668
	0.99999	34	0	0.14	52	64 [23.1; 44.9]	116
UKNet [3.25; 0.9995]	0.999	230	0.17	108000.1	605*	605 [0; 36.4]	951
	0.9999	416	4.38	108000.11	920*	922 [0.2; 39]	1511
	0.99999	420	4.5	108000.38	922*	926 [0.4; 39.1]	1521
ItalNet [3.64; 0.9994]	0.999	294	0.4	108004.69	988*	988 [0; 32]	1453
	0.9999	420	1.99	108000.15	1115*	1116 [0.1; 38.1]	1803
	0.99999	342	0.96	108000.17	721*	742 [2.9; 42.1]	1281

(*) Best objective obtained after time-limit (30 hours).

Table 6: The hetereogeneous scenario for networks without wavelength conversion

Topology $[\lvert\bar{R}\rvert; \bar{p_l}]$	A^{Target}	PS	GAP%	RT[s]	Number of backup wavelengths		
					LB	ILP$[\delta_{ILP}^{LB};\delta_{ILP}^{UB}]$	UB
Eurocore [2.8; 0.9984]	0.999	98	0	0.27	124	124 [0; 44.1]	222
	0.9999	110	0	0.27	126	126 [0; 48.8]	246
	0.99999	110	0.9	1.5	126	140 [11.1; 43.1]	246
EON [3.02; 0.997]	0.999	372	0.66	108009.32	768*	768 [0; 40.3]	1286
	0.9999	380	2.83	108000.58	772*	796 [3.1; 39]	1304
	0.99999	380	0	1.29	772*	1210 [56.7; 7.2]	1304
NSFNet [3.25; 0.9981]	0.999	172	0	27736.46	295*	295 [0; 53.3]	632
	0.9999	182	0.33	65899.95	303*	303 [0; 54.6]	668
	0.99999	182	0.21	39657.23	303*	504 [66.3; 24.6]	668
UKNet [3.25; 0.9995]	0.999	230	0.29	108022.18	605	605 [0; 36.4]	951
	0.9999	416	5.73	108000.85	920*	926 [0.6; 38.7]	1511
	0.99999	420	4.96	108000.82	922*	926 [0.4; 39.1]	1521
ItalNet [3.64; 0.9994]	0.999	294	0.35	108005.54	988*	988 [0; 32]	1453
	0.9999	420	2.09	108000.77	1115*	1115 [0; 38.2]	1803
	0.99999	420	2.73	108000.3	1115*	1122 [0.6; 37.8]	1803

(*) Best objective obtained after time-limit (30 hours).

It can be seen that the homogeneous and heterogeneous cases including the wavelength continuity constraint behave similarly to the corresponding cases with wavelength conversion. The most important observation is that taking the wavelength continuity into account does not significantly increase the backup wavelength requirements. Such observation was also made in the context of routing and wavelength assignment algorithms, with and without failure restoration capability [1]. This behavior highlights the fact that, given the time to find an optimal solution, significant savings in wavelength converters could be made when designing availability-aware networks.

Further research on the implications of the results presented in Tables 5 and 6 is part of future work.