
http://research.gold.ac.uk/26642/

The version presented here may differ from the published, performed or presented work. Please go to the persistent GRO record above for more information.

If you believe that any material held in the repository infringes copyright law, please contact the Repository Team at Goldsmiths, University of London via the following email address: gro@gold.ac.uk.

The item will be removed from the repository while any claim is being investigated. For more information, please contact the GRO team: gro@gold.ac.uk
The effects of gender inequality, wages, wealth concentration and fiscal policy on macroeconomic performance

Özlem Onaran*, Cem Oyvat, Eurydice Fotopoulou

Acknowledgements: This paper received funding from the Rebuilding Macroeconomics, ESRC Network+. The authors are grateful for comments from Diane Elson, Sue Himmelweit, Jerome De Henau, İpek İlkkaracan and Engelbert Stockhammer and the participants of the Rebuilding Macroeconomics ESRC Social Science Festival on Women in Economics on earlier versions. Greenwich Papers in Political Economy are intended to make results of research at the University of Greenwich available to other researchers in preliminary form for discussion and suggestions for revision before publication.

*Corresponding author: University of Greenwich, Institute of Political Economy, Governance, Finance and Accountability, Old Royal Naval College, Park Row, Greenwich, London SE10 9LS, Email:o.onaran@gre.ac.uk

Year: 2019

No: GPERC71
The effects of gender inequality, wages, wealth concentration and fiscal policy on macroeconomic performance

Executive summary
The aim of this paper is to develop a macroeconomic model to analyse the effects of multiple dimensions of inequalities and fiscal policies on macroeconomic outcomes. The theoretical novelty is to develop a unified model, integrating i) the impact of three dimensions of inequalities –functional income distribution between wages and profits, gender inequality, and wealth concentration, and their interactions; ii) the impact of fiscal policies, particularly the effects of government spending in social vs. physical infrastructure, and different types of taxation; iii) both the demand and supply-side effects; iv) effects on both output and employment.

We build a three sector gendered model with social sector (health, social care, education, child care), the rest of the market economy, and unpaid care sectors and three types of factors of production - male and female labour, and capital. On the demand side, we model behavioural equations determining consumption, private investment, exports, imports and government spending. On the supply side, productivity changes in the medium-run as an outcome of changes in wages, public and private expenditure and unpaid care. Hours of employment in the social sector and the rest of the economy are determined by output and labour productivity in the relevant sectors, and social norms about occupational segregation determines hours of employment of women and men in both sectors. Wealth concentration depends on functional income distribution and wealth tax.

We estimate this general model econometrically for the UK using time series data for the period of 1970-2016. For the medium-run estimation of productivity we use panel data of 18 industries for the period of 1970-2015. We find that an upward convergence in wages, i.e. increasing wages with closing gender pay gap in both sectors leads to higher output in both the short and the medium-run. The UK is both wage-led and gender equality-led, and hence equality-led. However the positive impact on productivity is stronger in the medium-run than on output, which leads to a fall in employment of both men and women. The positive impact of public social infrastructure investment on both output and employment is much higher, and despite a strong positive effect on productivity, employment of both men and women increase in the medium-run as well. A policy mix of upward convergence in wages and public social infrastructure investment has a strong positive impact on output and women’s employment, but men’s employment decreases in the medium-run. Public debt/GDP also falls as an outcome of this policy mix. A policy mix of upward convergence in wages and public investment in both social and physical infrastructure leads to a higher increase in output, and employment of both men and women increase both in the short and the medium-run. However, public debt/GDP increases marginally in the medium-run in this policy mix, and an increase in tax rates is required to improve public debt/GDP. An increase in the progressivity of income taxation in the form of increasing tax rate on capital income and decreasing tax rate on labour income increases output, men’s and women’s employment, and decreases public debt/GDP in both the short and the medium-run. An increase in the tax rate on wealth decreases wealth concentration, and has a positive and the strongest impact on output, employment and the budget.

Keywords: gender wage and employment gap, functional income distribution, wealth concentration; fiscal policy, social infrastructure, productivity, employment, growth, Post-Keynesian economics, feminist macroeconomic models
1. Introduction

The aim of this paper is to develop a macroeconomic model to analyse the effects of multiple dimensions of inequalities, and fiscal policies on macroeconomic outcomes. The theoretical novelty is to develop a unified model, integrating i) the impact of three dimensions of inequalities –functional income distribution between wages and profits, gender inequality, and wealth concentration, and their interactions; ii) the impact of labour market policies which effect wage rates and gender pay gap, and fiscal policies, particularly the effects of government spending in social vs. physical infrastructure, and different types of taxation; iii) both the demand and supply-side effects; iv) effects on both output and employment. An implicit aim is to contribute to gendering macroeconomics by incorporating gendered behavioural differences and the role of social norms in the model.

A distinctive feature of the last four decades prior to the Great Recession has been a rise in multiple dimensions of inequality. There has been a sharp polarization of personal income distribution (Atkinson et al., 2011) as well as a significant change in functional income distribution, i.e. a fall in the share of labour income in national income in both developed and developing countries (IMF, 2009; ILO, 2012; Stockhammer, 2016; Onaran and Galanis, 2014). There has been also a remarkable increase in wealth concentration (Piketty, 2014; Piketty and Zucman, 2014; Alvaredo, et al., 2017; Goda et al, 2016). Meanwhile, despite improvements in legal rights and education, gender gaps in income and employment remain very high and women do the vast majority of unpaid domestic care, which reinforces gender gaps in employment and wages and occupational segregation further (ILO, 2018a, b).

There is a growing recognition that inequality deters growth and stability, and has been an important contributing factor to the financial crisis in 2008 (Stiglitz, 2011; Kumhof et al., 2015, 2012; Kumhof and Ranciere, 2010; Rajan, 2010; Milanovic, 2011; Fitoussi and Saraceno, 2010; Goda and Lysandrou, 2014). One strand of research on the impact of inequality focuses on personal income distribution, e.g. recent research at the OECD and the IMF (IMF, 2009; Berg et al., 2012; Cingano 2014), based on reduced form econometric analysis of macro panel data, argue that inequality may impede growth due to supply side factors such as barriers to human capital accumulation or political risk, building on new institutionalist political economy (Galor and Zeira, 1993; Alesina and Rodrik, 1994; Persson and Tabellini, 1994; Alesina and Perotti, 1996). While this literature moves beyond the Kuznets (1955) hypothesis on inequality and growth, demand side factors which are crucial in economies with excess capacity, are not analysed.
Another strand focuses on the impact of functional income distribution on the demand side, in particular on consumption and private investment building on post-Keynesian/post-Kaleckian demand-led macroeconomic models (Bhaduri and Marglin, 1990; Onaran and Galanis, 2014; Onaran and Obst, 2016; Onaran et al, 2011; Stockhammer et al., 2009; Hein and Vogel, 2008; Naastepad and Storm, 2006/7; Stockhammer and Onaran, 2004). These models and the empirical estimations allow for both positive and negative effects of a fall in the labour share on the components of aggregate demand. Extensions to these demand and distribution-led macroeconomic models further integrate the impact of public spending and taxes (Blecker, 2002; Mott and Slattery, 1994; Hein, 2018; Palley, 2009; 2013a; 2014a; You and Dutt, 1996; Dutt, 2013a; Tavani and Zamparelli, 2017a; Allain, 2015; Ko, 2018; Commendatore et al., 2011; Obst et al., 2019). Going beyond the short-run demand effects in this strand of models, a series of papers also integrate the interaction of income distribution and productivity into demand-led macroeconomic models (Palley, 1996, 2012a, 2013b, 2014b; Casetti, 2003; Dutt, 2006, 2010, 2011, 2013b; Naastepad, 2006; Setterfield, 2006; Hein and Tarassow, 2010; Tavani and Zamparelli, 2017b); however these models do not include the public sector. Seguino (2010; 2012) extends these models by integrating public spending and productivity and incorporates the effects of gender wage gaps, however does not present an empirical analysis.

Among the macro models on the impact of gender inequality on growth, one strand again focuses on the supply side effects of gender inequality and intra household bargaining on fertility, savings of the household and the accumulation of human capital within the context of endogenous growth models (Becker et al., 1990; Doepke and Tertilt, 2009, 2014, 2016; Agenor and Agenor, 2014; Agenor and Canuto, 2015; Cavalcanti and Tavares, 2016). Fukui et al. (2019) develop a formal multi-region general equilibrium model, augmenting the real business cycle models with unpaid home production along the lines of Benhabib et al. (1991) and Greenwood and Hercowitz (1991), and show that increases in female employment during the “Grand Gender Convergence” in employment over the past half-century in the US translate into increases in total employment with little (and statistically insignificant) crowding out of men in the labour market. One important finding is that the entrance of women into the market sector has much smaller income effects on the labour supply of men compared to findings in earlier research by Jones et al., (2015), Heathcote et al., (2017), Knowles (2013), when the utility for men from women’s unpaid household production is incorporated to the model.
Cross-country empirical analysis regarding the impact of gender equality, mostly based on reduced form aggregate estimations of growth, focus on the supply side effects of equality in education and labour force participation, via the direct and indirect/intergenerational effects on productivity, because women are assumed to spend more on children’s education and health relative to men (Lundberg and Pollak, 1996; Phipps and Burton, 1998; Esteve-Volart, 2000; Knowles, et al., 2002; Morrison et al., 2007; Klasen and Lamanna 2009; Amin, et al., 2015; Gonzales et al., 2015; Cuberes and Teignier, 2014; Seguino, 2017). Reductions in labour market imperfections such as wage discrimination and occupational segregation are expected to stimulate growth in most cases. However, Seguino (2017) highlights that most of these models do not account for problems related to the demand side to ensure increases in female education and labour force participation are matched by sufficient labour demand.

Among gendered macro models, Braunstein et al. (2011) and Seguino (2010, 2012) incorporate both demand and supply side analysis within post-Kaleckian theoretical models, albeit without an empirical analysis, allowing for both positive and negative effects of gender equality on the demand side depending on the structural features of the economy and incorporating the positive effects on the supply side. Braunstein et al. (2011) model the impact of gender differences in income on consumption, investment, output, and productivity, but do not present an explicit modelling of the government sector. Informed by contributions from gender studies and sociology, this strand of feminist analysis of the macro economy treats labour as a produced means of production, as opposed to conventional macroeconomic models: the reproduction of labour is carried out by both paid and unpaid work, and women have a disproportionate share of unpaid work relative to men. Braunstein et al. (2018) analyse how care models, globalization and macroeconomic policy stance shape the development trajectories of different economies using a principle component analysis.

Another body of empirical research focusing on the demand side effects of gender gaps, uses input-output tables to analyse the impact of public spending in social care and education, and show the stronger effect of this type of fiscal spending on female employment as well as total employment compared to public investment in physical infrastructure (Antonopoulos et al., 2010; Ilkkaracan et al., 2015; Ilkkaracan and Kim, 2018; De Henau et al., 2016). Antonopoulos et al. (2010) and Ilkkaracan et al. (2015) extend this analysis using micro household data to match the macro labour demand with personal characteristics of the population to analyse the effects on employment of women and men. However, these studies are based on a static analysis, and do not take the medium-run productivity effects into account.
Pollitt et al (2017) use a demand-led Post-Keynesian econometric model to simulate the impact of gender pay gaps on growth, however changes in income distribution has only supply side effects and does not affect consumption and demand directly in their model; similarly wages or government spending in social infrastructure does not affect productivity. In a similar vein, Bargawi and Cozzi (2017) use a global demand-led model (Cambridge Alphametrics Model) without gendered variables to assess the impact of government expenditure in social infrastructure in what they call a gendered expansionary growth scenario.

Regarding the impact of wealth inequality, Boyer (2000), Lavoie and Godley (2001-2), van Treeck (2009) Skott and Ryoo (2008), Ryoo and Skott (2013) and Hein (2018) incorporate wealth effects in Post-Keynesian macroeconomic models, however, theoretical macro models integrating the impact of wealth inequality on output are only newly emerging (Taylor et al., 2015, 2018; Petach and Tavani, 2018; Palley, 2012b; 2017; Ederer and Rehm, 2018; Zamparelli, 2016; Botta et al, 2019), and the few exceptions in the macroeconometric analysis of inequality on output only integrate the impact of total wealth on consumption or investment rather than wealth inequality (e.g. Onaran et al., 2011; Stockhammer and Wildauer, 2016; Stockhammer et al., 2018; Kim et al., 2015; Zezza, 2009). This misses an important channel as micro-econometric evidence shows that marginal propensity to consume out of wealth differs across the wealth distribution (Arrondel et al., 2015; Mian et al., 2013). Wealth concentration is likely to have significant consequences also for investment due to both credit constraints and its impact on the financialization of the real economy, and is essential for a full analysis of the impact of inequalities on the macro economy. Moreover, greater wealth concentration could also be reflected in the market concentration in different sectors by restricting middle and upper-middle income individuals’ capability of entry. A high market concentration reduces the incentives to invest and innovate as shown in Gutiérrez and Philippon (2017) and the IMF (2019).

Synthesizing these different strands in an integrated general model, this paper aims at developing a novel gendered macroeconomic analysis building on structuralist, Post-Keynesian, and feminist economics and gender studies. Our aim is to develop a general theory, which allows for the possibility of different outcomes depending on the values of the behavioural parameters; e.g. inequalities may increase or decrease output and employment, or government spending may lead to lower or higher investment and productivity, both potentially with different effects in the short and medium-run.
We present a three sector gendered model with social sector (health, social care, education, child care), the rest of the market economy, and unpaid care sectors and three types of factors of production - male and female labour, and capital. On the demand side, we model behavioural equations determining consumption, private investment, exports, imports and government spending. On the supply side, productivity changes in the medium-run as an outcome of changes in wages, public and private expenditure and unpaid care. Hours of employment in the social sector and the rest of the economy are determined by output and labour productivity in the relevant sectors and social norms about occupational segregation determines hours of employment of women and men in both sectors.

We estimate this general model econometrically for the UK using time series data for the period of 1970-2016. For the medium-run estimation of productivity we use panel data of 18 industries for the period of 1970-2015. The estimated parameters are used to develop an empirical analysis of the impacts of inequalities and policies via both the demand and supply side effects, moving beyond the mainstream analysis (e.g. IMF, 2009; Cingano 2014), which is focused on the supply-side and personal income inequality.

In terms of the macroeconomic outcomes we analyse the effects on output, employment of men and women, public debt, private net wealth (net wealth), and productivity. The analysis of employment, gender employment gaps, and inequalities, rather than simply output, thereby broadening the policy impact analysis beyond the narrow measure of GDP is another aim of the paper. Other UK empirical macro models, such as those used by the Bank of England (see Burgess et al., 2013) or the Office for Budget Responsibility (OBR, 2013) do not address the impacts of inequalities.

The rest of the paper is structured as follows: Section 2 presents the model. Section 3 and 4 analyse the impact of labour market and fiscal policies based on the theoretical model. Section 5 presents the data, estimation methodology and estimation results. Section 6 analyses the effects of changes in wages, gender pay gap, public spending in social and physical infrastructure and tax rates on capital income, wealth and labour income based on the empirical estimations, and presents scenarios based on a mix of labour market and fiscal policies. Section 7 concludes.

2. The model

The model is structuralist, i.e. structural features of the economy and society, such as the existence of excess capacity, involuntary unemployment, oligopolistic market structure and price setting by firms, the structure of production and resulting price elasticities, inequalities (income and wealth distribution and gender inequality), social norms, the distribution of
unpaid domestic care labour between men and women, and the form and extent of gendered job segregation (e.g. women’s association with paid care work) play a crucial role in determining economic behaviour and macroeconomic outcomes.

Features regarding gendered behaviour and social norms such as reciprocity, caring, and non-selfish motives, which are largely absent from macroeconomics, are integrated to the behavioural specifications. Social norms, individual motivation of men and women, and public preferences, alongside the structure of the social welfare state, are important in determining the different gendered behaviour of men and women (Seguino, 2017). Consequently, a change in gender pay gap or public spending in social vs. physical infrastructure may have gendered short and medium-run impacts on employment and income.

Wage rates are determined exogenously as an outcome of a bargaining process between employers and workers, and labour market institutions. Gender pay gap is also determined exogenously depending on relative bargaining power of women, social norms, occupational segregation effected by these norms, availability of social care, labour market policies and legislation, as well as differences in personal characteristics such as education which in turn are affected by social norms. Gender gaps may be the result of women’s disproportionate responsibility for unpaid care work, stereotypes that lead to occupational segregation, or wage gaps in favour of men leading families to select the lowest-paid adult to provide unpaid care work (Seguino, 2017).

Functional income distribution is determined endogenously, as the wage share of men and women and profit share change when wages, output, employment and productivity change. Wealth concentration changes endogenously depending on after-tax functional income distribution and wealth tax.

We do not model the impact of wage inequality between other types of workers, such as low vs. high skilled or managerial workers, in order to focus on the impact of gender, functional income and wealth distribution. However, the theoretical framework can be extended to analyse behavioural differences among other heterogeneous agents, e.g. different types of workers. Moreover, from an empirical point of view, the distribution of income between wages and profits has significant consequences for personal income distribution, hence the latter is largely captured by functional income distribution.

In the rest of this section we present the structure of the model. Appendix 1 presents the list of variables and definitions. Aggregate output \( Y_t \) is the sum of total male wage bill \( WB_t^M \), total female wage bill \( WB_t^F \), and profits \( R_t \).
\[ Y_t = WB_t^M + WB_t^F + R_t \]  

The total wage bill for female workers \((WB_t^F)\) is a function of female wages in the social sector \((w_t^{HF})\), female employment in the social sector \((E_t^{HF})\), female wages in the rest of the economy \((w_t^{NF})\), and female employment in the rest of the economy \((E_t^{NF})\), and \(H\) denotes the social sector and \(N\) the rest of the economy:

\[ WB_t^F = w_t^{HF} E_t^{HF} + w_t^{NF} E_t^{NF} \]  

Similarly the total wage bill for male workers \((WB_t^M)\) is a function of male wages in the social sector \((w_t^{HM})\), male employment in the social sector \((E_t^{HM})\), male wages in the rest of the economy \((w_t^{NM})\), and male employment in the rest of the economy \((E_t^{NM})\):

\[ WB_t^M = w_t^{HM} E_t^{HM} + w_t^{NM} E_t^{NM} \]  

All wage rates are defined as hourly real wages and employment is defined as total hours worked by persons engaged in the respective sector. Working with hours of employment as opposed to headcount figures is important for a gendered macro analysis to reflect the high share of women in part-time work.

As shown in Figure 1, the average wages in both \(H\) and \(N\) sectors are significantly larger for male workers in the UK. Following this, we define gender wage gaps \((\alpha_t)\) for wages in \(H\) and \(N\) sectors as below:

\[ \alpha^N_t = \frac{w_t^{NM}}{w_t^{NF}} > 1, \quad \alpha^H_t = \frac{w_t^{HM}}{w_t^{HF}} > 1 \]  

**Figure 1 here**

The aggregate output in the market economy (GDP, excluding unpaid activities) is:

\[ Y_t = C_t^N + C_t^H + I_t + G_t^H + G_t^C + I_t^G + X_t - M_t \]  

where \(C_t^N\) is households’ social expenditures\(^1\), \(C_t^H\) is consumption in the rest of the economy, \(I_t\) is private investment expenditures, \(G_t^H\) is government’s social infrastructure expenditures (in health, social care, education, child care), \(G_t^C\) is government’s consumption expenditures, \(I_t^G\) is public investments other than investments in the social sector\(^2\), \(X_t\) is exports of goods and services and \(M_t\) is imports of goods and services. In line with the

---

\(^1\) While theoretically household consumption of social services amount to investment in human infrastructure and affects productivity in our model, as discussed below, we preserve the term “consumption” for this category consistent with the definitions in national accounts.

\(^2\) Government’s social infrastructure expenditures are classified as current spending on labour services in the national accounts. The physical infrastructure associated with providing social infrastructure such as schools and hospitals are counted as physical infrastructure. Hence part of \(I_t^G\) also contributes to social infrastructure. However, our classification is important for a gendered analysis of the employment impact of different fiscal policy decisions as \(G_t^H\)is very female labour intensive while construction, just as most other parts of \(I_t^G\) is male labour intensive.
feminist economics literature emphasizing the importance of government’s social expenditures on productivity and social fabric, we refer to $G^H_t$ as public social infrastructure investment in the rest of the paper (Elson, 2016, 2017; Women’s Budget Group, 2015). The public social expenditures is a fiscal policy decision targeted as a share of aggregate output ($\kappa^H_t$), and constitutes the public social sector output ($Y^H_t$). The rest of the GDP is the market output in the rest of economy ($Y^N_t$):

$$Y^H_t = G^H_t = \kappa^H_t Y_t$$  \hspace{1cm} (6)
$$Y^N_t = Y_t - G^H_t = Y_t (1 - \kappa^H_t)$$  \hspace{1cm} (7)

The share of government’s consumption expenditures ($G^C_t$) and public investments other than social infrastructure investment in the social sector ($I^C_t$) are also determined by government as a share of aggregate output and are respectively $\kappa^C_t$ and $\kappa^C_t$:

$$G^C_t = \kappa^C_t Y_t$$  \hspace{1cm} (8)
$$I^C_t = \kappa^C_t Y_t$$  \hspace{1cm} (9)

Hours of employment in the social sector and the rest of the economy are determined by output and labour productivity in the relevant sectors and social norms about occupational segregation determines the share of men and women in total hours of employment in both sectors. Analysing the effects on not just output but also employment is a novel feature of the paper compared to previous research on the macroeconomic impact of inequalities, which we believe is crucial to go beyond a narrow GDP focused policy impact analysis and broaden the targets to include other factors that affect wellbeing and social cohesion. The real world structuralist features of the model reflect that employment is demand-constrained in an economy where there is excess capacity and involuntary unemployment, and supply constraints are discussed as part of labour supply behaviour below.

The employment in sector N is output over labour productivity sector N ($T^N_t$):

$$E^N_t = \frac{Y^N_t}{T^N_t} = \frac{(1 - \kappa^H_t) Y_t}{T^N_t}$$  \hspace{1cm} (10)

In our model, the share of female employment in sector N is exogenously determined by social norms determining occupational segregation, and is demonstrated by $\beta^N_t$. The male workers in sector N constitute $(1 - \beta^N_t)$ of the sector:

---

3 For simplicity, we assume that sector H only consists of the public social sector. The employment and supply in this sector is entirely financed by public social expenditures. The households’ private social consumption (see equation 21) is supplied by the private market output in the rest of economy ($Y^N_t$). Hence, private social consumption do not directly contribute to the generation of employment in sector H; however, they affect labour productivity positively as discussed below.
We assume that the wage bill paid to male and female workers in the social sector constitutes the public social expenditures and the social sector is not making profits. Any non-labour inputs used constitute part of government consumption \((G^C)\). Following this, the public social expenditure can be written as a function of employment \((E^H_t)\), average female wage \((w^FH_t)\), average male wage \((w^HM_t)\), female employment share \((\beta^H_t)\) and male employment share \((1 - \beta^H_t)\) in the social sector.

\[
G^H_t = \kappa^H_t Y_t = \beta^H_t E^H_t w^FH_t + (1 - \beta^H_t) E^H_t w^HM_t
\]  
\[(13)\]

Figure 1 also shows that female employment share in H has been larger than female employment share in N in the UK. Onaran et al (2019) observed a larger female employment share in H for 13 emerging economies. Therefore, in our model we assume that \(\beta^H_t\) is larger than \(\beta^N_t\).

Using equations (13) and (4), we can write the total employment \((E^H_t)\), female employment \((E^{HF}_t)\) and male employment \((E^{HM}_t)\) in the social sector as a function of public social expenditures and female wages in the social sector.

\[
E^H_t = \frac{G^H_t}{w^FH_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)} = \frac{\kappa^H_t Y_t}{w^FH_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)}
\]  
\[(14)\]

\[
E^{HF}_t = \frac{\beta^H_t \kappa^H_t Y_t}{w^{FH}_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)}
\]  
\[(15)\]

\[
E^{HM}_t = \frac{\beta^H_t \kappa^H_t Y_t}{w^{FH}_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)}
\]  
\[(16)\]

We model the unpaid domestic care labour \((U_t)\) within the households as

\[
\log \frac{U_t}{N_t} = q_0 + q_G \log \frac{(G^H_t + C^H_t)}{N_t}
\]  
\[(17)\]

For a given demographic structure and population \((N)\) which determines the care needs of a society, \((q_0)\), higher per capita government expenditures or household consumption in the social sector are expected to reduce the need in households for unpaid care; therefore, it would lead to lower per capita unpaid labour \((q_G < 0)\). We specify the equation in logs, since the impact of social expenditures on the time spent on unpaid domestic care might be non-linear, i.e. the negative impact might be decreasing in absolute values as it gets increasingly more difficult to substitute unpaid care at lower levels of unpaid care. The potential squeeze
in unpaid care due to paid employment is excluded to simplify the model. The effect of \( G_t^H \) and \( C_t^H \) as determinants of employment only partially reflects this effect.

Next, we define the profits (\( R \)) in N. The income in N is distributed between wages and profits.\(^4\) The profit income is the operating surplus in N after wage payments.

\[
R_t = Y_t^N - w_t^{NF} E_t^{NF} - w_t^{NM} E_t^{NM} = Y_t^N - E_t^N (\beta_t^N + \alpha_t^N - \beta_t^N \alpha_t^N) w_t^{NF} = (1 - \kappa_t^H) Y_t - E_t^N (\beta_t^N + \alpha_t^N - \beta_t^N \alpha_t^N) w_t^{NF})
\]

The profit share in N (\( \pi_t \)) is the share of profits in output in N. The profit share can also be written as a function of female wages and labour productivity in N:

\[
\pi_t = \frac{Y_t^N - w_t^{NF} E_t^{NF} - w_t^{NM} E_t^{NM}}{Y_t^N} = 1 - \frac{(\beta_t^N + \alpha_t^N - \beta_t^N \alpha_t^N) w_t^{NF}}{Y_t^N}
\]

On the demand side of the economy, we model behavioural specifications for consumption, private investment, exports, imports, tax revenues and government spending. The next set of equations present the behavioural equations defining the demand side.

Household consumption behaviour is a function of after tax income, which in turn is income from female and male waged employment and profits, and private net wealth (private assets-debt) of two groups, the net wealth of the top 1% (\( PW1 \)) and the bottom 99% (\( PW99 \)). Due to data availability, it is not possible to disaggregate wealth alongside functional income distribution categories such as those earning wage vs. profit income or by gender. It is also not possible to disaggregate debt along personal or functional income distribution categories; therefore we use net wealth disaggregated by income percentiles to analyse the impact of wealth inequality and private debt. Total household consumption in two types of goods and services produced in the social sector and the rest of the economy depends on the differences in the marginal propensities to consume out of female and male wage income and capital income, and will be affected by changes in wages, functional income distribution and gender gaps. Both accounting for gendered categories of wage earners and wealth inequality in the consumption function are novel features.

Aggregate consumption of households in goods and services in the rest of the economy is:

\[
\log c_t^N = c_0 + c_r \log[R_t (1 - t_t^R)] + c_f \log[(w_t^{NF} E_t^{NF} + w_t^{HF} E_t^{HF})(1 - t_t^W)] + c_m \log[(w_t^{NM} E_t^{NM} + w_t^{HM} E_t^{HM})(1 - t_t^W)] + c_{PW1} \log(PW1_t (1 - t_t^{PW})) + c_{PW99} \log(PW99_t (1 - t_t^{PW}))
\]
where $t^R_t$ is the implicit tax rate (ITR) on capital income, $t^W_t$ is the ITR on labour income, and $t^{PW}_t$ is the ITR on wealth. The marginal propensity to consume in $N$ is assumed to be different for male and female workers, reflecting the gender pay gaps as well as differences in behaviour. We discuss this in more detail below while presenting the analysis of the model.

With respect to the effect of household wealth in the form of both financial and real estate assets on consumption, there are several channels to consider. In the 1990s the wealth effect in the consumption function has been rediscovered, motivated by the increase in private consumption expenditures in the USA, which was attributed to the rise in the value of financial assets during the stock market boom (Onaran et al., 2011; Boone et al. 1998). In 2000s, the focus turned to the effect of booming house prices on consumption. In line with the fact that residential property is more frequently accepted as collateral, there is empirical evidence that the marginal propensity to consume out of property wealth is substantially higher than out of financial assets, in particular in the UK (Case et al 2001; Catte et al. 2004; Girouard et al. 2006; Linder, 2014; Slacalek, 2009; Goodhart and Hoffman, 2008). Rising inequality also plays a role. As wages have stagnated in many countries, but consumption norms have increased, many households have been driven into debt (Cynamon and Fazzari, 2008; Brown, 2008).

Barba and Pivetti (2009) and Cynamon and Fazzari (2008) argue that increasing house prices help households with risky mortgages (part of the PW99 in our model) to get refinance loans, and are thereby able to relax their budget constraint for consumption. Unfortunately we cannot disaggregate financial and household wealth and debt for different percentiles. Overall we expect a positive effect of an increase in the private net wealth of both groups due to access to credit and improved consumer confidence, and the effects are likely to be more significant for PW99, i.e. the wealth of the more budget constrained group. There are also offsetting mechanisms; e.g. Buiter (2010) argues that the positive effects of higher house prices for owners may be offset by higher costs for renters.

The households’ social expenditures ($C_t^H$) is also a function of after tax profits and wage bills of female and male workers sectors, net private wealth, and governments’ social expenditures:

$$
\log C^H_t = z_0 + z_R \log (R_t (1 - t^R_t))
+ z_L \log [(w^NFE_t^{NF} + w^{HEFH}_t)(1 - t^W_t)]
+ z_M \log [(w^{NM}E^{NM}_t + w^{HM}E^{HM}_t)(1 - t^W_t)]
+ z_{PW1} \log (PW1_t (1 - t^{PW}_t)) + z_{PW99} \log (PW99_t (1 - t^{PW}_t))
$$

(21)
The marginal propensity to consume social goods is different for male and female workers. The governments’ social expenditures ($G^H_t$) is part of the wage bill in $H$ and can \textit{i}) increase households’ social expenditures by providing wage income in the social sector, \textit{ii}) decrease households’ social expenditures by reducing the need for these expenditures. We assume that the demand for $C^H_t$ is provided by the private sector in the market economy as part of the output in $N$, as mentioned above.

Finally, households’ social expenditures and other consumption are interdependent decisions and will be analysed as part of a system in the empirical analysis, as discussed in the next section.

An alternative specification, where relative prices in $N$ and $H$ also affect household consumption in $H$ and $N$ is not presented, as empirical analysis shows that price elasticities are insignificant. This is possibly a result of the composition of the two sectors in our case. Demand for $H$ is likely to be very inelastic and it is also a very small part of total household spending (4.5% in 1985 when records start and 3.6% in 2017). Consumption in $N$ constitutes the vast majority of spending and thereby aggregate price deflator is dominated by prices in $H$. The systems estimation methodology is expected to capture any potential common shocks, which the dependent variables do not account for, via the correlation of the errors between the two decisions. Finally, as prices depend on unit labour costs, the effects of the wage income and its ratio to the profit income (and hence to total income), capture the price effects of higher wages as well. The exclusion of the insignificant explicit price elasticities in the model also helps to reduce the complexity in the analytical solution of the model.

We model private investment as a behavioural function of the share of profits in national income (reflecting expected profitability and availability of internal funds), GDP (capturing demand effects), private net wealth of different groups, and public debt to GDP ratio, which in turn affects the interest rate. An important novelty of this proposal is that profitability is affected by not only wage costs, but also changes in productivity in the medium-run, integrating the interaction between the demand and supply sides, as we discuss in more detail below. Both public spending in physical infrastructure such as transport or information and communication technology, or spending in education and health, are widely expected to affect productivity. The model thereby integrates dual and conflicting effects of government spending on investment via a potentially negative crowding out effect, if higher public borrowing leads to higher interest rate; and a potentially positive crowding in effect in the medium-run, if profitability increases due to higher productivity. Another novelty is to
account for the impact of wealth concentration on investment. Hence, private investment \((I_t)\) is

\[
\log I_t = i_0 + i_1 \log Y_t + i_2 \log \left[ \pi_t (1 - t^R_t) \right] + i_3 \log \left( \frac{D_t}{Y_t} \right) + i_4 \log (PW1_t (1 - t^{PW}_t)) + i_5 \log (PW99_t (1 - t^{PW}_t))
\]

(22)

where \(D\) is the public debt. The private investment is expected to increase as a result of higher aggregate output \((i_1 > 0)\). \(\pi_t (1 - t^R_t)\) is the after tax share of disposable profits in the N sector. Following Bhaduri and Marglin (1990) and Blecker (1989), we expect the profit share to have a positive direct impact on private investment \((i_2 > 0)\); e.g. You and Dutt, 1996; Hein and Vogel, 2008; Stockhammer et al., 2009; Seguino, 2012; Onaran and Galanis, 2014; Onaran and Obst, 2016).\(^5\) We assume that firms consider after-tax profits in making investment decisions as widely assumed in the literature (e.g. You and Dutt, 1996; Blecker, 2002; Seguino, 2012). We use the ratio of public debt to GDP, \((D/Y)_t\), to consider the possible negative crowding out effects of rising public debt on the interest rate and, thereby, private investment \((i_3 < 0)\). Finally, we incorporate the net household wealth (private asset-debt) of different groups, \(PW1\) and \(PW99\). Rising wealth and/or asset prices is expected to lower the cost of finance, which in turn is expected to have a positive effect on investment (Stockhammer et al., 2018). If an increase in assets, in particular financial assets, lead to improved access to credit as well as more optimistic business expectations about future profitability, a rise in net wealth is expected to increase private investment \((i_4 \text{ and } i_5 > 0)\). However, if a higher private net wealth implies higher business liabilities, or if firms react more strongly to liabilities than assets (even if assets are increasing faster than liabilities), private net wealth could have a negative effect on investment (Stockhammer et al., 2018). Furthermore, \(PW99\) and \(PW1\) can also have different effects. \(PW99\) can have positive effects in particular on residential investment. Recent firm level evidence on financialization suggests that the share of financial assets in total assets of the non-financial companies has increased substantially in the past decades, and the effect of non-operating income originating from financial assets crowds out the physical investment of the large non-financial companies, while it has a positive effect on the investment of the relatively cash constrained smaller companies (Tori and Onaran, 2018, 2019; Orhangazi, 2008; Demir, 2009). \(^5\) Alternatively, investment can be modeled as a function of the profit rate (e.g. Rowthorn, 1981; Dutt, 1984; Taylor, 1985; van Treeck, 2009; Carvalho and Rezai, 2016; Tavani and Zamparelli, 2017a). However, Blecker (2002) and Bhaduri and Marglin (1990) argue that the inclusion of the profit rate instead of the profit share in the investment function would increase the possibility of obtaining a wage-led demand regime, because the profit rate can be decomposed into capacity utilisation and the profit share, and the inclusion of both variables would double count the impact of capacity utilisation on investment.

---

\(^5\) Alternatively, investment can be modeled as a function of the profit rate (e.g. Rowthorn, 1981; Dutt, 1984; Taylor, 1985; van Treeck, 2009; Carvalho and Rezai, 2016; Tavani and Zamparelli, 2017a). However, Blecker (2002) and Bhaduri and Marglin (1990) argue that the inclusion of the profit rate instead of the profit share in the investment function would increase the possibility of obtaining a wage-led demand regime, because the profit rate can be decomposed into capacity utilisation and the profit share, and the inclusion of both variables would double count the impact of capacity utilisation on investment.
evidence would suggest that the effect of PW99, which includes ownership of small and medium enterprises, is expected to be positive \((i_5 > 0)\), while the effect of PW1 is expected to be negative as firms direct their activities to financial accumulation as opposed to their core businesses \((i_4 < 0)\). Furthermore, the increase in PW1 might go along with increasing market concentration in different sectors. This would also create an impediment on private investment, since high market concentration would increase the barriers to entry and reduce firms’ incentives to invest and innovate (IMF, 2019; Gutiérrez and Philippon, 2016).

The public debt at time \(t\) \((D_t)\) is the public debt accumulated from the public debt in the previous period \((D_{t-1})\) with an interest rate of \(r_{t-1}\), plus the total government expenditures at \(t\), minus the taxes collected from profits, wages, wealth and consumption at time \(t\).

\[
D_t = (1 + r_{t-1})D_{t-1} + G_t^H + G_t^C + I_t^G - t_t^W(WB_t^F + WB_t^N) - t_t^R R_t
- t_t^{PW} PW_t - t_t^C (C_t^H + C_t^N)
\]

\[
D_t = (1 + r_{t-1})D_{t-1} + \frac{Y_t^N(N_t^H + N_t^C + N_t^G)}{1 - \kappa_t^H} - w_t^{NF}(\alpha_t^N E_t^{NM} + E_t^{NF})t_t^W
- w_t^{HF}(\alpha_t^H E_t^{HM} + E_t^{HF})\alpha_t^M - t_t^R (Y_t^N - w_t^{NF} (E_t^{NF} + \alpha_t^N E_t^{NM}))
- t_t^{PW} PW_t - t_t^C (C_t^H + C_t^N)
\]

where \(t_t^C\) is the ITR on consumption, and \(r_t\) is the interest rate on public debt.

Exports are modelled as a function of prices of exports relative to foreign prices and foreign income \((Y_{world})\) and the exchange rate \((\varepsilon)\); imports are a function of demand in \(N\) and domestic prices relative to import prices. For simplicity we assume that marginal propensity to import in \(H\) is zero. Imports depend on domestic prices relative to import prices, the exchange rate and aggregate demand in \(Y^N\). Appendix 2 shows the links between domestic and import prices, nominal unit labour cost, real unit labour cost, which is the wage share, and thereby presents the effect of the profit share on exports and imports via the pass through from wage share to nominal unit labour costs and prices and the price elasticity of exports and imports. Hence, to simplify the model, in reduced form, exports and imports can be written as

\[
\log X_t = x_0 + x_1 \log Y_t^{World} + x_2 \log \pi_t + x_3 \log \varepsilon_t
\]

\[
\log M_t = n_0 + n_1 \log Y_t^N + n_2 \log \pi_t + n_3 \log \varepsilon_t
\]

Based on empirical evidence, Lysandrou (2011) shows that wealth distribution and hence individuals at the top of wealth distribution are the main drivers of the demand for toxic assets in the US before the 2008 Economic Crisis.
Labour productivity is constant in the short run and changes endogenously in the medium-run in the rest of the economy, as we assume technological change or adoption of new techniques take time. We assume productivity in the social sector is constant, and simply equal to output per hour of employment in both the short and the medium-run.\(^7\) Labour productivity in the N sector \((T_t^N)\) is

\[
\log T_t^N = t_0 + t_1 \log \left( \frac{C_{t-1}^H + C_{t-1}^H}{N_{t-1}} \right) + t_2 \log \left( \frac{I_{t-1}^G}{N_{t-1}} \right) + t_3 \log Y_{t-1}^N + t_4 \log w_{t-1}^{NF} + t_5 \log (a_{t-1}^N w_{t-1}^{NF}) + t_6 \log \left( \frac{U_{t-1}}{N_{t-1}} \right)
\]

In the medium-run, the labour productivity is likely to be positively affected by lagged values of social infrastructure investment provided by the government as well as households’ consumption expenditures in marketized social services \((C^H)\), and physical public investment \((t_1, t_2 > 0)\). We also expect domestic unpaid care labour to affect labour productivity positively \((t_6 > 0)\). Substituting equation (17) for unpaid care labour, we are able to model the effect indirectly via the effect of public and private spending in H.\(^8\) We expect the effects of these to be realised over a longer time period, namely in the next period. Higher output would also lead to higher labour productivity due to Verdoorn effect (Naastepad, 2006; Hein and Tarassow, 2010), as greater scale can lead to more efficient allocation of sources \((t_3 > 0)\). Moreover, following Marx (1867) and later the theoretical contributions and empirical findings of Naastepad (2006), Taylor (2004) and Hein and Tarassow (2010), we expect that higher female and male wages in N increases the firm’s preference for labour-saving technologies, which in turn increases labour productivity \((t_4, t_5 > 0)\). This is also consistent with the new Keynesian efficiency wage theories (Shapiro and Stiglitz, 1984). Higher output and higher wages also have a lagged effect, since the change in technology and/or techniques pushed by these factors would require time. This is also consistent with Hein and Tarassow (2010) who estimate lagged positive effects of wages and output on labour productivity. The next period is a sufficiently long time period for these effects to be realised, e.g. five years or more;\(^9\) furthermore the time required for

---

\(^7\) Output in H is simply equal to the wage bill in H, as there is no profit in H. Productivity in H is \(w_t^{HF}(\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)\) as defined in Equation 14. Increasing productivity in H is less related to the availability of technology or better skills, as the quality of these services is more important and is in many cases requires more hours of nurses, care workers, teachers per patient or student.

\(^8\) This simplification is also imposed by the unavailability of time series data for unpaid care labour at the stage of empirical analysis.

\(^9\) In Section 5 in the empirical analysis, we take the five year sum (non-overlapping average of explanatory variables starting from 1970 and of the dependent variable starting from 1971) of the explanatory variables,
these different factors to affect productivity is an empirical question, e.g. the impact of public investment in childcare may take longer than the impact of other types of government spending or higher wages. In the theoretical model, we abstract from differences in the lag structure of the effects, and represent the long-time period with a single lagged effect. Using (17) and (27) we can further simplify productivity as in (27’), for the purposes of the analytical solution in the next section

\[
\log T_t^N = h_0 + h_1 \log \left( \frac{G_{t-1}^H + C_{t-1}^H}{N_{t-1}} \right) + h_2 \log \left( \frac{I_{t-1}^G}{N_{t-1}} \right) + h_3 \log Y_{t-1}^N + h_4 \log w_{t-1}^{NF} + h_5 \log \alpha_{t-1}^N
\]  

(27’)

where \( h_0 = t_0 + g_0 t_6 \) and \( h_1 = t_1 + g_1 t_6 \).

Unpaid domestic care labour, \( U_t \), is shared between women (\( U_t^F \)) and men (\( U_t^M \)), where \( \beta_d \) is the share of \( U_t^F \) in \( U \), and is exogenously determined by social norms:

\[
U_t^F = \beta_d U_t
\]  

(28)

\[
U_t^M = (1 - \beta_d) U_t
\]  

(29)

In case of extreme gender inequality \( \beta_d = 1 \).

Female and male labour force participation rates (labour force as a ratio to population, \( N_t^F \) and \( N_t^M \)) are positive functions of average wages, benefits and social infrastructure and negative functions of \( U_t \). Hence female and male labour force is

\[
L_t^F = (l_{1F}(w_t^{FH} + w_t^{FN}) + l_{2F}(G_t^H + l_{3F}U_t^F)N_t^F
\]  

(30)

\[
L_t^M = (l_{1M}(w_t^{MH} + w_t^{MN}) + l_{2M}(G_t^H + l_{3M}U_t^M)N_t^M
\]  

(31)

For simplicity we consider benefits as part of \( G_t^H \).

If employment grows faster than the labour force for a particular type of worker, unemployment rate will decrease, and vice versa. If demand for employment, \( E \), for a particular type of worker is not met by an increase in labour supply due to constraints in supply, e.g. a low female labour supply due to lack of provision of public social infrastructure for care, either there will be an exogenous increase in labour supply due to migration, or gender norms, and occupational segregation coefficients will change or wages will adjust.

Changes in population via increased migration, to relax labour supply constraints in the care economy due to rising need for care work along with rising female employment is not analysed in this model, and is assumed to be exogenous. Similarly, to simplify the model we

which is a proxy for the human capital stock. Investment in the sector is also added in the panel data analysis, whereas in the aggregate theoretical model here, it is part of aggregate output.
also do not model the impact of social infrastructure and unpaid care on fertility or mortality rates. Both are potential extensions for future research.

While in our model for simplicity we ignore the feedback effects of changes in labour supply and consequently unemployment on wages, it is realistic to assume that, in the long-run, changes in labour demand vs. labour supply can lead to changes in wages, however to simplify the model wages are set exogenously based on bargaining power, institutions and social norms.

Similarly, a rise in wages in a particular sector, e.g. H as an outcome of higher public social infrastructure, or a faster increase in wages in the social sector compared to wages in the rest of the economy is likely to lead to higher labour supply of both men and women. This would lead to also changes in the sectoral segregation ratios in the social sector and the rest of the economy, as well as a change in social gendered norms and the distribution of unpaid domestic labour. While these are interesting extensions, they are outside the scope of this paper, where our primary aim is to analyse the impact of exogenous changes in wages and gender pay gap on employment of women and men and fiscal policies.

Finally, net wealth will change with changes in income and net saving propensity of different income groups.

\[
\log(PW_t(1 - t^P)) = a_0 + a_F \log(WB^F_t (1 - t^W)) + a_M \log(WB^M_t (1 - tW)) + aR\log(Rt1- tR) + ac\log(PWt- 11 - t- 1PW)
\]

We model the wealth concentration, i.e. \(PW1/PW(\lambda_t)\) as a function of functional income distribution (after tax profit share), gender wage gap, wealth tax, and lagged value of wealth

\[
\log(\lambda_t) = s_0 + s_1 \log(\pi_t(1 - t^R)) + s_2 \log(t^P_t) + s_3 \log(\alpha_t^N) + s_4 \log(\alpha_t^H) + s_5 \log(\lambda_{t-1})
\]

The effect of the profit share captures the effect of the different marginal propensity to save from profit vs. wage income as well as the scale and type effects due to differences in the assets and liabilities of households earning predominantly capital vs. labour income (Gabaix et al. 2016; Benhabib et al., 2011, 2019; Piketty, 2014, 2015; Fagereng et al. 2016; Kaplan et al., 2018). The effects of gender pay gaps capture such differences between male and female workers. Tax on wealth is expected to affect wealth concentration if it has a
progressive nature. Finally, wealth concentration is expected to have a strong path dependency, and be significantly correlated to its past values.\footnote{Public borrowing (debt/GDP) could have an effect on wealth and its concentration but as the empirical estimations for the UK did not indicate significant effects, we did not include it in the theoretical model for simplicity. Similarly, asset prices are not included as exogenous variables in modeling wealth in order to focus on the interaction of wealth and income distribution in the model.}

The net wealth of top 1% \( (PW_{1\%}) \) and bottom 99% \( (PW_{99\%}) \) are given by

\[
\log(PW_{1\%}(1 - t^{PW}_{t})) = \log(PW_{t}(1 - t^{PW}_{t})) + \log(\lambda_{t}) \tag{34}
\]

\[
\log(PW_{99\%}(1 - t^{PW}_{t})) = \log(PW_{t}(1 - t^{PW}_{t})) + \log(1 - \lambda_{t}) \tag{35}
\]

3. The impact of labour market policies affecting wages and the gender pay gap

3.1 The effects of a change in female and male wages in N

We first analyse the impact of rising female and male wages in N in the short-run with a constant ratio between male and female wages \( (\alpha^{N}) \).\footnote{We abstract from changes in inter-sectoral labour supply in response to changes in wages in N with constant wages in H. In Section 5, we combine this static analysis with the case when wages increase both in N and H at the same rate, which is a more realistic scenario.} A change in wage rates, affects the share of wages (and profits) in national income, which in turn affects wealth concentration. Following Bowles and Boyer (1995), we classify the regimes in which rising wages (rather than wage shares as in Bhaduri and Marglin (1990)) leads to an increase in aggregate output as wage-led, and we classify the regimes in which higher wages reduce aggregate output as profit-led.

The effect of rising wages in N on aggregate output in the short-run \( (\Psi^{NF}_{t} \text{ in Appendix 3}) \) is through the effects on consumption in N and H, private investment, exports, imports and the consequent multiplier effects \( \left( \frac{1}{1 - \varphi^{NF}_{t}} \right) \). Details of the effects coming through each component of demand are shown in Appendix A3.1. We define a demand regime as wage-led in the short-run if the impact of a simultaneous increase in female and male wages in N on aggregate demand is positive \( (\Psi^{NF}_{t} > 0) \) and as profit-led in the short-run if the impact is negative \( (\Psi^{NF}_{t} < 0). \footnote{While the definition of short-run demand regimes are comparable to the previous literature based on Bhaduri and Marglin (1990), the medium-run effects combine both demand and supply-side effects, and therefore refers to the properties of the economy rather than just the demand regime.}

For constant total output, a rise in female and male wages in N does not have a partial impact on female and male employment in N and H, since an increase in labour productivity through switching to labour saving technologies takes place only with a lag. Hence, the total impact on female and male employment in N and H is only due to the change in total output.
Moreover, for constant output, an increase in female and male wages in N squeezes the profit share \(\left(\frac{\partial \pi_t}{\partial w_t^{P,F}}\right)_{Y_t,A_t^N} < 0\) as the partial impact on employment in N is zero.

An increase in female and male wages in N has a partial impact on consumption in N and H through changing distribution of income and the impact of this on wealth in the short-run. For constant income, we expect rising wages in N to have a positive partial impact on the total consumption (in N and H) in the short-run through declining profit share, as the marginal propensity to consume out of wage income is expected to be larger than that out of profit income, based on empirical estimations (see Hein and Vogel, 2008; Onaran and Galanis, 2014; Onaran and Obst, 2016; Obst et al., 2019 for the UK). Higher wages in N are expected to reduce the wealth share of the top 1% while increasing total wealth, which together affect the consumption in N and H through \(PW1\) and \(PW99\).

Female and male wages in N have a partial negative impact on private investments through squeezing profit share in the short-run. Higher wages in N also affect private investment through their effects on \(PW1\) and \(PW99\). Furthermore they have a modest short-run partial effect on public debt/GDP if the tax rates on labour and capital income aren’t the same, because the change in income distribution affects tax revenues from labour and capital income.

Finally, higher female and male wages in N lead to rising unit labour costs (falling profit share), and have a partial negative on exports and a positive impact imports in the short-run.

In the next period, a rise in female and male wages in N affect labour productivity, and has further effects on aggregate output through changes in consumption in N and H, private investment, export, imports, government expenditures and the consequent multiplier effects \(\frac{1}{1-\varphi_{NF}}\).

Figure 2 summarizes these channels. As discussed above, we expect higher wages in N to increase labour productivity. The rest of the effects are via changes in aggregate output in the short-run. If demand is wage-led in the short-run, higher wages in N also leads to higher labour productivity in the next period through higher aggregate output due to the Verdoorn effects. Moreover, we expect higher output to have further positive impact on the medium-term productivity through increasing consumption in H, public social expenditures and other public expenditures. The effects via aggregate output are the opposite if demand is profit-led in the short-run.

**Figure 2 here**
For a constant aggregate output, female and male wages in N have a negative partial medium-run effect on female and male employment in N if their influence on the labour productivity is positive, since higher labour productivity is expected to reduce the demand for labour for a constant output. Under these conditions, the medium-run partial effect of higher female and male wages in N on the profit share is also positive due to declining wage costs. Nevertheless, higher female and male wages in N could also have a negative partial medium-run impact on the labour productivity and profit share if demand is profit-led in the short-run and the effects via aggregate output are sufficiently large.

The medium-run partial impact of female and male wages in N on consumption in N and H will be via changes in wage income, profits and PW1 and PW99. Nevertheless, the effect of higher female and male wages in N is ambiguous due to the ambiguity of its medium-run effect on the profit share. The medium-run partial impact of female and male wages in N on private investment is via the effect on the profit share, public debt/GDP and PW1 and PW99. If demand is wage-led in the short-run, the medium-run effect of wages in N is also more likely to be positive as its effects through the profit share, public debt/GDP and wealth are more likely to be positive.

For a constant aggregate output, the medium-run effect of female and male wages in N on trade depends on the medium-run effect on labour productivity. If the impact of labour productivity is positive, higher female and male wages in N have a positive partial impact on exports and a negative effect on imports in the medium-run.

Table 1 summarises different regimes and their conditions. In the short-run, an increase in female wages in N with a constant gender wage gap is likely to have a positive partial effect on consumption in N and H and a negative partial effect on private investment and net exports. Therefore, the size of the effect on consumption relative to private investment and net exports determines whether the regime is profit-led or wage-led. If the sum of effects in the short-run and the next period is positive \((\Psi_{t(t-1)}^{NF} + \Psi_{t(t-1)}^{NF}) > 0\), we define this regime as wage-led in the medium-run. If the total effect is negative \((\Psi_{t(t-1)}^{NF} + \Psi_{t(t-1)}^{NF}) < 0\), we define this regime as profit-led in the medium-run.

Table 1 here

As the impact of wages in N on labour productivity and the profit share in the next period is ambiguous, we cannot predict their effect on each component of aggregate output in the next period and the likely conditions that would make an economy wage-led or profit-led in the medium-run without knowing the size of the effects on productivity. Theoretically, an
economy that is profit-led in the short-run could be wage-led in the medium-run, if the increase in wages has a sufficiently large positive effect on labour productivity in the next period, because increasing labour productivity could offset the squeeze on the profit share. Similarly, an economy that is wage-led in the short-run could theoretically be profit-led in the medium-run, if higher wages lead to a significant shift to labour-saving techniques, which would substantially reduce employment and hence labour income. Nevertheless, if an economy is wage-led (profit-led) in the short-run, it is likely that the economy will be wage-led (profit-led) in the medium-run too.

Finally, Figure 3 summarizes the impact of a simultaneous increase in female and male wages in N on employment. A simultaneous increase in female and male wages in N increases female and male employment in N and H in the short-run, if the economy is wage-led in the short-run. Similarly, in the next period changes in aggregate output affects employment, depending on whether output increases or decreases. Nevertheless, a simultaneous increase in female and male wages also affects employment in N in the next period via an increase in labour productivity. Therefore, an economy that is wage-led in the medium-run could also experience a decline in both female and male total employment along with higher female and male wages in N.

**Figure 3 here**

### 3.2 The effects of a change in the gender wage gap in N

In this section, we analyse the effects of closing the gender wage gap in N through a rise in female wages in the rest of the economy. This can be achieved via an upwards convergence, i.e. female wages increasing faster than male wages or downward convergence, i.e. female wages decreasing slower than male wages, or with only female wages increasing. In this section, we focus on the latter case where the male wages in N are constant (\(w^{NM} = w^{NM'}\)) and only female wages in N increase. We analyse the effects of the former case in section 5 empirically by combining the analysis in sections 3.1 and 3.2.

The short-run effect of closing the gender wage gap in N on aggregate output (\(\Psi_{it}^{Ne}\)) also depends on its effects on consumption in N and H, private investment, export, imports and the multiplier (\(\frac{1}{1-\varphi_{NF}}\)). Details of these effects are in Appendix A3.2. We define a demand regime, in which a rise in only female wages with closing the gender gap in N leads to a higher aggregate output in the short-run (\(\Psi_{it}^{Ne} > 0\)) as female wage-led or gender equality-led in the short-run. If a rise in female wages with closing the gender gap in N leads to lower
aggregate output in the short-run ($\Psi_{tt}^{Na} < 0$), the demand regime is defined as *gender inequality-led in the short-run*.

For a constant aggregate output, the partial effects of a rise in female wages in N on employment are zero in the short-run. The short-run partial effect on the profit share is expected to be negative ($\left| \frac{\partial \pi_t}{\partial w_{t}^{NF}} \right|_{y_t} < 0$) although the magnitude of this effect is expected to be smaller compared to the short-run partial effect of a simultaneous increase in both female and male wages in N on the profit share.

For a constant aggregate output, the signs of the short-run partial effects on consumption in N ($\left| \frac{\partial c_t^N}{\partial w_{t}^{NF}} \right|_{y_t}$) and consumption in H ($\left| \frac{\partial c_t^H}{\partial w_{t}^{NF}} \right|_{y_t}$) depend on the marginal propensity to consume in N and H out of female wage income relative to capital income and also the effects through the changes in PW1 and PW99. A higher positive effect on consumption in H ($\left| \frac{\partial c_t^H}{\partial w_{t}^{NF}} \right|_{y_t}$) is more likely than on consumption in N ($\left| \frac{\partial c_t^N}{\partial w_{t}^{NF}} \right|_{y_t}$), because women tend to devote a larger share of their income on social expenditures like education and healthcare compared to men as shown in numerous studies (Phipps and Burton, 1998; Lundberg and Pollak, 1996; Morrison et al., 2007).

For a constant aggregate output, the effect of closing the gender wage gap in N in the short-run on investment ($\left| \frac{\partial i_t}{\partial w_{t}^{NF}} \right|_{y_t}$) depends on changes in the profit share, public debt/GDP ($D/Y$), PW1 and PW99. Again, the magnitude of this effect is expected to be smaller than the partial effect of a simultaneous increase in female and male wages, since it reduces the profit share less, which in turn is expected to affect $D/Y$ and wealth less. For a constant aggregate output, closing the gender wage gap in N reduces exports and increases imports; however, the magnitude of the effect on both is smaller than the effects of a simultaneous increase in female and male wages in N.

Table 2 summarises the alternative demand regimes in the short-run. An economy could be wage-led and female wage-led/gender equality-led or profit-led and gender inequality-led in the short-run. We define the former as *equality-led demand regime* and the latter as *inequality-led regime*. However, an economy could also be wage-led and gender inequality-led or profit-led and female wage-led/gender equality-led in the short-run at the same time depending on the marginal propensities to consume out of female and male wage income and capital income.

*Table 2 here*
The effect of closing the gender wage gap in N in the next period is mainly through its influence on labour productivity in N. The effects are very similar to those that were depicted in Figure 2 except that the shock is through only rising female wages in N. The direct impact of female wages in N on labour productivity through labour-saving technologies is smaller compared to the case of a simultaneous rise in both female and male wages in N. If the economy is female wage-led/gender equality-led in the short-run, closing the gender wage gap in N increases labour productivity in the next period as its effects through aggregate output and consumption in H and government expenditures are also expected to be positive. However, theoretically, the gender wage gap in N could also reduce labour productivity in the next period if the economy is gender inequality-led in the short-run.

The medium-run partial effects of rising female wages in N on female and male employment in N \((\left|\frac{\partial E_{t}^{NF}}{\partial w_{t-1}^{NF}}\right|_{Y_t}, \left|\frac{\partial E_{t}^{NM}}{\partial w_{t-1}^{NF}}\right|_{Y_t})\) are negative, if the effect of wages on labour productivity in N is positive. In this case female wages in N also have a positive partial effect on the profit share in the next period \((\left|\frac{\partial \pi_{t}^{F}}{\partial w_{t-1}^{NF}}\right|_{Y_t} > 0)\).

The medium-run partial effects of closing the gender wage gap in N on the components of demand depend on the same channels as in the effects of simultaneous increases in female and male wages in N except for the differences in the magnitudes. If closing the gender wage gap in N leads to higher labour productivity, then the partial effect on consumption in N \((\left|\frac{\partial c_{t}^{N}}{\partial w_{t-1}^{NF}}\right|_{Y_t})\) and H \((\left|\frac{\partial c_{t}^{H}}{\partial w_{t-1}^{NF}}\right|_{Y_t})\) in the next period is negative. The rest of the effects on consumption in N and H are through the medium-run effects on PW1 and PW99.

The medium-run partial effects of closing the gender wage gap in N on private investment \((\left|\frac{\partial i_{t}}{\partial w_{t-1}^{NF}}\right|_{Y_t})\) depend on the changes in the profit share, public debt/GDP, PW1 and PW99. The partial effects on the profit share are positive or negative depending on whether higher female wages increase or decrease labour productivity in the next period. Similarly, the medium-run partial effects on export and imports depend on productivity effects.

We define an economy in which the sum of the short-run and medium-run impact of an increase in female wages in N on output is positive as female wage-led/gender equality-led in the medium-run \((\psi_{t(t-1)}^{Na} + \psi_{tt}^{Na} > 0)\). The case in which the sum of the short-run and medium-run impacts is negative is defined as gender inequality-led in the medium-run \((\psi_{t(t-1)}^{Na} + \psi_{tt}^{Na} < 0)\).
Finally, the effects of closing the gender wage gap in N on employment are similar to the case depicted in Figure 3. If the economy is female wage-led/gender equality-led in the short-run, closing the gender wage gap in N increases female and male employment in the short-run. The impact of closing the gender wage gap in N in the next period affects the female and male employment via changes in labour productivity and aggregate output in the medium-run and could lead to a fall in female and/or male employment in the medium-run even if the economy is wage-led in the medium-run.

4. The effects of fiscal policy

4.1 The effects of an increase in public social infrastructure investment by increasing employment in H

In this section, we analyse the case where social expenditure as a share of GDP (κ_{H}^{E}) increases solely through the new public sector employment in the social sector with constant wages in this sector. In the UK, the share of female employment in the social sector (H) is significantly larger than the share of female employment in the rest of the economy (N). Therefore, we expect that with this policy larger employment for female workers would be generated in the short-run in the public social sector.

The short-run effect of higher public social infrastructure investment as a share of GDP on aggregate output depends on the effects on consumption in N and H, private investment, public expenditures and the consequent multiplier effects as described in Appendix A4.1. An increase in the public social infrastructure investment affects female and male employment in N and profit share only through the multiplier effects of changes in aggregate output in the short-run; i.e. the partial effects are zero \( \left| \frac{\partial E_{t}^{NF}}{\partial \kappa_{H}^{E}} \right|_{Y_{t}^{N}} = \left| \frac{\partial E_{t}^{NM}}{\partial \kappa_{H}^{E}} \right|_{Y_{t}^{N}} = \left| \frac{\partial \pi_{it}}{\partial \kappa_{H}^{E}} \right|_{Y_{t}^{N}} = 0 \)\(^{13}\). The short-run effects are summarised in Figure 4.

**Figure 4 here**

An increase in public social infrastructure investment has a direct positive effect on aggregate output in the short-run. Moreover, the generation of new employment in the public social sector is also expected to stimulate consumption in N and H in the short-run. The wage payments in the public social sector also have a positive partial impact on both \( PW1 \) and 99 \( \left| \frac{\partial PW1_{it}}{\partial \kappa_{H}^{E}} \right|_{Y_{t}^{N}} > 0 \) and \( \left| \frac{\partial PW1_{it}}{\partial \kappa_{H}^{E}} \right|_{Y_{t}^{N}} > 0 \), which also increases consumption in N and H.

\(^{13}\) For a constant aggregate output, higher public social infrastructure investment does not affect the profit share and thereby employment.
For a constant aggregate output in N, higher public social infrastructure investment has a positive impact on private investment in the short-run through rising total aggregate output. The possible positive effects on PW1 and PW99 also affect private investment in the short-run; the effect of PW1, in particular, is ambiguous as discussed in Section 2. However, an increase in public social infrastructure investment may partially crowd out private investment due to an increase in public debt/GDP in the short-run, if this leads to an increase in interest rates and if the investment is sensitive to interest rates.

Next, we analyse the effect of an increase in public social infrastructure investment on labour productivity in the next period as summarised in Figure 5. We expect higher public social infrastructure investment to increase labour productivity in the next period. Moreover, higher public social infrastructure investment also affects labour productivity in the next period through changes in aggregate output depending on whether the effects on aggregate output are positive or negative in the short-run. If higher public social infrastructure investment stimulates aggregate output, it also leads to an increase in households’ social expenditures and public physical infrastructure investment in the short-run which may also increase the labour productivity in the next period.

**Figure 5 here**

An increase in the share of public social infrastructure in GDP has an effect on aggregate output in the next period through changes in labour productivity, public debt/GDP and wealth as summarized in Figure 6. For a constant aggregate output in N, the medium-run partial effect of public social infrastructure on female and male employment is negative and the effect on the profit share is positive if higher public social infrastructure increases labour productivity in the next period. This also affects consumption in N and H, private investment, exports and imports in the next period as discussed in Sections 3.1-2. The effects of a possible increase in public debt/GDP in the short-run effects are transmitted to $D/Y$ in the next period. This reduces private investment unless there is a sufficient increase in GDP and tax revenues to offset the increase in debt. Finally, the short-run effects on wealth and medium-run effect on the labour productivity (hence employment and the profit share) also affect PW1 and PW99 in the next period, which affects consumption in N and H and private investment in the next period.

**Figure 6 here**

We depict the overall effect of higher public social infrastructure on employment in Figure 7. Higher public social infrastructure directly generates female and male employment in the
social sector in the short-run; however, it also is likely to generate further employment in the whole sector by increasing the GDP in the short-run. It is also expected to increase the labour productivity in the next period, which has a direct negative effect on the employment and might lead to a positive or negative effect through aggregate output in the next period.

**Figure 7 here**

Finally, we analyse the impact of higher public social infrastructure investment on public debt as a ratio to GDP. Higher public social infrastructure investment has an ambiguous effect on public debt/GDP in the short run as both debt and GDP increase. Higher public social infrastructure investment can lead to lower $D/Y$ if the effect on GDP is sufficiently large. The rise in GDP increases both the denominator and tax revenues.

**4.2 The effects of an increase in public social infrastructure investment via a simultaneous increase in female and male wages in H**

In this section, we analyse the effect of an increase in public social infrastructure investment as a ratio to GDP due to an increase in female and male wages in H with a constant $\alpha^w$ as opposed to directly increasing employment in H. The short-run effects of this change are very similar to the case where public social infrastructure expenditures increase via increasing employment in H with constant wages. Similar to Section 4.1, higher public social infrastructure expenditures due to an increase in female and male wages in H has a direct positive impact on aggregate output. Moreover, it affects the economy due to rising total wage payments in H, which in turn stimulates consumption in N and H, $PW1$ and $PW99$, which have further effects on consumption in N and H and private investment. However, if the public debt/GDP increases, this leads to a negative effect on private investment.

The channels in the medium-run are identical to the effects of an increase in public social expenditures due to higher employment in H. Higher female and male wages in H would increase labour productivity in the next period as described in Figure 5. It has a direct positive effect on labour productivity and also is likely to increase labour productivity due to Verdoorn effects and the increases in households’ social expenditure and other public expenditures stimulated by higher aggregate output in the short-run. Higher labour productivity may have either positive or negative effects on aggregate output in the next period due to the channels discussed in Sections 3.1-2 and 4.1.

If public debt/GDP increases in the short-run, the public debt/GDP in the next period will be affected as well, and in turn will lead to a negative effect on private investments in the
next period. However, this effect might be alleviated or even reversed if greater female and male wages in H increase the aggregate output and tax revenues sufficiently.

4.3 Effects of a change in the gender wage gap in H

This section analyses the effect of an increase in public social expenditures by closing the gender wage gap in H by increasing female wages with a constant male wage. Figure 8 describes the channels in which closing the gender wage gap in H affects aggregate output in the short-run. Compared to a simultaneous rise in female and male wages in H, the short-run effects on consumption in N and H are smaller for the same amount of increase in wages. However, the difference between the effects of these two changes on consumption is smaller than the difference between the effects of a simultaneous increase in female and male wages in N and closing the gender wage gap in N, since female workers constitute a larger part of employment in H.

Figure 8 here

The medium-run effects of closing the gender wage gap in H on labour productivity in the next period would be the same as in Figure 5 as discussed in Section 4.2.

4.4 Effects of an increase in public physical infrastructure investment

This section analyses the effects of a rise in public physical infrastructure investment as a share of GDP. In the short-run, higher public physical infrastructure investment has a direct positive effect on aggregate output. However, it also has a positive partial impact on public debt/GDP, which in turn leads to a negative partial effect on private investment \( \left( \frac{\partial \pi^d_t}{\partial x_p t} \right)_{y_t} < 0 \).

The combination of the direct effect of public physical infrastructure investment and its influence through public debt/GDP along with their multiplier effects determines the sign of the effect of public physical infrastructure investment in the short-run as shown in Appendix A4.4.

The medium-run effect of public physical infrastructure investment is determined by changes in labour productivity and public debt/GDP in the next period. As discussed in Section 5 below, we don’t find a positive direct effect of public physical infrastructure investment on labour productivity in the UK. However, public physical infrastructure investment can still affect labour productivity through changes in the aggregate output in the short-run. For a constant output, the medium-run effect of public physical infrastructure investment on public debt/GDP \( \left( \frac{\partial (D/Y)_t}{\partial x_p t} \right)_{y_t} \) depends on the changes in public debt and tax
revenues. If public debt/GDP increases in the next period, this also will affect private investment negatively.

4.5 The effects of taxes on wealth, profits and wages

This section analyses the effects of an increase on tax rates on wealth, profit and wage income. The details of the effects are in Appendix 4.5.

An increase in the tax rate on wealth leads to not only a decline in after-tax wealth, but also could reduce wealth concentration, i.e. the share of the top 1% in total wealth tax, if taxation on wealth has a progressive character. The changes in PW1 and PW99 have short-run effects on consumption in N and H and private investment. Furthermore, an increase in the tax rate on wealth leads to a decrease in public debt, which has a positive effect on private investment in the short-run unless the decrease in the total wealth and consequently consumption leads to a substantial decrease in tax revenues. The short-run effects of wealth tax on aggregate output also affect labour productivity in the next period, which in turn affect aggregate output in the next period through the channels discussed in sections 3-4. An increase in the tax rate on wealth has further medium-run effects on total wealth, wealth concentration and public debt/GDP, which in turn affect aggregate output in the next period.

An increase in the tax rate on profit income has a negative short-run impact on total wealth \( \left( \frac{\partial PW_t}{\partial t_t} \right)_{Y_t} < 0 \) and PW1 \( \left( \frac{\partial PW1_t}{\partial t_t} \right)_{Y_t} < 0 \), for a constant total output. However, its effect on PW99 is ambiguous as a lower wealth concentration has a positive effect on the wealth of the bottom 99%. The effects due to changes in PW1 along with tax rate on profit income have negative effects on consumption in N and H in the short-run. A higher tax rate on profit leads to a decline in the after-tax profit share and has a negative partial effect on investment in the short-run, for a constant aggregate output. However, it may also have a positive effect on private investment due to a decline in public debt/GDP, unless the decline in wealth and consumption leads to a substantial decline in tax revenues. The changes in PW1 and PW99 also affect private investment (either positively or negatively) in the short-run. The increase in the tax rate on profit affects the aggregate output in the next period through its effects on labour productivity, total wealth, wealth concentration and public debt/GDP.

Finally, we analyse the impact of an increase in the tax rate on wage income. For a constant total output, this has a negative short-run impact on the total wealth \( \left( \frac{\partial PW_t}{\partial t_t} \right)_{Y_t} < 0 \) and it also influences the wealth share of the top 1% in the short-run. The rise in the tax rate on wage income affects consumption in N and H in the short-run due to the negative effects
on the disposable incomes of workers as well as PW1 and PW99. The change in PW1 and PW99 also affect private investment in the short-run. Moreover, as in the case of other taxes, an increase in the tax rate on wage income may reduce public debt/GDP in the short-run which in turn may have a positive effect on private investment in the short-run, unless the decrease in wealth and consumption substantially reduces tax revenues. There are further effects on GDP in the next period due to changes in labour productivity, total wealth, wealth concentration and public debt/GDP.

5. Data, estimation methodology and results

The behavioural specifications are econometrically estimated using time series data for the UK. The definitions of all variables and data sources are in Appendix 1. The hourly wage and hours of work data are calculated based on data supplied by the EUKLEMS database for the period of 1970-2015. The national accounts time series data is based on the Annual Macro Economic database of the European Commission (AMECO) and the OECD for the period of 1970-2016. The implicit tax rates are based on Eurostat data. The ratio of consumption in the social sector to total consumption is based on ONS (2016a) data.

The data on wealth concentration for the share of the top 1% vs. the 99% in net private wealth is provided by the World Wealth and Income Database, which gathers information from national accounts, surveys, fiscal data and wealth rankings. We relied on data provided by Credit Suisse (2014-17) for the missing years in the former database. More detailed wealth data, which could allow distinction along with functional income categories or gender, based on household surveys for the UK (Wealth and Assets Survey) cover only a very short period of 4 waves during 2006-14, and therefore does not provide sufficient observations for macroeconomic analysis.

The stylised facts of our data are presented in Appendix 5. Figure 1 above presents the variables related to gender inequality measured by the ratio of the hourly wage rate of men/women (α) and the share of women in hours worked (β) in the social sector (H) and the rest of the economy (N) in the UK. Despite an improvement in all measures since the early 1980s, as of 2015 α\textsuperscript{H} and α\textsuperscript{N} are still as high as 1.313 and 1.230 respectively (own calculations based on EUKLEMS data). The gender composition of employment has been relatively more stable, and the share of women in hours worked in N (β\textsuperscript{N}) is still as low as 0.406 and women still constitute the vast majority of employment in H (β\textsuperscript{H}) with a ratio of 0.752 in 2015 (own calculations based on EUKLEMS data).
Figure 9 shows the functional income distribution, i.e. the share of wages in national income (labour compensation/GDP at factor cost, adjusted for the labour compensation for each self-employed equivalent to the average compensation of the dependent employees) and wealth concentration (share of the top 1% in total net wealth, λ) in the UK. The share of wages in GDP fell from its peak of 0.706 to 0.584 in 1996 and despite a recovery since then, it is 4%-point below its peak at 0.665 as of 2016. Wealth concentration, measured by the share of the top 1% in total net wealth, has fallen from 0.283 in 1972 to 0.152 in 1984 and has risen sharply since then to 0.233 as of 2016.

**Figure 9 here**

There is no time series data dating back to 1970s for unpaid care work and its gender distribution; however, there is time use survey data for selected years. In 2014 women carried out 61.5% of all the hours of unpaid work at the household and 69.3% of the unpaid care work (in adult care and child care, laundry, cleaning and housework) in the UK (ONS, 2016b). Hence, the gender composition of hours of unpaid care work is similar to the composition of paid care work.

The productivity in N is estimated using panel data of 18 industries based on EUKLEMS for the period of 1970-2015. In order to reflect medium-term effects a non-overlapping five years average of explanatory variables starting from 1970 and of the dependent variable starting from 1971 are used to account for the lagged effects. Clustered standard errors are used. Different from equation (27’) for the aggregate economy, the sector’s own investment (I_t) is also included, as at the panel data level sectors value added (Y_{it}) does not include sector’s I_{it}, while at the aggregate level Y^N includes private investment. The use of panel data helps to model the medium-run effects on productivity, which may not be easy to detect using time series due to the short time dimension of data. The integration of time series and panel data analysis to estimate the parameters of the structural behavioural specifications is a methodological novelty of the proposal.

Households’ social and other consumption expenditures are estimated as interdependent decisions as part of a system using seemingly unrelated regression (SUR). We estimate separate single equations for investment, exports, and imports. We choose the single equation

---

14 The last year is determined by data availability. Electricity, gas and water; construction; public administration and defence, compulsory social security; agriculture, forestry and fishing and mining and quarrying (as well as education and health and social work) are excluded due to the difficulty in measuring productivity in these sectors. The results are rather robust to the inclusion of these sectors. The results are also robust to estimations excluding the post-2008 Great Recession period.

15 As discussed in Section 2, the use 5-year sum (average) serves as a proxy for capital stock in terms of both private and public human and physical capital.
approach because it allows a clearer interpretation of the results. We also present two-stage least square estimations (and three-stage LS in the case of consumption) using instrumental variables (IV) to test robustness, where instruments are contemporaneous, first and second lagged values of hourly wage rates and employment shares of women and gender pay ratios in H and N, tax rates on labour and profit income and wealth (and the GDP of the rest of the world in the exports equation). Endogeneity issues could also be tackled by using a VAR method, however, this would require a large number of observations, and would make it difficult to individually specify each behavioural equation (Onaran and Obst, 2016).

Unit root tests suggest that all our variables except for wealth concentration are integrated of order one.\(^\text{16}\) We first estimate error-correction models (ECM), except for wealth concentration. If no cointegration is found, the equations are estimated in differences.\(^\text{17}\) We start with general specifications with contemporaneous and lagged effects and arrive at the most parsimonious specification. In the case of wealth concentration, which is stationary, we estimate an ARDL (autoregressive distributed lag model). In the simulation analysis, we treat the coefficients as non-zero even when the p-value of the t-statistic is greater than 0.10; in these cases, the p-values are often around 0.20.\(^\text{18}\) In order to test for autocorrelation, we use the Breusch-Godfrey test. We derive the long-run elasticities if ECM is significant.

### 5.1. Estimation results

SUR results for households’ social and other consumption expenditures (equations (21-22)) are given in Table 3. Multiplying elasticities with consumption in the relevant category the relevant income category, we find that the marginal propensity to consume (MPC) in N out of men’s wage income (0.73) is larger than the MPC out of women’s wage income (0.50), which in turn is larger than the MPC out of profit income (0.20). MPC in H is however highest out of women’s wage income (0.04), followed by MPC out of men’s wage income (0.02), and the MPC in H out of profit income is again the smallest (0.01). To our knowledge, this is the first empirical comparison of the marginal propensity to consume out of female and male wage income and profits. The results are consistent with other estimations showing that the marginal propensity to consume out of total wages are higher than that out of profits (see Onaran and Galanis, 2014 for a review) as well as micro-level evidence that the propensity to

---

\(^{16}\) Results are available upon request.

\(^{17}\) The t-ratios reported by Banerjee et al. (1998) are used for the speed of adjustment coefficient to test whether there is cointegration among the variables.

\(^{18}\) We follow this methodology because in our simulations in Section 6, we do not prefer to treat our variables that have intuitively expected signs and are statistically insignificant (at 10%) as zero. The problems of dismissing the effects coming through variables that are statistically insignificant at commonly accepted levels are discussed in Ziliak and McCloskey (2004; 2008) in detail.
save is higher for female workers than male workers and women tend to devote a larger share of their incomes on social expenditures like education and healthcare compared to men (Seguino and Floro, 2013; Stotsky, 2006; Morrison et al, 2007). MPC out of the wealth of the bottom 99% is 0.03 and the MPC out of the wealth of the top 1% is 0.01 in N, while the MPC out of wealth of both groups in H is negative, which we treat as zero in our simulations, due to the perverse sign as well as the very low level of statistical significance. However, the explanatory power of the estimations for C in H is rather low.

Table 3 here

Table 4 presents the estimation results for private investment based on equation (22). After-tax profit share has a significant positive effect, but there is no cointegration relationship between the profit share and private investment. Public debt as a ratio to GDP has a significant negative effect, which reflects some negative crowding out effects of public borrowing on private investment. There is a strong significant and co-integrated long-run effect of aggregate demand on private investment. In terms of wealth effects, while the net wealth of the 99% has a positive significant, albeit short term, effect on private investment, the net wealth of the top 1% is co-integrated with private investment and has a negative significant effect. While the former can be interpreted as positive effects of wealth related to the residential investment or business investment of small and medium enterprises, the latter can be interpreted as evidence of the effect of financialization and concentration. This is consistent with the firm level evidence that the rise in non-operating income originating from financial assets crowds out the physical investment of the large non-financial companies, although it has a positive effect on the investment of the relatively cash constrained, smaller companies (see Tori and Onaran, 2018 for the UK; Orhangazi, 2008 for the US and Tori and Onaran, 2019 for the EU15). The results are also in line with Stockhammer et al. (2018) who find negative significant effect of the total wealth on private investment in the UK. As a rise in the profit share leads to an increase in PW1 as we discuss below, the negative effect of PW1 on investment offsets part of the positive effect of the profit share on investment.

Table 4

Table 5 and Table 6 present the estimation results for exports and imports based on equations 25-26. The rest of the world’s GDP has a statistically significant positive impact on exports, and an increase in profit share also leads to higher international competitiveness, and thereby higher exports, although the p-value is not very high (0.29). The increase in the output in the rest of the economy leads to a significant increase in imports, and the two
variables are co-integrated. A higher profit share leads to lower imports, again reflecting the impact of higher international competitiveness.

**Tables 5-6 here**

The panel data estimation results for productivity in N based on equation (27’) are presented in Table 7. The value added and the hourly wage rates in the sector (as indicated by the female wage rate and the gender pay ratio) have a statistically significant positive effect on productivity. At the macro level, the sum of the per capita public and private spending in the social sector has a positive statistically significant effect on productivity in N. The high effect of public spending in education, childcare, health and social care on productivity in the rest of the economy provides supporting evidence that this spending serves the purpose of infrastructure investment. The effect of the sector’s own investment and government physical infrastructure investment are statistically highly insignificant and are treated as zero in the simulations, as the negative sign is rather perverse.

**Table 7**

Table 8 and Table 9 present the estimation results for total private net wealth and its concentration based on equations (32-33) respectively. An increase in the wage income of men and women and profit income lead to a rise in total wealth and there is also a strong lagged effect of past wealth. The concentration of wealth increases with increasing after-tax profit share\(^{19}\) and its own lagged value, and decreases with tax on wealth. Empirical estimations failed to indicate a statistically significant effect of the gender wage gap or the share of women in total wage income on wealth concentration. While gender wage gaps are expected to lead to gender differences in the net wealth of men and women wage earners, the share of the top 1% in net wealth seems to be related to functional income distribution, rather than gender wage gaps. However, any increase in female wages at a rate faster than male wages, which closes the gender pay gap, also decreases the profit share, and thereby affects wealth concentration. The results show how functional income inequality leads to wealth concentration, which also has a strong path dependency. Redistributive tax policies on capital income and wealth also decrease wealth concentration.

**Tables 8-9**

Appendix 6 presents the specifications in Tables 1-4 with the IV approach. While the results are very robust, in some cases the significance of the variables is lower in the IV estimations, which reflects the weakness of the instruments. In the next section, we will base

\(^{19}\) The statistically insignificant lags of the profit share are kept in the specification due to the ARDL structure because of the existence of unit root in the profit share.
the simulation analysis for the impact of changes in wages, gender pay gaps or fiscal policies on the parameters estimated by the estimations in Tables 3-6.

6. Policy analysis: effects of the labour market and fiscal policies

In this section we use the empirically estimated parameters of the model to simulate the effects of changes in wages, gender pay gap, different types of public spending, and taxes on wages, profits and wealth. Changes in wages in turn affect functional income distribution between wage and profit income. Changes in the share of profit in national income (after-tax) as well as wealth tax affect wealth concentration. In each case, the simulations assume that the increase in the wage rate, public spending/GDP ratio or tax rates take place in the first period, and then the relevant variables (e.g. the wage rate) stay constant in the next period in order to compare the effects in the short and medium-run.

Table 10 shows the total (post-multiplier) effects of changes in wages and gender pay gap on the components of aggregate demand, consumption in N and H, private investment, exports, imports, government investment in social and physical infrastructure and government current spending (all as a ratio to GDP), GDP, employment and public debt as a ratio to GDP. The details of the calculations are in Appendices 3-4. The medium-run (MR) is defined as the cumulative of the effects in the short-run (SR) and the next period when productivity in N changes endogenously.

Table 10 here

Scenario (A) presents the effects of a 1% increase in both female and male hourly (real) wage rate in N; scenario (B) presents the effects of a 1% increase in only female wages in N with constant male wages (1% decline in \(\pi^N\)); i.e. closing the gender pay gap in N by 1%. In both cases, all components of demand except exports increase both in the short and the medium-run. Exports decrease as an increase in wages in N lead to a decline in the profit share. The multiplier \(\left(\frac{1}{1-\varphi_{NF}}\right)\) is 2.234. In scenario (A), GDP increases by 0.244% in the short-run and by 0.146% in the medium-run; hence the economy is wage-led, although the effect is economically small. The increase in GDP the medium-run in all scenarios is smaller as in the next period the increase in productivity in N leads to a decline in employment in N and therefore offsets some of the demand effects. In scenario (B), GDP increases by 0.062% in the short-run and by 0.027% in the medium-run; hence the economy is gender equality-led, but the effects are even smaller than in the case when both wages increase. Hours of

---

20 Wherever required, the elasticities in the estimations in Tables 3-9 are converted to marginal effects using the averages of the relevant variables for the estimation period.
employment of both men and women increase in the short-run in both scenarios (A) and (B), but decrease in the medium-run (by 0.556% in (A) and 0.105% in (B)) as the productivity increase in N in the medium-run (0.812% in (A) and 0.153% in (B)) is stronger than the increase in GDP.

Scenario (C) presents the effects of a 1% increase in both female and male wages in H and scenario (D) presents the effects of a 1% increase in only female wages in H with constant male wages (1% decline in $\alpha^H$); i.e. closing the gender pay gap in H by 1%. Demand increases again with both higher wages and gender equality both in the short and the medium-run. Compared to the effects of increasing wages in N, the total effects on GDP are higher for a variety of reasons: The increase in consumption in H is higher because a rise in wages in H have a more substantial effect on the female wage bill and the marginal propensity to consume in H out of the female wage income is higher compared to the male wage income. The increase in private investment is higher because a rise in wages in H does not squeeze private profits. For this reason, the rise in imports in the short run is smaller and exports do not fall in the short run and increase in the medium-run, as a rise in productivity in N by 0.433% increases the profit share. The multiplier $\left(\frac{1}{1-\varphi_R}\left(\frac{1}{1-\varphi_T}\right)\right)$ is 2.245. In scenario (C) GDP increases by 0.427% in the short-run and by 0.333% in the medium-run, and in Scenario (D) GDP increases by 0.298% in the short-run and by 0.232% in the medium-run. In both scenarios, employment of women increases not only in the short-run but also in the medium-run albeit by a small amount (0.022% and 0.012% respectively), but the employment of men increase only in the short-run and decreases slightly in the medium-run due to strong productivity gains (by 0.071% and 0.054% respectively).

Finally, scenario (E) presents the effects of a 1% increase in female and male wages in both N and H, which is the sum of the effects in scenarios (A) and (C), and (F) presents an upward convergence scenario of closing the gender pay gaps with female wages increasing faster than male wages, i.e. a 2% increase in female wages and 1% increase in male wages in both N and H, which is the sum of the effects in simulations (A), (B), (C) and (D). An example of the latter scenario would be to increase average wages via an increase in the minimum wage or collective bargaining coverage while at the same time enforcing equal pay legislation and aiming at higher rates of increases in occupations at the bottom end of the pay

---

21 The increase in hourly real wage rate in N and H in GBP is comparable. A 1% increase in female wages in H and N are £0.18 and £0.17 respectively, and a 1% increase in male wages in H and N are £0.24 and £0.21 respectively in 2015.
scale, where women constitute a large share of the workforce. In the upward convergence scenario, GDP increases by 1.030% in the short-run and by 0.736% in the medium-run, but despite an increase in employment of both women and men in the short run, employment decreases in the medium-run for both (by 0.528% for women and 0.865% for men). Total employment, as well as employment of both men and women, are wage-led and gender equality-led in the short-run but not in the medium-run when wages increase in both sectors.

Along with the increase in GDP, public debt as a ratio to GDP decreases in all scenarios, including in (C)-(F), all of which include a direct increase in public social infrastructure spending via higher wage rates in H; e.g. in (F) public debt/GDP decreases by 0.354%-points in the short-run and 0.327%-points in the medium-run.

The results in scenario (A) are comparable to previous research, albeit based on the impact of the profit share on aggregate output only, which find that the UK is a wage-led economy (Bowles and Boyer, 1995; Stockhammer and Onaran, 2004; Naastepad and Storm, 2007; Hein and Vogel, 2008; Onaran and Galanis, 2014; Onaran and Obst, 2016; Obst et al., 2019; Jump and Mendieta-Muñoz, 2017; Oyvat et al., 2018). Based on our short-run results for the rise in the wage rate of both women and men in N, a 1%-point fall in the profit share leads to 0.378% increase in GDP after the multiplier, which is comparable for the previous estimation results for the UK based on aggregate data.

Table 11 shows the total (post-multiplier) effects of fiscal policies. The details of the calculations are in Appendix 4. Scenario (A) presents the effects of a 1%-point increase in public social infrastructure investment as a ratio to GDP (κ^H), i.e. hours of employment in H (e.g. more teachers, nurses, social care workers) with a constant wage rate in H. Following Ilkkaracan (2013), who coined the term “purple” economy for public social infrastructure to chime with the green economy, we label this policy as purple public social infrastructure investment. Scenario (B) presents the effects of a 1%-point increase in public physical infrastructure investment/GDP (κ^G). To indicate the priority of investment in renewable energy, public transport, and housing insulation we label this investment as green public investment. In both cases, all components of demand increase, and the increase in the medium-run is slightly smaller due to the increase in productivity. Exports increase only in the medium-run due to the increase in the profit share. A 1%-point increase in public investment in social infrastructure increases productivity (output per hour) in the rest of the economy by 3.3% percent in the medium run. The increase in productivity is substantially higher in the case of higher social infrastructure investment (3.272%) compared to the case of
higher physical infrastructure investment (0.510%). This is mostly due to the strong direct positive impact of social infrastructure on productivity as well as the higher rate of increase in household consumption in H, as more jobs are created for women in scenario (A) in H, which predominantly hires women. In the case of higher social infrastructure spending, GDP increases more (3.585% in the short-run and 2.707% in the medium-run) than the case of physical infrastructure investment (2.046% in the short-run and 1.999% in the medium-run) not only in the short-run but also the medium-run. The GDP and employment impact are substantially higher than the effects of increasing wages in Table 10. Despite productivity increases, employment increases not only in the short-run but also the medium-run for both men and women in both scenarios. However, the increase in women’s employment is much stronger compared to men in the case of social infrastructure investment due to occupational segregation and concentration of women in the social sector. Women’s employment increases by 6.722% in the short-run and 3.238% in the medium-run while men’s employment increases by 4.437% in the short-run and only 0.420% in the medium-run in (A), whereas in (B) employment of both men and women increase at a rather similar rate (2.210% for women and 2.109% for men in the short-run and 1.764% for women and 1.576% for men in the medium-run). Our short-run results are comparable to the input-output table based analysis in De Henau et al. (2016) for the UK suggesting that the positive impact of public social infrastructure investment on male employment is at least as high as the effect of public physical infrastructure investment; however when the increase in productivity in the medium-run is included in our analysis, the effect on male employment in the former case is substantially smaller. The magnitudes of the effects are not comparable as De Henau et al. (2016) focus on only construction for physical infrastructure and childcare and social care for social infrastructure.

Table 11

In both scenarios, public debt/GDP decreases in the short-run (by 0.981%-point in (A) and 0.213%-point in (B)) but increases marginally in the medium-run (by 0.497%-point in (A) and 0.550%-point in (B)). But even in the medium-run, increasing public spending funds about half of itself by generating higher output and tax revenues. It is also worth emphasizing that private investment increases overall, despite the partial negative effect of higher government borrowing thanks to the positive demand and productivity effects.

Scenario (C), (D) and (E) in Table 11 present the effects of a 1%-point increase in the implicit tax rate on capital income, wealth and labour income respectively. Increasing taxes on both capital and labour income lead to a decline in all components of demand and overall
GDP, productivity in N as well as employment for both men and women in both the short and the medium-run. However, the negative effects on demand are much larger in the case of taxes on labour, even in the case of private investment, owing to a stronger negative effect on demand and productivity in N, and thereby public debt/GDP increases in the medium-run despite a rise in the tax rate. In contrast, a 1%-point increase in the implicit tax rate on wealth has positive and very large effects on both GDP and employment of men and women; however, we have to emphasize that a 1%-point increase in the implicit tax rate on wealth is almost doubling the current rate, which stands at 0.989% in 2016 taking it back to its peak in 1970; hence an economically much more substantial increase than the 1%-point increase in the implicit tax rate on capital income. The most important effect of increasing wealth tax by 1%-point is the fall in wealth concentration by 0.876%-point\(^{22}\), which in turn decreases PW1 and increases PW99 in both the short and the medium-run. Both of these developments lead to a significant increase in private investment due to the positive effects of the increase in PW99 and the decrease in PW1 as well as higher consumption due to higher marginal propensity to consume in N out of PW99. As a consequence, GDP increases by 0.902% in the short-run and 4.285% in the medium-run; total employment increases by 0.949% in the short-run and 4.134% in the medium-run with comparable effects for both men and women. Public debt/GDP falls by 4.264%-point in the short-run and 10.268%-point in the medium-run.

Finally, in Table 12 we present the impact of policy mixes. Scenario (A) shows the effects of a 1%-point increase in purple public social infrastructure investment/GDP and closing the gender gaps via upward convergence with a 2% increase in female wages and a 1% increase in male wages in both N and H. This sums up the effects in scenarios (A) in Table 11 and (F) in Table 10. GDP increases substantially in both the short-run (4.615%) and the medium-run (3.443%). Employment of women increases both in the short-run and the medium-run (7.835% and 2.710% respectively); however, employment of men increases only in the short-run (5.500%) but decrease in the medium-run (0.445%) due to productivity gains in N, where most male employment is generated. Public debt/GDP decreases (by 1.543%-point in the short-run and 0.010%-point in the medium-run) when fiscal expansion takes the form of both hiring more people and paying them a higher hourly wage rate in H combined with increasing wages and gender equality in also the rest of the economy.

\(^{22}\) The elasticity in the estimation in Table 9 is converted to this marginal effect using the averages of the variables for the estimation period.
Scenario (B) in Table 12 adds to (A) also a 1%-point increase in public green physical infrastructure investment/GDP ($\kappa^G$). Hence, scenario (B) is the case of purple and green public investment and upward convergence in wages, summing up the effects in scenarios (A) and (B) in Table 11 and (F) in Table 10. The effects on GDP are even stronger than in policy mix (A) and employment of both men and women increase both in the short (7.609% and 10.044%) and the medium-run (1.132% and 4.475%).

To summarize, the effects of higher wages and gender equality on GDP are positive in both the short and the medium-run, albeit small; however, the effect of higher wages and gender equality on productivity is much stronger in the medium-run and therefore the impact on employment is negative. Hence, achieving both higher wages and gender equality and employment for both men and women requires a stimulus to demand in the form of higher public spending in both H and N. However, in this scenario, while public debt/GDP decreases in the short-run (1.756%-point), it increases marginally in the medium-run (0.540%-point).

Scenario (C) in Table 12 presents a policy of progressive income taxation, i.e. increasing tax rates on capital income and decreasing tax rates on labour income by 1%-point, which is equivalent to the effects in simulations (C) minus (E) in Table 11. This leads to higher GDP, private investment and employment for both men and women and lower public debt/GDP in both the short and the medium-run. In the medium-run, GDP increases by 1.129%, women’s employment increases by 0.840%, men’s employment increases by 0.698% and public debt/GDP decreases by 0.531%-point.

Finally, scenario (D) in Table 12 presents a policy mix of purple and green public investment, upward convergence in wages, and progressive income and wealth taxation via a 1%-point increase in public social and physical infrastructure investment/GDP ($\kappa^H$ and $\kappa^G$) and closing the gender gaps via upward convergence in wages with a 2% increase in female wages and a 1% increase in male wages in both N and H, a 1%-point increase in the tax rate on profit income ($t^R$), a 1%-point decrease in the tax rate on wages ($t^W$) and a 1%-point increase in the tax rate on wealth ($t^{PW}$), which is equivalent to the effects in simulations (A) plus (B) plus (C) plus (D) minus (E) in Table 11 plus (F) in Table 10. In the medium-run, GDP increases by 10.856%, women’s employment increases by 9.607%, men’s employment increases by 5.836%, and public debt/GDP decreases by 10.259%-point. The results indicate that taxation of wealth is a particularly effective policy to fund purple and green public investment; e.g. inheritance tax may be a suitable tool for funding long term elderly care.
7. Conclusion

This paper develops a unified macroeconomic model to analyse the effects of changes in wages, gender pay gaps and wealth concentration and fiscal policies on output, employment of women and men, productivity and public debt/GDP.

The results indicate that there is a significant interaction between functional income, gender and wealth equality. An increase in wages, including via closing gender pay gap with upward convergence leads to an increase in the wage share and functional income equality, which in turn decreases wealth concentration, which also has a strong path dependency. Redistributive tax policies on capital income and wealth also decrease wealth concentration. Similarly public spending affects inequalities as well by effecting employment and wage income. Furthermore, changes in inequalities have crucial effects on macroeconomic outcomes such as output, employment, productivity and budget balance.

We find that an upward convergence in wages, i.e. increasing wages with closing gender pay gap in both the social sector and the rest of the economy, leads to higher output in both the short and the medium-run. The UK is both wage-led and gender equality-led, and hence equality-led. However, the positive impact on productivity is stronger in the medium-run than on output, which leads to a fall in employment of both men and women.

The positive impact of public social infrastructure investment on both output and employment is much higher, and despite a strong positive effect on productivity, employment of both men and women increases in the medium-run as well.

The high effect of public spending in education, childcare, health and social care on productivity in the rest of the economy provide supporting evidence that this spending serves the purpose of infrastructure investment. Our analysis challenges conventional thinking about the categorization of public spending in health and social care, education and child care in national accounts. Day to day spending in these sectors, e.g. wages of teachers, nurses or social care workers, is considered as current spending, thus not as investment, in our national accounts; however public spending in these social sectors has long term benefits to the society as a whole, with substantial productivity impact in all other sectors of the economy by increasing the skills, health and innovative capacity of people. Crucially, they improve gender equality, and reverse one of the most persistent dimensions of inequality in our societies, as they provide crucial services which are otherwise provided by the unpaid invisible domestic labour of women. Public supply of these services helps women to participate in social and economic life more equally. This in turn further increases
productivity by unleashing the hidden potential of women. Moreover, in the current gendered occupationally segregated labour markets, these sectors employ predominantly women, and more social public spending helps closing the gender gap in employment.

Recognizing the vast amount and importance of the time women spend on unpaid care at the household, which is not accounted for in the standard national accounts and measures such as GDP, is crucial for designing policies to increase gender equality. A fiscal policy stance, which aims to publicly provide the necessary social services, would radically decrease the amount of unpaid domestic care. E.g. universal free child care and nurseries open for sufficiently long hours benefit mothers and fathers by giving them an equal chance to balance work and life, and also benefit the society by decreasing inequality between children from different backgrounds, and improving the creative capacity of children. Needless to say there will always be the need and desire for care provided by family members for children or the elderly in the domestic private sphere; regulations such as parental leave for both mothers and fathers, and working time arrangements that facilitate combining care and work for both men and women should ensure that time for caring can be equally shared between men and women.

A policy mix of upward convergence in wages and public social infrastructure investment has a strong positive impact on output and women’s employment, but men’s employment decreases in the medium-run. Public debt/GDP also falls as an outcome of this policy mix.

A policy mix of upward convergence in wages and public investment in both social and physical infrastructure leads to a higher increase in output and the employment of both men and women increase both in the short and the medium-run. To summarize, the effects of higher wages and gender equality on GDP are positive in both the short and the medium-run, albeit small; however, the effect of higher wages and gender equality on productivity is much stronger in the medium-run and therefore the impact on employment is negative. Hence, achieving both higher wages, gender equality and employment for both men and women requires a stimulus to demand in the form of higher public spending in both the social sector and the rest of the economy along with an upward convergence in wages. However, public debt/GDP increases marginally in the medium-run in this policy mix and an increase in tax rates is required to improve public debt/GDP.

An increase in the progressivity of income taxation in the form of increasing the tax rate on capital income and decreasing the tax rate on labour income increases output, men’s and women’s employment, and decreases public debt/GDP in both the short and the medium-run.
An increase in the tax rate on wealth decreases wealth concentration and has a positive and the strongest impact on output, employment and the budget.

Progressive taxation, which improves after tax equality in terms of income, wealth and gender, is also important in the context of public spending on non-means-tested services such as universal health and social care, education and child care. A higher tax rate on higher incomes is a way for those who can afford to contribute more towards universally provided public services. The results indicate that taxation of wealth is a particularly effective policy to fund purple and green public investment; e.g. inheritance tax may be a suitable tool for funding long term elderly care.

In this paper, we analysed the effect of the various labour market and fiscal policies on hours of employment of women and men; however, we did not analyse the changes in working time regulations. Future research can analyse the effect of a further change in labour market regulation leading to a shortening of the working week, where a given number of hours of employment can be shared among a higher number of employees. A scenario of upward convergence in hourly wage rates along with a downward convergence in weekly working hours between men and women, i.e. men working shorter hours than the current circumstances, while more women increasing their hours of work, is expected to reduce both gender pay and employment gaps. Higher hourly wage rates may make a reduction in weekly working hours appealing for the current full-time employees, and the provision of high quality public social infrastructure may make higher hours of work appealing for the current part-time workers, who are predominantly women.

There is also an important complementarity between gender equality, shorter working hours and green development (Onaran, 2016; İlkkaracan, 2013). A larger proportion of the society’s time spent caring for each other is also a greener alternative, whether that is in paid or unpaid time, as these activities are much lower in terms of their carbon intensity. Furthermore social infrastructure services are very labour-intensive and therefore public investment in this area is a vehicle for generating more employment for a given rate of growth in national output—a target more consistent with low carbon emissions.

Our analysis points at a number of further questions for future research. In this paper, we treated gender norms as exogenous; however, changes in employment patterns and gender gaps can have crucial effects on gender norms. Future research can analyse both exogenous and endogenous changes in occupational and sectoral gender segregation. To simplify the model, we also did not analyse the changes in population related to the impact of income, employment, social infrastructure and unpaid care on fertility and mortality rates as well as
migration. In particular, the gendered character of in the care sector and the intersection of gender, ethnic and racial inequalities present an important further research question.

These findings hint at policy insights to address some urgent destabilizing economic and social issues in the UK and the world such as stagnation in productivity, unemployment, unhealthy growth driven by private debt or demographic and care crisis. An appropriate mix of the labour market and fiscal policies may help to tackle the multiple dimensions of inequalities with an aim to achieve both a stable macroeconomic environment and social cohesion.
References


LINDNER, F. 2014 The housing wealth effect on consumption reconsidered. Economics Discussion Papers.


Office for Budget Responsibility 2013. The macroeconomic model, Briefing paper No.5, OBR, London


Oyvat, C., Öztunalı, O. and Elgin, C. 2018, Wage-led vs. profit-led growth: a comprehensive empirical analysis, Greenwich Papers in Political Economy, University of Greenwich, #GPERC65


Figure 1: The ratio of hourly wage rate of men/women (\(\alpha\)) and share of women in hours worked (\(\beta\)) in the social sector (H) and the rest of the economy (N) in the UK

Source: Own calculations based on EU KLEMS database

Figure 2: The effects of female and male wages in N on labour productivity in the next period
Figure 3: The effects of a simultaneous increase in female and male wages’ in N on total employment in the short-run and in the next period

- Female and male wages in N sector
- Aggregate output in the short-run
- Labour productivity in the next period
- Aggregate output in the next period
- Employment in the short-run
- Employment in the next period
Figure 4: The effects of an increase in public social infrastructure investment on total output in the short-run

* Based on Figure 1, the positive partial impact of public social expenditures is expected to be relatively larger for female employment compared to the partial impact from expenditures in N sector.

** The impact of total wage payments in H sector is through their impact on wage and wealth taxes.

*** The impact of total wage payments in H sector is through their impact on the wealth of top 1% and bottom 99%.
Figure 5: The effects of an increase in public social infrastructure investment on labour productivity in the next period

Figure 6: The effects of an increase in public social infrastructure investment on total output in the next period

* The effects from the wealth of top 1% on aggregate output is ambiguous and the impact of aggregate output on the wealth of top 1% is positive.
Figure 7: The effects of public social infrastructure investment on total employment in the short-run and in the next period
Figure 8: The effects of closing the gender wage gap in H on total output in the short-run

* The impact of total wage payments in H sector is through their impact on wage and wealth taxes.
** The impact of total wage payments in H sector is through their impact on the wealth of top 1% and bottom 99%.
Figure 9: The share of wages in GDP (adjusted, at factor cost) and wealth concentration (share of top 1% in total net wealth, $\lambda$) in the UK

Source: AMECO for wage share and WIID for wealth concentration
Table 1: The regimes and their conditions in the case of an increase in female and male wages in N with a constant gender wage gap ($\alpha^N_t = \alpha^N_{t-1}$, $\alpha_{t-1}^N = \alpha_{t-1}^N$)

<table>
<thead>
<tr>
<th>Case</th>
<th>Growth Regime</th>
<th>Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi^N_{tt} &gt; 0$</td>
<td>Wage-led in the short-run</td>
<td>$\left( \frac{\partial C^N_t}{\partial w^N_t} \right)<em>{Y_t,\alpha^N_t} + \left( \frac{\partial C^M_t}{\partial w^N_t} \right)</em>{Y_t,\alpha^N_t} &gt; \left( \frac{\partial X_t}{\partial w^N_t} \right)<em>{Y_t,\alpha^N_t} - \left( \frac{\partial M_t}{\partial w^N_t} \right)</em>{Y_t,\alpha^N_t}$</td>
</tr>
<tr>
<td>$\psi^N_{tt} &lt; 0$</td>
<td>Profit-led in the short-run</td>
<td>$\left( \frac{\partial C^N_t}{\partial w^N_t} \right)<em>{Y_t,\alpha^N_t} + \left( \frac{\partial C^M_t}{\partial w^N_t} \right)</em>{Y_t,\alpha^N_t} &lt; \left( \frac{\partial X_t}{\partial w^N_t} \right)<em>{Y_t,\alpha^N_t} - \left( \frac{\partial M_t}{\partial w^N_t} \right)</em>{Y_t,\alpha^N_t}$</td>
</tr>
<tr>
<td>$(\psi^N_{tt} + \psi^N_{t(t-1)}) &gt; 0$</td>
<td>Wage-led in the medium-run</td>
<td>Ambiguous due to effects on productivity</td>
</tr>
<tr>
<td>$(\psi^N_{tt} + \psi^N_{t(t-1)}) &lt; 0$</td>
<td>Profit-led in the medium-run</td>
<td>Ambiguous due to effects on productivity</td>
</tr>
</tbody>
</table>

Table 2: The demand regimes in the short-run

<table>
<thead>
<tr>
<th>Female wage-led/ gender equality-led in the short-run</th>
<th>Wage-led in the short-run</th>
<th>Profit-led in the short-run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact of $w^N_t$ &amp; $w^M_t$ (constant $\alpha^N_t$) on total consumption</td>
<td>[Impact of $w^N_t$ &amp; $w^M_t$ (constant $\alpha^N_t$) on total consumption]</td>
<td>[Impact of $w^N_t$ &amp; $w^M_t$ (constant $\alpha^N_t$) on total consumption]</td>
</tr>
<tr>
<td>&gt;</td>
<td>&gt;</td>
<td>&gt;</td>
</tr>
<tr>
<td>Impact of $w^N_t$ &amp; $w^M_t$ (constant $\alpha^N_t$) on investment + net exports</td>
<td>[Impact of $w^N_t$ &amp; $w^M_t$ (constant $\alpha^N_t$) on investment + net exports]</td>
<td>[Impact of $w^N_t$ &amp; $w^M_t$ (constant $\alpha^N_t$) on investment + net exports]</td>
</tr>
<tr>
<td>&amp;</td>
<td>&amp;</td>
<td>&amp;</td>
</tr>
<tr>
<td>Impact of $w^N_t$ on total consumption</td>
<td>[Impact of $w^N_t$ on total consumption]</td>
<td>[Impact of $w^N_t$ on total consumption]</td>
</tr>
<tr>
<td>&gt;</td>
<td>&gt;</td>
<td>&gt;</td>
</tr>
<tr>
<td>Impact of $w^N_t$ on investment + net exports</td>
<td>[Impact of $w^N_t$ on investment + net exports]</td>
<td>[Impact of $w^N_t$ on investment + net exports]</td>
</tr>
<tr>
<td>&gt;</td>
<td>&gt;</td>
<td>&gt;</td>
</tr>
<tr>
<td>Gender inequality-led in the short-run</td>
<td>[Impact of $w^N_t$ &amp; $w^M_t$ (constant $\alpha^N_t$) on total consumption]</td>
<td>[Impact of $w^N_t$ &amp; $w^M_t$ (constant $\alpha^N_t$) on total consumption]</td>
</tr>
<tr>
<td>&gt;</td>
<td>&gt;</td>
<td>&gt;</td>
</tr>
<tr>
<td>Impact of $w^N_t$ &amp; $w^M_t$ (constant $\alpha^N_t$) on investment + net exports</td>
<td>[Impact of $w^N_t$ &amp; $w^M_t$ (constant $\alpha^N_t$) on investment + net exports]</td>
<td>[Impact of $w^N_t$ &amp; $w^M_t$ (constant $\alpha^N_t$) on investment + net exports]</td>
</tr>
<tr>
<td>&gt;</td>
<td>&gt;</td>
<td>&gt;</td>
</tr>
<tr>
<td>Impact of $w^N_t$ on investment + net exports</td>
<td>[Impact of $w^N_t$ on investment + net exports]</td>
<td>[Impact of $w^N_t$ on investment + net exports]</td>
</tr>
<tr>
<td>&gt;</td>
<td>&gt;</td>
<td>&gt;</td>
</tr>
</tbody>
</table>
### Table 3: Estimation results for consumption in N and H

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\Delta \log C^N_t$</th>
<th>$\Delta \log C^H_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>p-value</td>
<td>Coeff.</td>
</tr>
<tr>
<td>Constant</td>
<td>0.007</td>
<td>0.011</td>
</tr>
<tr>
<td>$\Delta \log (R_t(1-t^{R_t}))$</td>
<td>0.085</td>
<td>0.063</td>
</tr>
<tr>
<td>$\Delta \log (WB^F_t(1-t^{W^{F}_t}))$</td>
<td>0.150</td>
<td>0.304</td>
</tr>
<tr>
<td>$\Delta \log (WB^M_t(1-t^{W^{M}_t}))$</td>
<td>0.375</td>
<td>0.244</td>
</tr>
<tr>
<td>$\Delta \log (PW_{99}(1-t^{PW_{99}}))$</td>
<td>0.132</td>
<td>-0.072</td>
</tr>
<tr>
<td>$\Delta \log (PW_{1}(1-t^{PW_{1}}))$</td>
<td>0.017</td>
<td>-0.053</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.735</td>
<td>0.134</td>
</tr>
<tr>
<td>DW statistic</td>
<td>1.529</td>
<td>1.394</td>
</tr>
</tbody>
</table>

Estimation method: SUR

### Table 4: Estimation results for private investment

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\Delta \log I_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coeff.</td>
<td>p-value</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.947</td>
</tr>
<tr>
<td>$\Delta \log (\pi(1-t^{R_t}))$</td>
<td>0.196</td>
</tr>
<tr>
<td>$\Delta \log Y_t$</td>
<td>1.282</td>
</tr>
<tr>
<td>$\Delta \log (PW_{1}(1-t^{PW_{1}}))$</td>
<td>-0.058</td>
</tr>
<tr>
<td>$\Delta \log (PW_{99}(1-t^{PW_{99}}))$</td>
<td>0.389</td>
</tr>
<tr>
<td>$\Delta \log (D/Y)_t$</td>
<td>-0.289</td>
</tr>
<tr>
<td>$\log I_{t-1}$</td>
<td>-0.276</td>
</tr>
<tr>
<td>$\log Y_{t-1}$</td>
<td>0.403</td>
</tr>
<tr>
<td>$\log (PW_{1}(1-t^{PW_{1}}))$</td>
<td>-0.074</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>0.694</td>
</tr>
<tr>
<td>DW statistic</td>
<td>2.031</td>
</tr>
<tr>
<td>Sample</td>
<td>1971-2016</td>
</tr>
</tbody>
</table>

Estimation method: Error correction model
### Table 5: Estimation results for exports

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.014</td>
<td>0.213</td>
</tr>
<tr>
<td>Δlog(πₜ)</td>
<td>0.124</td>
<td>0.299</td>
</tr>
<tr>
<td>ΔlogYₜ^{World}</td>
<td>1.741</td>
<td>0.000</td>
</tr>
<tr>
<td>Adj. R²</td>
<td>0.418</td>
<td></td>
</tr>
<tr>
<td>DW statistic</td>
<td>1.778</td>
<td></td>
</tr>
</tbody>
</table>

Sample: 1971-2016

Estimation method: OLS in first differences

### Table 6: Estimation results for imports

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-2.261</td>
<td>0.005</td>
</tr>
<tr>
<td>Δlog(πₜ)</td>
<td>-0.182</td>
<td>0.129</td>
</tr>
<tr>
<td>ΔlogYₜ^{N,t-1}</td>
<td>1.591</td>
<td>0.000</td>
</tr>
<tr>
<td>logMₜ₋₁</td>
<td>-0.259</td>
<td>0.005</td>
</tr>
<tr>
<td>logYₜ₋₁</td>
<td>0.534</td>
<td>0.005</td>
</tr>
<tr>
<td>Adj. R²</td>
<td>0.678</td>
<td></td>
</tr>
<tr>
<td>DW statistic</td>
<td>2.615</td>
<td></td>
</tr>
</tbody>
</table>

Sample: 1971-2016

Estimation method: Error correction model
Table 7: Estimation results for productivity in N

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>logT&lt;sub&gt;t&lt;/sub&gt;</th>
<th>Coeff.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>logY&lt;sub&gt;(t-1)&lt;/sub&gt;</td>
<td>0.231</td>
<td>0.011</td>
<td></td>
</tr>
<tr>
<td>logI&lt;sub&gt;(t-1)&lt;/sub&gt;</td>
<td>-0.100</td>
<td>0.149</td>
<td></td>
</tr>
<tr>
<td>logw&lt;sub&gt;F(t-1)&lt;/sub&gt;</td>
<td>0.679</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>logα&lt;sub&gt;(t-1)&lt;/sub&gt;</td>
<td>0.564</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>log(G&lt;sub&gt;H(t-1)+C&lt;sub&gt;H(t-1))/N&lt;sub&gt;t-1&lt;/sub&gt;</td>
<td>0.267</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td>log(I&lt;sub&gt;G(t-1))/N&lt;sub&gt;t-1&lt;/sub&gt;</td>
<td>-0.029</td>
<td>0.293</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.534</td>
<td>0.230</td>
<td></td>
</tr>
</tbody>
</table>

Adj. R<sup>2</sup> 0.920

Number of observations 162

Number of sectors 18

Sample 1971-2016

Estimation method: Fixed effects panel regression

Note: The time indicator t refers to five year non-overlapping average of explanatory variables starting from 1970 and of the dependent variable starting from 1971.

Table 8: Estimation results for private net wealth

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>ΔlogPW&lt;sub&gt;t&lt;/sub&gt;</th>
<th>Coeff.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.002</td>
<td>0.776</td>
<td></td>
</tr>
<tr>
<td>Δlog(WB&lt;sub&gt;F(t-1)W&lt;sub&gt;t&lt;/sub&gt;))</td>
<td>0.496</td>
<td>0.016</td>
<td></td>
</tr>
<tr>
<td>Δlog(WB&lt;sub&gt;M(t-1)W&lt;sub&gt;t&lt;/sub&gt;))</td>
<td>0.420</td>
<td>0.091</td>
<td></td>
</tr>
<tr>
<td>Δlog(R&lt;sub&gt;t(1-t&lt;sub&gt;R&lt;/sub&gt;))</td>
<td>0.213</td>
<td>0.000</td>
<td></td>
</tr>
<tr>
<td>Δlog(PW&lt;sub&gt;t-1(1-t&lt;sub&gt;PW&lt;/sub&gt;t-1))</td>
<td>0.333</td>
<td>0.016</td>
<td></td>
</tr>
</tbody>
</table>

Adj. R<sup>2</sup> 0.606

DW statistic 1.842

Sample 1972-2015

Estimation method: OLS in first differences
Table 9: Estimation results for private net wealth concentration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.081</td>
<td>0.671</td>
</tr>
<tr>
<td>(\log(\pi_{t,1}(1-t^R_{t,1})))</td>
<td>0.108</td>
<td>0.452</td>
</tr>
<tr>
<td>(\log(\pi_{t,2}(1-t^R_{t,2})))</td>
<td>-0.229</td>
<td>0.227</td>
</tr>
<tr>
<td>(\log(\pi_{t,3}(1-t^R_{t,3})))</td>
<td>0.244</td>
<td>0.095</td>
</tr>
<tr>
<td>(\log\lambda_{t,1})</td>
<td>0.854</td>
<td>0.000</td>
</tr>
<tr>
<td>(\log t^{PW}_{t,1})</td>
<td>-0.058</td>
<td>0.075</td>
</tr>
</tbody>
</table>

| Adj. R² | 0.809 |
| DW statistic | 2.282 |

Sample: 1973-2016

Estimation method: Autoregressive distributed lag model (ARDL)
Table 10: The total (post-multiplier) effects of changes in wages and gender pay gap on the components of aggregate demand (as a ratio to GDP), GDP, employment and public debt/GDP

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔC/Y</td>
<td>0.036</td>
<td>0.013</td>
<td>0.046</td>
<td>0.004</td>
<td>0.018</td>
<td>0.013</td>
<td>0.006</td>
<td>0.007</td>
<td>0.007</td>
<td>0.004</td>
<td>0.146</td>
<td>-0.556</td>
<td>-0.472</td>
<td>-0.623</td>
<td>-0.208</td>
<td></td>
</tr>
<tr>
<td>ΔC^2/Y</td>
<td>0.133</td>
<td>0.002</td>
<td>0.067</td>
<td>0.008</td>
<td>0.085</td>
<td>0.041</td>
<td>0.015</td>
<td>0.020</td>
<td>0.015</td>
<td>0.027</td>
<td>-0.105</td>
<td>-0.089</td>
<td>-0.118</td>
<td>-0.069</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔC^4/Y</td>
<td>0.091</td>
<td>0.006</td>
<td>0.013</td>
<td>0.007</td>
<td>0.007</td>
<td>0.013</td>
<td>0.009</td>
<td>0.011</td>
<td>0.007</td>
<td>0.027</td>
<td>-0.010</td>
<td>-0.009</td>
<td>-0.024</td>
<td>-0.014</td>
<td>-0.028</td>
<td></td>
</tr>
<tr>
<td>ΔC^6/Y</td>
<td>0.048</td>
<td>0.003</td>
<td>0.011</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.027</td>
<td>-0.105</td>
<td>-0.089</td>
<td>-0.118</td>
<td>-0.069</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔE/E</td>
<td>0.091</td>
<td>0.006</td>
<td>0.013</td>
<td>0.007</td>
<td>0.007</td>
<td>0.013</td>
<td>0.009</td>
<td>0.011</td>
<td>0.007</td>
<td>0.027</td>
<td>-0.105</td>
<td>-0.089</td>
<td>-0.118</td>
<td>-0.069</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔE^4/E</td>
<td>0.133</td>
<td>0.002</td>
<td>0.067</td>
<td>0.008</td>
<td>0.085</td>
<td>0.041</td>
<td>0.015</td>
<td>0.020</td>
<td>0.015</td>
<td>0.027</td>
<td>-0.105</td>
<td>-0.089</td>
<td>-0.118</td>
<td>-0.069</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔE^6/E</td>
<td>0.091</td>
<td>0.006</td>
<td>0.013</td>
<td>0.007</td>
<td>0.007</td>
<td>0.013</td>
<td>0.009</td>
<td>0.011</td>
<td>0.007</td>
<td>0.027</td>
<td>-0.105</td>
<td>-0.089</td>
<td>-0.118</td>
<td>-0.069</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔΔE/M</td>
<td>0.048</td>
<td>0.003</td>
<td>0.011</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.003</td>
<td>0.027</td>
<td>-0.105</td>
<td>-0.089</td>
<td>-0.118</td>
<td>-0.069</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: (i) Column (9)=(1)+(2)+(3)+(4)-(5)+(6)+(7)+(8). In each column, the effects in Appendices 3-4 are multiplied by the wage rate in the relevant sector and divided by Y.
(ii) SR: short run. MR: medium-run, defined as the cumulative of the effects in the short-run and the next period when productivity changes.
(iii) Sum of the effects in simulations (A) and (C)
(iv) Sum of the effects in simulations (A), (B), (C) and (D)
Table 11: The total (post-multiplier) effects of changes in fiscal policies on the components of aggregate demand (as a ratio to GDP), GDP, employment and public debt/GDP

<table>
<thead>
<tr>
<th></th>
<th>%-point change in consumption in N/GDP</th>
<th>%-point change in consumption in H/GDP</th>
<th>%-point change in private investment /GDP</th>
<th>%-point change in imports in N /GDP</th>
<th>%-point change in public social infrastructure investment /GDP</th>
<th>%-point change in public physical infrastructure investment /GDP</th>
<th>% Change in GDP</th>
<th>% change in total employment</th>
<th>% change in female employment</th>
<th>% change in male employment</th>
<th>%-point change in public debt /GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>ΔC(N)/Y</td>
<td>ΔC(H)/Y</td>
<td>ΔI/Y</td>
<td>ΔX/Y</td>
<td>ΔG(H)/Y</td>
<td>ΔG(C)/Y</td>
<td>ΔY/Y</td>
<td>ΔE/E</td>
<td>ΔE(F)/E(F)</td>
<td>ΔE(M)/E(M)</td>
<td>ΔD/Y</td>
</tr>
<tr>
<td>(2)</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
<td>(10)</td>
<td>(11)</td>
</tr>
</tbody>
</table>

A. The effects of a 1%-point increase in public purple social infrastructure investment/GDP (\(\kappa^{H}\))

SR (ii) 1.847 0.071 0.960 0.000 1.200 1.435 0.365 0.107 3.585 5.454 6.722 4.437 -0.981
MR (ii) 0.649 0.018 0.753 0.148 0.545 1.328 0.276 0.081 2.707 1.674 2.398 0.420 0.497

B. The effects of a 1%-point increase in public green physical infrastructure investment/GDP (\(\kappa^{G}\))

SR 0.985 0.034 0.512 0.000 1.003 0.249 0.208 0.161 2.046 2.154 2.210 2.109 -0.213
MR 0.916 0.027 0.472 0.023 0.945 0.243 0.204 1.060 1.999 1.660 1.764 1.576 0.550

C. The effects of a 1%-point increase in the tax rate on profit income (\(t_{R}\))

SR -0.194 -0.006 -0.057 0.000 -0.102 -0.025 -0.021 -0.006 -0.208 -0.219 -0.224 -0.214 -0.200
MR -0.230 -0.005 -0.009 -0.005 -0.094 -0.025 -0.021 -0.006 -0.207 -0.127 -0.143 -0.114 -0.478

D. The effects of a 1%-point increase in the tax rate on wealth (\(t_{PW}\))

SR 0.298 0.015 0.802 0.000 0.442 0.110 0.092 0.027 0.902 0.949 0.974 0.930 -4.264
MR 1.986 0.066 3.199 0.020 2.070 0.521 0.436 0.128 4.285 4.134 4.293 4.006 -10.268

E. The effects of a 1%-point increase in the tax rate on wage income (\(t_{W}\))

SR -1.080 -0.038 -0.321 0.000 -0.570 -0.142 -0.119 -0.035 -1.164 -1.226 -1.257 -1.200 0.212
MR -1.156 -0.034 -0.394 -0.027 -0.614 -0.162 -0.136 -0.040 -1.335 -0.888 -0.983 -0.812 0.053

Notes: (i) Column (9) = (1)+(2)+(3)+(4)-(5)+(6)+(7)+(8). In each column, the marginal effects in Appendix 4 are divided by Y.
(ii) SR: short run. MR: medium-run, defined as the cumulative of the effects in the short-run and the next period when productivity in N changes endogenously;
Table 12: The total (post-multiplier) effects of mix of labour market and fiscal policies on the components of aggregate demand (as a ratio to GDP), GDP, employment and public debt/GDP

<table>
<thead>
<tr>
<th></th>
<th>% point change in consumption in N/GDP</th>
<th>% point change in consumption in H/GDP</th>
<th>% point change in private investment/GDP</th>
<th>% point change in imports in N/GDP</th>
<th>% point change in public social infrastructure investment/GDP</th>
<th>% point change in government current expenditure/GDP</th>
<th>% point change in public physical infrastructure investment/GDP</th>
<th>% Change in GDP</th>
<th>% change in total employment</th>
<th>% change in female employment</th>
<th>% change in male employment</th>
<th>% point change in public debt/GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Purple public investment and upward convergence in wages: The effects of a 1% -point increase in public social infrastructure investment/GDP (κ^H) and closing gender gaps via upward convergence in wages via 2% increase in female wages and 1% increase in male wages in both N and H (ii)</td>
<td>ΔC^N/Y</td>
<td>ΔC^H/Y</td>
<td>ΔY/Y</td>
<td>ΔM/Y</td>
<td>ΔG^Y/H/Y</td>
<td>ΔG^C/H/Y</td>
<td>ΔG^P/H/Y</td>
<td>ΔG^P/E</td>
<td>ΔE/E</td>
<td>ΔE^F/E</td>
<td>ΔE^M/E</td>
<td>ΔD/Y</td>
</tr>
<tr>
<td>SR</td>
<td>2.658</td>
<td>0.205</td>
<td>1.226</td>
<td>-0.059</td>
<td>1.719</td>
<td>1.696</td>
<td>0.470</td>
<td>0.138</td>
<td>4.615</td>
<td>6.539</td>
<td>7.835</td>
<td>5.500</td>
</tr>
<tr>
<td>MR</td>
<td>0.941</td>
<td>0.126</td>
<td>1.018</td>
<td>0.161</td>
<td>0.809</td>
<td>1.554</td>
<td>0.351</td>
<td>0.103</td>
<td>3.443</td>
<td>0.959</td>
<td>2.710</td>
<td>-0.445</td>
</tr>
<tr>
<td>B. Purple and green public investment and upward convergence in wages: The effects of a 1% -point increase in public social and physical infrastructure investment/GDP (κ^H and κ^G) and closing gender gaps via upward convergence in wages in both N and H (iii)</td>
<td>ΔC^N/Y</td>
<td>ΔC^H/Y</td>
<td>ΔY/Y</td>
<td>ΔM/Y</td>
<td>ΔG^Y/H/Y</td>
<td>ΔG^C/H/Y</td>
<td>ΔG^P/H/Y</td>
<td>ΔG^P/E</td>
<td>ΔE/E</td>
<td>ΔE^F/E</td>
<td>ΔE^M/E</td>
<td>ΔD/Y</td>
</tr>
<tr>
<td>SR</td>
<td>3.643</td>
<td>0.239</td>
<td>1.738</td>
<td>-0.059</td>
<td>2.722</td>
<td>1.945</td>
<td>0.678</td>
<td>1.199</td>
<td>6.661</td>
<td>8.693</td>
<td>10.044</td>
<td>7.609</td>
</tr>
<tr>
<td>MR</td>
<td>1.856</td>
<td>0.153</td>
<td>1.490</td>
<td>0.184</td>
<td>1.754</td>
<td>1.797</td>
<td>0.554</td>
<td>1.163</td>
<td>5.443</td>
<td>2.619</td>
<td>4.475</td>
<td>1.132</td>
</tr>
<tr>
<td>C. Progressive income tax: The effects of a 1% -point increase in the tax rate on profit income (t_R) and a 1% -point decrease in the tax rate on wages (t_W) (iv)</td>
<td>ΔC^N/Y</td>
<td>ΔC^H/Y</td>
<td>ΔY/Y</td>
<td>ΔM/Y</td>
<td>ΔG^Y/H/Y</td>
<td>ΔG^C/H/Y</td>
<td>ΔG^P/H/Y</td>
<td>ΔG^P/E</td>
<td>ΔE/E</td>
<td>ΔE^F/E</td>
<td>ΔE^M/E</td>
<td>ΔD/Y</td>
</tr>
<tr>
<td>SR</td>
<td>0.887</td>
<td>0.032</td>
<td>0.264</td>
<td>0.000</td>
<td>0.469</td>
<td>0.116</td>
<td>0.097</td>
<td>0.029</td>
<td>0.956</td>
<td>1.007</td>
<td>1.033</td>
<td>0.986</td>
</tr>
<tr>
<td>MR</td>
<td>0.926</td>
<td>0.029</td>
<td>0.385</td>
<td>0.022</td>
<td>0.519</td>
<td>0.137</td>
<td>0.115</td>
<td>0.034</td>
<td>1.129</td>
<td>0.761</td>
<td>0.840</td>
<td>0.698</td>
</tr>
<tr>
<td>D. Purple and green public investment, upward convergence in wages, and progressive income and wealth taxation: a 1% -point increase in public social and physical infrastructure investment/GDP (κ^H and κ^G) and closing gender gaps via upward convergence in wages via 2% increase in female wages and 1% increase in male wages in both N and H (v)</td>
<td>ΔC^N/Y</td>
<td>ΔC^H/Y</td>
<td>ΔY/Y</td>
<td>ΔM/Y</td>
<td>ΔG^Y/H/Y</td>
<td>ΔG^C/H/Y</td>
<td>ΔG^P/H/Y</td>
<td>ΔG^P/E</td>
<td>ΔE/E</td>
<td>ΔE^F/E</td>
<td>ΔE^M/E</td>
<td>ΔD/Y</td>
</tr>
<tr>
<td>SR</td>
<td>4.827</td>
<td>0.286</td>
<td>2.804</td>
<td>-0.059</td>
<td>3.632</td>
<td>2.171</td>
<td>0.867</td>
<td>1.255</td>
<td>8.519</td>
<td>10.649</td>
<td>12.051</td>
<td>9.525</td>
</tr>
<tr>
<td>MR</td>
<td>4.767</td>
<td>0.248</td>
<td>5.074</td>
<td>0.226</td>
<td>4.344</td>
<td>2.455</td>
<td>1.105</td>
<td>1.325</td>
<td>10.856</td>
<td>7.514</td>
<td>9.607</td>
<td>5.836</td>
</tr>
</tbody>
</table>

Notes: (i) Column (9)=(1)+(2)+(3)+(4)-(5)+(6)+(7)+(8)
(ii) Sum of the effects in simulations (A) in Table 11 and (F) in Table 10.
(iii) Sum of the effects in simulations (A) and (B) in Table 11 and (F) in Table 10.
(iv) The effects in simulations (C) minus (E) in Table 11.
(v) The effects in simulations (A) plus (B) plus (C) plus (D) minus (E) in Table 11 plus (F) in Table 10.
### Appendix 1: Variables and data sources

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Variable name</th>
<th>Source</th>
<th>Time period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>Aggregate output, GDP (real), in billions</td>
<td>AMECO</td>
<td>1970-2016</td>
</tr>
<tr>
<td>W/H</td>
<td>Total wage bill, labour compensation adjusted for the labour income of the self-employed (real), in billions</td>
<td>AMECO, own calculations</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/F</td>
<td>Total wage bill for female workers (real, adjusted labour compensation), in billions</td>
<td>AMECO, own calculations</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/M</td>
<td>Total wage bill for male workers (real, adjusted labour compensation), in billions</td>
<td>Own calculations based on data from AMECO and EUROPEMS (1)</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/HM</td>
<td>Total employment in the public social sector (total hours worked by persons engaged in education and health &amp; social work categories of the industrial classification of EUROPEMS), in billions</td>
<td>Own calculations based on data from AMECO and EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/FM</td>
<td>Total employment in the rest of the economy, in billions</td>
<td>Own calculations based on data from AMECO and EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/FH</td>
<td>Hours of Employment of women in the public social sector, in billions</td>
<td>Own calculations based on data from AMECO and EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/MH</td>
<td>Hours of Employment of men in the public social sector, in billions</td>
<td>Own calculations based on data from AMECO and EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/FH</td>
<td>Hours of Employment of women in the rest of the economy, in billions</td>
<td>Own calculations based on data from AMECO and EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/MH</td>
<td>Hours of Employment of men in the rest of the economy, in billions</td>
<td>Own calculations based on data from AMECO and EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/Hr</td>
<td>Average female hourly wage rate in the public social sector (real)</td>
<td>Own calculations based on data from AMECO and EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/mr</td>
<td>Average male hourly wage rate in the public social sector (real)</td>
<td>Own calculations based on data from AMECO and EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/HH</td>
<td>Average female hourly wage rate in the rest of the economy (real)</td>
<td>Own calculations based on data from AMECO and EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/mH</td>
<td>Average male hourly wage rate in the rest of the economy (real)</td>
<td>Own calculations based on data from AMECO and EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/Hm</td>
<td>Ratio between male and female wages in the public social sector</td>
<td>Own calculations based on data from EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/Hhm</td>
<td>Ratio between male and female wages in the rest of the economy</td>
<td>Own calculations based on data from EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>W/PHS</td>
<td>Households’ private social expenditures (real), in billions</td>
<td>Own calculations based on data from Office of National Statistics (ONS) (2016) and AMECO (2)</td>
<td>1970-2016</td>
</tr>
<tr>
<td>W/PS</td>
<td>Private consumption of goods and services in the rest of the economy (real), in billions</td>
<td>Own calculations based on data from Office of National Statistics (ONS) (2016) and AMECO (2)</td>
<td>1970-2016</td>
</tr>
<tr>
<td>W/PI</td>
<td>Private investment (real), in billions</td>
<td>AMECO, own calculations</td>
<td>1970-2016</td>
</tr>
<tr>
<td>W/PSG</td>
<td>Government’s consumption expenditures (real), in billions</td>
<td>Own calculations based on data from OECD National Accounts and AMECO</td>
<td>1970-2016</td>
</tr>
<tr>
<td>W/PSO</td>
<td>Public investment other than investments in the social sector (real), in billions</td>
<td>AMECO, own calculations</td>
<td>1970-2016</td>
</tr>
<tr>
<td>W/PSI</td>
<td>Government’s social infrastructure expenditures (real), in billions</td>
<td>Own calculations based on data from OECD National Accounts and AMECO</td>
<td>1970-2016</td>
</tr>
<tr>
<td>X</td>
<td>Imports (real), in billions</td>
<td>AMECO</td>
<td>1970-2016</td>
</tr>
<tr>
<td>X/E</td>
<td>Exports (real), in billions</td>
<td>AMECO</td>
<td>1970-2016</td>
</tr>
<tr>
<td>T</td>
<td>Total expenditure in the social sector (real), in billions</td>
<td>Y/Y</td>
<td>1970-2016</td>
</tr>
<tr>
<td>T/E</td>
<td>Total expenditure in the rest of the economy (real), in billions</td>
<td>Y/Y</td>
<td>1970-2016</td>
</tr>
<tr>
<td>T/G</td>
<td>Share of government spending on the social sector in total output</td>
<td>(T/E)/Y</td>
<td>1970-2016</td>
</tr>
<tr>
<td>T/GP</td>
<td>Share of government’s consumption expenditures in total output</td>
<td>(T/E)/Y</td>
<td>1970-2016</td>
</tr>
<tr>
<td>T/T</td>
<td>Total output</td>
<td>Y/Y</td>
<td>1970-2016</td>
</tr>
<tr>
<td>T/P</td>
<td>Productivity in the rest of the economy (real)</td>
<td>(T/E)/Y</td>
<td>1970-2016</td>
</tr>
<tr>
<td>T/W</td>
<td>Share of women employed in the rest of the economy</td>
<td>Own calculations based on data from EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>T/WP</td>
<td>Share of women employed in the public social sector</td>
<td>Own calculations based on data from EUROPEMS</td>
<td>1970-2015</td>
</tr>
<tr>
<td>U</td>
<td>Unpaid domestic core labour</td>
<td>ONS (2016)</td>
<td>2014</td>
</tr>
<tr>
<td>P</td>
<td>Gross operating surplus (real), in billions</td>
<td>AMECO, own calculations</td>
<td>1970-2016</td>
</tr>
<tr>
<td>P/E</td>
<td>Profit share in the rest of the economy (P/E)</td>
<td>AMECO, own calculations</td>
<td>1970-2016</td>
</tr>
<tr>
<td>P/E</td>
<td>Implicit tax rate on labour, %</td>
<td>Taxpayers Commission, Banque and Osnam (2012)</td>
<td>1970-2016</td>
</tr>
<tr>
<td>P/K</td>
<td>Implicit tax rate on capital income, %</td>
<td>Taxpayers Commission, Banque and Osnam (2012)</td>
<td>1970-2016</td>
</tr>
<tr>
<td>P/W</td>
<td>Implicit tax rate on wealth, %</td>
<td>Own calculations based on Banque, EC (2009) and Osnam (2012)</td>
<td>1970-2016</td>
</tr>
<tr>
<td>P/C</td>
<td>Implicit tax rate on consumption, %</td>
<td>Taxpayers Commission, Banque and Osnam (2012)</td>
<td>1970-2016</td>
</tr>
<tr>
<td>V</td>
<td>General government consolidated debt/Y</td>
<td>AMECO, own calculations</td>
<td>1970-2016</td>
</tr>
<tr>
<td>V/F1</td>
<td>Net private wealth of top 1% (real), in billions</td>
<td>World Wealth and Income Database and Credit Suisse (2014-17), own calculations</td>
<td>1970-2016</td>
</tr>
<tr>
<td>R</td>
<td>Real exchange rate</td>
<td>World Development Indicators</td>
<td>1970-2016</td>
</tr>
<tr>
<td>R/W</td>
<td>Rest of the world income</td>
<td>Own calculations based on World Bank World Development Indicators</td>
<td>1970-2016</td>
</tr>
</tbody>
</table>

Notes: (1) The data in 2018 release is linked back with data in 2012 and 2009 releases
(2) The ONS data for the composition of C starts in 1985; for the years before 1985 we assumed C/H/C to be constant.
<table>
<thead>
<tr>
<th>Definitions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta p$</td>
<td>Short-run impact of simultaneous increase in female and male wages in N on total output</td>
</tr>
<tr>
<td>$\Delta p_{-1}$</td>
<td>Partial effect of simultaneous increase in female and male wages N on public debt/GDP in the short-run</td>
</tr>
<tr>
<td>$\Delta p_{-2}$</td>
<td>Impact of simultaneous increase in female and male wages in N on total output in the next period</td>
</tr>
<tr>
<td>$\Delta p_{-3}$</td>
<td>Partial effect of simultaneous increase in female and male wages in N on public debt/GDP in the next period</td>
</tr>
<tr>
<td>$\Delta p_{-4}$</td>
<td>Short-run impact of increase in female wages (decline in gender wage gap) in N on total output</td>
</tr>
<tr>
<td>$\Delta p_{-5}$</td>
<td>Partial effect of female wages (decline in gender wage gap) in N on public debt/GDP in the short-run</td>
</tr>
<tr>
<td>$\Delta p_{-6}$</td>
<td>Impact of increase in female wages (decline in gender wage gap) in N in the next period</td>
</tr>
<tr>
<td>$\Delta p_{-7}$</td>
<td>Partial effect of increase in female wages (decline in gender wage gap) in N on public debt/GDP in the next period</td>
</tr>
<tr>
<td>$\Delta p_{-8}$</td>
<td>Short-run impact of rising share of social expenditures in GDP on total output</td>
</tr>
<tr>
<td>$\Delta p_{-9}$</td>
<td>Partial effect of rising share of social expenditures in GDP on public debt/GDP in the short-run</td>
</tr>
<tr>
<td>$\Delta p_{-10}$</td>
<td>Impact of rising share of social expenditures in GDP on total output in the next period</td>
</tr>
<tr>
<td>$\Delta p_{-11}$</td>
<td>Partial effect increase rising share of social expenditures on public debt/GDP in the next period</td>
</tr>
<tr>
<td>$\Delta p_{-12}$</td>
<td>Short-run impact of increase in female wages (decline in gender wage gap) in H on total output</td>
</tr>
<tr>
<td>$\Delta p_{-13}$</td>
<td>Partial effect of increase in female wages (decline in gender wage gap) in H on public debt/GDP in the short-run</td>
</tr>
<tr>
<td>$\Delta p_{-14}$</td>
<td>Impact of increase in female wages (decline in gender wage gap) in H in the next period</td>
</tr>
<tr>
<td>$\Delta p_{-15}$</td>
<td>Partial effect increase in female wages (decline in gender wage gap) in H on public debt/GDP in the next period</td>
</tr>
</tbody>
</table>
Appendix 2: Deriving the reduced form export and import functions

Exports are a function of relative prices of exports to imports, the GDP of the rest of the world and exchange rate.

\[ \log X = x_0 + x_1 \log Y_{rw} + x_{pxm} \log \left( P_x / P_m \right) + x_3 \log E \] (A2.1)

Imports depend on domestic prices relative to import prices, the exchange rate and aggregate demand in \( Y^N \).

\[ \log M = n_0 + n_1 \log Y_N + n_{ppm} \log \left( P / P_m \right) + n_3 \log E \] (A2.2)

Domestic prices (\( P \)) and export prices (\( P_x \)) are set as a mark-up on unit labour costs and other imported input costs depending on the oligopolistic market power of firms in an imperfectly competitive economy as follows:

\[ \log P = p_0 + p_{ulc} \log (ulc) + p_m \log P_m \] (A2.3)

\[ \log P_x = p_{x0} + p_{xulc} \log (ulc) + p_{xm} \log P_m \] (A2.4)

where \( ulc \) denotes nominal unit labour costs, \( P_m \) stands for import prices.

As nominal unit labour costs are real unit labour costs multiplied by domestic prices, and the wage share is identical to real unit labour costs (corrected for the ratio of GDP at factor cost to GDP at market prices), a fall in the wage share, i.e. a rise in the profit share, leads to a fall in relative prices and improves net exports, depending on the labour intensity of exports, the pass through from labour costs to export prices and domestic prices and the price elasticity of exports and imports.

Hence in reduced form, the marginal effect of \( \pi \) on exports/GDP and imports/GDP is given by:

\[ \frac{\partial NX}{\partial \pi} = \frac{\partial X}{\partial \pi} - \frac{\partial M}{\partial \pi} \] (A2.5)

where

\[ \frac{\partial X}{\partial \pi} = \frac{\partial \log X}{\partial \log P_x} \frac{P_x}{\partial \log \left( \log \left( \frac{P_x}{P_m} \right) \right)} \frac{\partial \log \left( \frac{P_x}{P_m} \right)}{\partial \log \pi} \frac{X/Y}{\pi} = -\left( \frac{\exp e_{P_{ulc}}}{1 - e_{P_{ulc}}} \right) \frac{Y_f}{Y} \frac{X/Y}{\partial \log \pi} \] (A2.6)

\[ \frac{\partial M}{\partial \pi} = \frac{\partial \log M}{\partial \log P} \frac{\partial \log \left( \frac{P}{P_m} \right)}{\partial \log \pi} \frac{M/Y}{\pi} = -\left( \frac{\exp e_{P_{ulc}}}{1 - e_{P_{ulc}}} \right) \frac{Y_f}{Y} \frac{M/Y}{\partial \log \pi} \] (A2.7)

\( x_2 \) and \( n_2 \) in the reduced form in Equations (25)-(26) in Section 2 are a direct way of modelling these chain derivatives.
Appendix 3: The effects of wages and gender pay gap

A3.1 The effects of a change in female and male wages in N

A3.1.1 The short-run effect of a change in female and male wages in N on aggregate output

\[
\psi_{tt}^{NF} = \frac{dY_t}{dW_t^{NF}} \\
= \frac{\left( \frac{\partial C_t^N}{\partial w_t^{NF}} \right)_{y_t,a_t^N} + \left( \frac{\partial C_H}{\partial w_t^{NF}} \right)_{y_t,a_t^N} + \left( \frac{\partial I_t}{\partial w_t^{NF}} \right)_{y_t,a_t^N} + \left( \frac{\partial X_t^N}{\partial w_t^{NF}} \right)_{y_t,a_t^N} - \left( \frac{\partial M_t}{\partial w_t^{NF}} \right)_{y_t,a_t^N}}{1 - \phi_{NF}} \tag{A3.1}
\]

where \( \phi_{NF} \) is

\[
\phi_{NF} = \left( \frac{\partial E_t^{NF}}{\partial Y_t} \right)_{w_t^{NF},a_t^N} + \left( \frac{\partial E_t^{NM}}{\partial Y_t} \right)_{w_t^{NF},a_t^N} + \left( \frac{\partial E_t^{HF}}{\partial Y_t} \right)_{w_t^{NF},a_t^N} + \left( \frac{\partial E_t^{HM}}{\partial Y_t} \right)_{w_t^{NF},a_t^N} \tag{A3.2}
\]

which will be defined in A3.1.5.

\[
\left( \frac{\partial E_t^{NF}}{\partial w_t^{NF}} \right)_{y_t,a_t^N} = e_t^{NF} = 0 \tag{A3.3}
\]

\[
\left( \frac{\partial E_t^{NM}}{\partial w_t^{NF}} \right)_{y_t,a_t^N} = e_t^{NM} = 0 \tag{A3.4}
\]

\[
\left( \frac{\partial E_t^{HF}}{\partial w_t^{NF}} \right)_{y_t,a_t^N} = e_t^{HF} = 0 \tag{A3.5}
\]

\[
\left( \frac{\partial E_t^{HM}}{\partial w_t^{NF}} \right)_{y_t,a_t^N} = e_t^{HM} = 0 \tag{A3.6}
\]

\[
\left( \frac{\partial PW_t}{\partial w_t^{NF}} \right)_{y_t,a_t^N} = PW_t \left( \frac{1}{\frac{1}{PW_t} \left( \frac{\partial PW_t}{\partial w_t^{NF}} \right)_{y_t,a_t^N} + \frac{1}{\lambda_t} \left( \frac{\partial \lambda_t}{\partial w_t^{NF}} \right)_{y_t,a_t^N}} \right) \tag{A3.7}
\]

= \lambda_t \left( \frac{\partial PW_t}{\partial w_t^{NF}} \right)_{y_t,a_t^N} + PW_t \left( \frac{1}{\frac{\lambda_t}{\pi_t} \left( \frac{\partial \pi_t}{\partial w_t^{NF}} \right)_{y_t,a_t^N}} \right) \]
\[
\frac{\partial PW99_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} = PW99_t \left( \frac{1}{PW_t} \frac{\partial PW_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} + \frac{1}{(1 - \lambda_t)} \frac{\partial (1 - \lambda_t)}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} \right)
\]
\[
= (1 - \lambda_t) \left( \frac{\partial PW_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} \right)
- PW_t \left( \frac{\lambda_t}{\pi_t} \frac{\partial \pi_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} \right)
\]

where

\[
\frac{\partial PW_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} = PW_t \left( a_F \frac{E_{t}^{NF}}{W_B^F} + a_M \frac{\alpha_t^N (E_{t}^{NM})}{W_B^M} - a_R \frac{E_{t}^{NF} + \alpha_t^N E_{t}^{NM}}{R_t} \right)
\]

\[
\frac{\partial C_t^N}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} = C_t^N \left( c_F \frac{E_{t}^{NF}}{W_B^F} + c_M \frac{\alpha_t^N (E_{t}^{NM})}{W_B^M} - c_R \frac{E_{t}^{NF} + \alpha_t^N E_{t}^{NM}}{R_t} \right)
+ c_{PW1} \frac{1}{PW1_t} \left( \frac{\partial PW1_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} \right)
+ c_{PW99} \frac{1}{PW99_t} \left( \frac{\partial PW99_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} \right)
\]

\[
\frac{\partial C_t^H}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} = C_t^H \left( z_F \frac{E_{t}^{NF}}{W_B^F} + z_M \frac{\alpha_t^N (E_{t}^{NM})}{W_B^M} - z_R \frac{E_{t}^{NF} + \alpha_t^N E_{t}^{NM}}{R_t} \right)
+ z_{PW1} \frac{1}{PW1_t} \left( \frac{\partial PW1_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} \right)
+ z_{PW99} \frac{1}{PW99_t} \left( \frac{\partial PW99_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} \right)
\]

\[
\frac{\partial l_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} = l_t \left( i_2 \frac{\partial \pi_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} + i_3 \frac{d_{t}^F}{V_t} + i_4 \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} \right)
+ i_5 \frac{1}{PW99_t} \left( \frac{\partial PW99_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} \right)
\]

\[
\frac{\partial \pi_t}{\partial w_{t}^{NF}}_{Y_t,\alpha_t^N} = - \frac{(\alpha_t^N - \alpha_t^N \beta_t^N + \beta_t^N)}{T_t^N} < 0
\]
A3.1.2 The effect of a change in female and male wages in $N$ on aggregate output in the next period

\[
d_{t+1}^F = \left. \frac{\partial \left( \frac{D}{Y} \right)_t}{\partial w_{t+1}^{NF}} \right|_{Y_t, \alpha_t^N y_t, \alpha_t^N} = \frac{1}{Y_t} \left( t_t^F - t_t^W \right) (E_t^{NF} + \alpha_t^N E_t^{NM}) \\
- \frac{\partial C_t^N}{\partial w_{t+1}^{NF}}_{Y_t, \alpha_t^N} + \frac{\partial C_t^H}{\partial w_{t+1}^{NF}}_{Y_t, \alpha_t^N} - t_t^W \frac{\partial P_{t+1}^W}{\partial w_{t+1}^{NF}}_{Y_t, \alpha_t^N} \tag{A3.14}
\]

\[
\frac{\partial X_t}{\partial w_{t}^{NF}}_{Y_t, \alpha_t^N} = X_t \left( \frac{\left| \frac{\partial \pi_t}{\partial w_{t+1}^{NF}}_{Y_t, \alpha_t^N} \right|}{\pi_t} \right) > 0 \tag{A3.15}
\]

\[
\frac{\partial M_t}{\partial w_{t}^{NF}}_{Y_t, \alpha_t^N} = M_t \left( \frac{\left| \frac{\partial \pi_t}{\partial w_{t+1}^{NF}}_{Y_t, \alpha_t^N} \right|}{\pi_t} \right) > 0 \tag{A3.16}
\]

\[
\frac{\partial E_t^{NF}}{\partial w_{t-1}^{NF}}_{Y_t, \alpha_t^N} = e_t^{NF} = -\beta_t^N \left( 1 - \kappa_t^N \right) Y_t \frac{\partial T_t^N}{\partial w_{t-1}^{NF}}_{Y_t, \alpha_t^N_{t-1}} \tag{A3.17}
\]

\[
\frac{\partial E_t^{NM}}{\partial w_{t-1}^{NF}}_{Y_t, \alpha_t^N} = e_t^{NM} = -\left( 1 - \beta_t^N \right) \left( 1 - \kappa_t^N \right) Y_t \frac{\partial T_t^N}{\partial w_{t-1}^{NF}}_{Y_t, \alpha_t^N_{t-1}} \tag{A3.18}
\]

\[
\frac{\partial E_t^{HF}}{\partial w_{t-1}^{NF}}_{Y_t, \alpha_t^N} = e_t^{HF} = 0 \tag{A3.19}
\]

\[
\frac{\partial E_t^{HM}}{\partial w_{t-1}^{NF}}_{Y_t, \alpha_t^N} = e_t^{HM} = 0 \tag{A3.20}
\]

\[
\psi_{t+1}^{NF} = \frac{dY_t}{dw_{t-1}^{NF}} = \frac{\frac{\partial C_t^N}{\partial w_{t-1}^{NF}}_{Y_t, \alpha_t^N} + \frac{\partial C_t^H}{\partial w_{t-1}^{NF}}_{Y_t, \alpha_t^N} + \frac{\partial I_t}{\partial w_{t-1}^{NF}}_{Y_t, \alpha_t^N} + \frac{\partial X_t}{\partial w_{t-1}^{NF}}_{Y_t, \alpha_t^N} - \frac{\partial M_t}{\partial w_{t-1}^{NF}}_{Y_t, \alpha_t^N}}{1 - \varphi_{NF}} \tag{A3.21}
\]
\[
\left. \frac{\partial T_i^N}{\partial w_{t-1}^{nf}} \right|_{y_t, a_t^N} = T_i^N \left( h_1 \frac{\partial C^H_{t-1}}{\partial w_{t-1}^{nf}} \frac{\Psi_{(t-1)(t-1)}^F}{Y_{t-1}} + \frac{\kappa^H_{t-1} \Psi_{(t-1)(t-1)}^F}{C^H_{t-1} + \vartheta^H_{t-1}} \right) + (h_2 + h_3) \frac{\Psi_{(t-1)(t-1)}^F}{Y_{t-1}} \\
+ \frac{h_4}{w_{t-1}^{nf}} \tag{A3.22}
\]

where

\[
\Psi_{(t-1)(t-1)}^F = \frac{dY_{t-1}}{dw_{t-1}^{nf}} \tag{A3.23}
\]

and

\[
\left. \frac{\partial C^H_{t-1}}{\partial w_{t-1}^{nf}} \right|_{y_t, a_{t-1}^N} = \left. \frac{\partial C^H_{t-1}}{\partial w_{t-1}^{nf}} \right|_{y_t, a_{t-1}^N} + \left. \frac{\partial C^H_{t-1}}{\partial v_{t-1}} \right|_{y_t, a_{t-1}^N} \psi_{(t-1)}^{F(t-1)} \tag{A3.24}
\]

\[
\left. \frac{\partial C_i^N}{\partial w_{t-1}^{nf}} \right|_{y_t, a_{t-1}^N} = c_t^N \left( \frac{c_F^{nw}(e_F^{nw}(t-1)w_{t-1})}{WB_t^F} + c_M \frac{\alpha_t^N(e_M^{nw}(t-1)w_{t-1})}{WB_t^M} \\
- c_R \frac{(e^{nw}(F(t-1)) \alpha_t^N + e^{nw}(F(t-1))w_{t-1})}{R_t} + c_{pW1} \frac{1}{PW1_t} \frac{\partial PW1_t}{w_{t-1}^{nf}} \right|_{y_t, a_{t-1}^N} \\
+ c_{pW99} \frac{1}{PW99_t} \frac{\partial PW99_t}{w_{t-1}^{nf}} \right|_{y_t, a_{t-1}^N} \tag{A3.25}
\]

where

\[
\left. \frac{\partial PW1_t}{w_{t-1}^{nf}} \right|_{y_t, a_{t-1}^N} = PW_t \left( c_F \frac{\lambda_t(e_F^{nw}(t-1)w_{t-1})}{WB_t^F} + c_M \frac{\lambda_t \alpha_t^N(e_M^{nw}(t-1)w_{t-1})}{WB_t^M} \\
- c_R \frac{(e^{nw}(F(t-1)) \alpha_t^N + e^{nw}(F(t-1))w_{t-1})}{R_t} + c_c \frac{\lambda_t}{PW_t} \frac{\partial PW1_t}{w_{t-1}^{nf}} \right|_{y_t, a_{t-1}^N} \\
+ \frac{s_1}{\pi_t} \frac{\lambda_t}{w_{t-1}^{nf}} \right|_{y_t, a_{t-1}^N} \\
+ s_1 s_5 \frac{\lambda_t}{\pi_{t-1}} \frac{\partial \pi_{t-1}}{w_{t-1}^{nf}} \right|_{y_t, a_{t-1}^N} \tag{A3.26}
\]
\[
\frac{\partial PW99_t}{\partial w_{t-1}^{NF}} \bigg|_{Y_t,a_t^{N,-1}} = PW_t \left( c_F \frac{(1 - \lambda_t)(e_{F(t-1)}^{NF}w_t^{NF})}{WB_t^F} + c_M \frac{(1 - \lambda_t)\alpha_t^N(e_{F(t-1)}^{NM}w_t^{NF})}{WB_t^M} \right. \\
- c_R \frac{(e_{F(t-1)}^{NM}\alpha_t^N + e_{F(t-1)}^{NF}) (1 - \lambda_t)w_t^{NF}}{R_t} + a_c \frac{(1 - \lambda_t)}{PW_{t-1}} \left| \frac{\partial PW_{t-1}}{\partial w_{t-1}^{NF}} \right| \bigg|_{Y_t,a_t^{N,-1}} \\
- s_1 \frac{\lambda_t}{\pi_t} \frac{\partial \pi_t}{\partial w_{t-1}^{NF}} \bigg|_{Y_t,a_t^{N,-1}} \\
- s_1 s_5 \frac{\lambda_t}{\pi_{t-1}} \frac{\partial \pi_{t-1}}{\partial w_{t-1}^{NF}} \bigg|_{Y_t,a_t^{N,-1}} \\
\text{where}
\]
\[
\frac{\partial \pi_{t-1}}{\partial w_{t-1}^{NF}} \bigg|_{Y_t,a_t^{N,-1}} = - \frac{(\alpha_{t-1}^N - \alpha_{t-1}^N \beta_{t-1}^N + \beta_{t-1}^N)}{T_{t-1}^N} < 0
\] (A3.28)

and

\[
\frac{\partial PW_{t-1}}{\partial w_{t-1}^{NF}} \bigg|_{Y_t,a_t^{N,-1}} = PW_{t-1} \left( a_F \frac{(E_{t-1}^{FN} + (e_{Y(t-1)}^{NF}w_{t-1}^{NF} + e_{Y(t-1)}^{HF}w_{t-1}^{HF}) \psi_{t-1}(t-1)(t-1))}{WB_{t-1}^F} \right. \\
+ a_M \frac{(e_{MN}^{NM}z_{t-1} + (e_{Y(t-1)}^{NM}w_{t-1}^{NF}z_{t-1}^{NM} + e_{Y(t-1)}^{HM}w_{t-1}^{HF}z_{t-1}^{NM} \psi_{t-1}(t-1)(t-1))}{WB_{t-1}^M} \right. \\
+ a_R \frac{1 - \kappa_{t-1}^H}{\pi_{t-1}(t-1)(t-1)} \psi_{t-1}(t-1)(t-1) \\
- a_R \frac{(e_{t-1}^{FN} + e_{Y(t-1)}^{NF}w_{t-1}^{NF} \psi_{t-1}(t-1)(t-1)) + \alpha_{t-1}^N(E_{t-1}^{MN} + (e_{Y(t-1)}^{NM}w_{t-1}^{NF} \psi_{t-1}(t-1)(t-1)))}{R_{t-1}} \right) \\
\text{where } e_{Y(t-1)}^{NF}, e_{Y(t-1)}^{HF}, e_{Y(t-1)}^{NM}, e_{Y(t-1)}^{HM} \text{ are the effects of previous period’s output on previous period’s employment.}
\]

\[
\frac{\partial \pi_t}{\partial w_{t-1}^{NF}} \bigg|_{Y_t,a_t^{N,-1}} = \left(\frac{(\alpha_t^N - \alpha_t^N \beta_t^N + \beta_t^N)(w_t^{NF})}{(T_t^N)^2} \right) \frac{\partial T_t^N}{\partial w_{t-1}^{NF}} \bigg|_{Y_t,a_t^{N,-1}} \\
\frac{\partial c_t^H}{\partial w_{t-1}^{NF}} \bigg|_{Y_t,a_t^{N,-1}} = c_t^H \left( z_F \frac{(e_{F(t-1)}^{NF}w_t^{NF})}{WB_t^F} + z_M \frac{(e_{F(t-1)}^{NM}w_t^{NF})}{WB_t^M} \right. \\
- z_R \frac{(e_{F(t-1)}^{NM}\alpha_t^N + e_{F(t-1)}^{NF})}{R_t} w_t^{NF} + z_{PW1} \frac{1}{PW_t} \left| \frac{\partial PW_t}{\partial w_{t-1}^{NF}} \right| \bigg|_{Y_t,a_t^{N,-1}} \\
+ z_{PW99} \frac{1}{PW99_t} \left| \frac{\partial PW99_t}{\partial w_{t-1}^{NF}} \right| \bigg|_{Y_t,a_t^{N,-1}} \right)
\] (A3.30)
\[
\left| \frac{\partial l_t}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} \right| = I_t \left( i_2 \left| \frac{\partial \pi_t}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} \right| \frac{\pi_t}{\pi_t} + i_3 \frac{d_{t(t-1)}^F}{Y_t} + i_4 \frac{1}{PW_{1t}} \left| \frac{\partial PW_{1t}}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} \right| \right) + i_5 \frac{1}{PW_{99t}} \left| \frac{\partial PW_{99t}}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} \right|
\]

(A3.32)

\[
d_{t(t-1)}^F = \frac{\partial \left( \frac{D}{Y} \right)_t}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} = \frac{\partial D_t}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} \frac{1}{Y_t}
\]

(A3.33)

\[
= \left( \frac{\partial D_t}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} \right) (1 + r_{t-1}) - t_{t}^W \left( e_{Ft(t-1)}^{NM} \alpha_t^N + e_{Ft(t-1)}^{NF} \right) w_{t}^{NF} - t_{t}^P \left( e_{Ft(t-1)}^{NM} \alpha_t^N + e_{Ft(t-1)}^{NF} \right) w_{t}^{PF} - t_{t}^C \left( \frac{\partial C_t^N}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} + \frac{\partial C_t^H}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} \right) \frac{1}{Y_t}
\]

where

\[
\frac{\partial D_{t-1}}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} = Y_{t-1} \left| \frac{d \left( \frac{D}{Y} \right)_{t-1}}{d w_{t-1}^{NF} Y_t, \alpha_t^{N}} \right| + \psi_{t(t-1)(t-1)}^F D_{t-1} Y_{t-1}
\]

(A3.34)

\[
\left| \frac{\partial X_t}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} \right| = X_t \left( x_2 \left| \frac{\partial \pi_t}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} \right| \right)
\]

(A3.35)

\[
\left| \frac{\partial M_t}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} \right| = M_t \left( n_2 \left| \frac{\partial \pi_t}{\partial w_{t-1}^{NF} Y_t, \alpha_t^{N}} \right| \right)
\]

(A3.36)

A3.1.3 The effect of a change in female and male wages on employment

\[
\frac{d E_t^F}{d w_{t}^{NF} \alpha_t^N} = \beta_t^N \left( 1 - \kappa_t^H \right) \psi_t^{NF} + \frac{\beta_t^H \kappa_t^H}{w_t^{HF} \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right)} \psi_t^{HF}
\]

(A3.37)
\[
\left| \frac{dE^M}{d\omega_{tt}^{NF}} \right|_{\alpha_{t-1}^N} = (1 - \beta_i^N) \frac{(1 - \kappa^H)}{T_t^N} \psi_{tt}^{NF} + \frac{(1 - \beta_i^H)\kappa^H_t}{w_t^{HF}(\beta_i^H + \alpha_i^H - \beta_i^H \alpha_i^H)} \psi_{tt}^{NF} \tag{A3.38}
\]

\[
\left| \frac{dE_t}{d\omega_{tt}^{NF}} \right|_{\alpha_{t-1}^N} = \left( \frac{(1 - \kappa^H_t)}{T_t^N} + \frac{\kappa^H_t}{w_t^{HF}(\beta_i^H + \alpha_i^H - \beta_i^H \alpha_i^H)} \right) \psi_{tt}^{NF} \tag{A3.39}
\]

\[
\left| \frac{dE_t}{d\omega_{t(t-1)}^{NF}} \right|_{\alpha_{t-1}^N} = e_F^{N(t-1)} + \beta_i^N \frac{(1 - \kappa^H_t)}{T_t^N} \psi_{t(t-1)}^{NF} + \frac{\beta_i^H \kappa^H_t}{w_t^{HF}(\beta_i^H + \alpha_i^H - \beta_i^H \alpha_i^H)} \psi_{t(t-1)}^{NF} \tag{A3.40}
\]

\[
\left| \frac{dE^M}{d\omega_{t(t-1)}^{NF}} \right|_{\alpha_{t-1}^N} = e_F^{NM(t-1)} + (1 - \beta_i^N) \frac{(1 - \kappa^H_t)}{T_t^N} \psi_{t(t-1)}^{NF} + \frac{(1 - \beta_i^H)\kappa^H_t}{w_t^{NF}(\beta_i^H + \alpha_i^H - \beta_i^H \alpha_i^H)} \psi_{t(t-1)}^{NF} \tag{A3.41}
\]

\[
\left| \frac{dE_t}{d\omega_{t}^{NF}} \right|_{\alpha_{t-1}^N} = e_F^{NF(t-1)} + e_F^{NM(t-1)} + \left( \frac{(1 - \kappa^H_t)}{T_t^N} + \frac{\kappa^H_t}{w_t^{HF}(\beta_i^H + \alpha_i^H - \beta_i^H \alpha_i^H)} \right) \psi_{t(t-1)}^{NF} \tag{A3.42}
\]

**A3.1.4 The effect of a change in female and male wages on public debt**

\[
\left| \frac{d(D^Y)}{d\omega_{t}^{NF}} \right|_{\alpha_{t}^N} = d_{tt}^{WF} + d_{tt}^{Y} \psi_{tt}^{NF} \tag{A3.43}
\]

\[
\left| \frac{d(D^Y)}{d\omega_{t(t-1)}^{NF}} \right|_{\alpha_{t-1}^N} = d_{t(t-1)}^{WF} + d_{t(t-1)}^{Y} \psi_{t(t-1)}^{NF} \tag{A3.44}
\]

**A3.1.5 Income multiplier**

\[
Income\ multiplier = \frac{1}{1 - \phi_{NF}} \tag{A3.45}
\]

where \(\phi_{NF}\) is
\[ \varphi_{Nt} = \frac{\partial C_N}{\partial Y_t} w_{NF, \alpha_N} + \frac{\partial C_H}{\partial Y_t} w_{HF, \alpha_H} + \frac{\partial I_t}{\partial Y_t} w_{HF, \alpha_H} + \frac{\partial X_t}{\partial Y_t} w_{HF, \alpha_H} \]

(A3.46)

\[ \frac{\partial M_t}{\partial Y_t} = \kappa^H_t + \kappa^G_t \]

(A3.47)

\[ \frac{\partial E_{Nt}^N}{\partial Y_t} = \epsilon_{Nt}^N = \beta^N_t \frac{(1 - \kappa^H_t)}{T_t^N} > 0 \]

(A3.48)

\[ \frac{\partial E_{Nt}^M}{\partial Y_t} = \epsilon_{Nt}^M = (1 - \beta^N_t) \frac{(1 - \kappa^H_t)}{T_t^M} > 0 \]

(A3.49)

\[ \frac{\partial E_{Ht}^{HF}}{\partial Y_t} = \epsilon_{Ht}^{HF} = \frac{\beta^H_t \kappa^H_t}{w_{HF} (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)} > 0 \]

(A3.50)

\[ \frac{\partial C_N}{\partial Y_t} = C_T^N \left( C_F \frac{(e_{Nt} w_{Nt} + e_{Ht}^{HF} w_{HF})}{w_{Nt}^{HF}} + C_M (e_{Nt} w_{Nt} + e_{Nt}^{HF} w_{HF}^{HF}) \right) \]

(A3.51)

\[ + \frac{(1 - \kappa^H_t) - \alpha^N_t w_{HF} e_{Nt} - w_{HF} e_{Nt}}{R_t} \]

\[ + c_{PW1} \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial Y_t} + c_{PW99} \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial Y_t} \]

\[ \frac{\partial C_H}{\partial Y_t} = C_T^H \left( Z_F \frac{(e_{Nt} w_{Nt} + e_{Ht}^{HF} w_{HF})}{w_{HF}} + Z_M (e_{Nt}^{HF} w_{Nt} + e_{Nt}^{HF} w_{Nt}^{HF}) \right) \]

(A3.52)

\[ + \frac{(1 - \kappa^H_t) - \alpha^N_t w_{HF} e_{Nt} - w_{HF} e_{Nt}}{R_t} \]

\[ + z_{PW1} \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial Y_t} + z_{PW99} \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial Y_t} \]

where

\[ \frac{\partial PW1_t}{\partial Y_t} = PW1_t \left( \frac{1}{PW_t} \frac{\partial PW_t}{\partial Y_t} + \frac{1}{\lambda_t} \frac{\partial \lambda_t}{\partial Y_t} \right) \]

(A3.53)

\[ = \lambda_t \frac{\partial PW_t}{\partial Y_t} + PW_t \left( s_1 \frac{1}{\pi_t} \frac{\partial \pi_t}{\partial Y_t} \right) \]

\[ \frac{\partial PW99_t}{\partial Y_t} = PW99_t \left( \frac{1}{PW_t} \frac{\partial PW_t}{\partial Y_t} + \frac{1}{(1 - \lambda_t)} \frac{\partial (1 - \lambda_t)}{\partial Y_t} \right) \]

(A3.54)

\[ = (1 - \lambda_t) \frac{\partial PW_t}{\partial Y_t} - PW_t \left( s_1 \frac{1}{\pi_t} \frac{\partial \pi_t}{\partial Y_t} \right) \]

where
\[
\frac{\partial PW_t}{\partial Y_t} = PW_t \left( a_F \left( e_t^{NF} w_t^{NF} + e_t^{HF} w_t^{HF} \right) + a_M \frac{(e_t^{NM} w_t^{NF} \alpha_t^N + e_t^{HM} w_t^{HF} \alpha_t^H)}{WB_t^M} + a_R \frac{(1 - \kappa_t^H) - \alpha_t^N w_t^{NF} e_t^{NM} - w_t^{NF} e_t^{IF}}{R_t} \right)
\]

\[\frac{\partial I_t}{\partial Y_t = I_t \left( i_1 \frac{1}{Y_t} + i_2 \frac{\partial \pi_t}{\partial Y_t} + i_3 \left( \frac{\partial \left( \frac{D}{Y} \right)_t}{\partial Y_t} \right) + i_4 \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial Y_t} + i_5 \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial Y_t} \right)}\]

\[\frac{\partial \pi_t}{\partial Y_t} = 0\]

\[\frac{\partial \left( \frac{D}{Y} \right)_t}{\partial Y_t} = \frac{\frac{\partial D_t}{\partial Y_t} Y_t - D_t}{Y_t^2} = \frac{\partial D_t}{\partial Y_t} Y_t - \frac{D_t}{Y_t^2}\]

\[\frac{\partial \left( \frac{D}{Y} \right)_t}{\partial Y_t} = \left( \kappa_t^H + \kappa_t^C + \kappa_t^L \right) - t_t^R (1 - \kappa_t^H) - (t_t^W - t_t^F) w_t^{NF} (e_t^{NF} + \alpha_t^N e_t^{NM}) - t_t^W (e_t^{HF} + \alpha_t^H e_t^{HM})\]

\[\frac{\partial X_t}{\partial Y_t} = X_t \left( x_2 \frac{\partial \pi_t}{\partial Y_t} \right) = 0\]

\[\frac{\partial M_t}{\partial Y_t} = M_t \left( \frac{n_1}{Y_t} + n_2 \left( \frac{\partial \pi_t}{\partial Y_t} \right) \right) = \frac{M_t n_1}{Y_t} > 0\]

A3.2 The effects of a change in gender wage gap in N

A3.2.1 The short-run effect of closing the gender pay gap with rising female wages in N on aggregate output

As the male wages in N are constant, the rising female wages will reduce the gender pay gap in N in the following way:

\[
\frac{d \alpha_t^N}{d w_t^{NF}} = - \frac{\alpha_t^N}{w_t^{NF}}
\]
where

\[
\frac{\partial \psi_{tt}}{\partial w_{tt}^{NF}} = \frac{\frac{\partial C_t^N}{\partial w_{tt}^{NF}} y_t + \frac{\partial C_t^H}{\partial w_{tt}^{NF}} y_t + \frac{\partial I_t^H}{\partial w_{tt}^{NF}} y_t + \frac{\partial X_t^H}{\partial w_{tt}^{NF}} y_t - \frac{\partial M_t}{\partial w_{tt}^{NF}} y_t}{1 - \varphi_{NF}}
\]  
(A3.62)

\[
\frac{\partial E_{tt}^{NF}}{\partial w_{tt}^{NF}} = e_{at}^{NF} = 0
\]  
(A3.63)

\[
\frac{\partial E_{tt}^{NM}}{\partial w_{tt}^{NF}} = e_{at}^{NM} = 0
\]  
(A3.64)

\[
\frac{\partial E_{tt}^{HF}}{\partial w_{tt}^{NF}} = e_{at}^{HF} = 0
\]  
(A3.65)

\[
\frac{\partial E_{tt}^{HM}}{\partial w_{tt}^{NF}} = e_{at}^{HM} = 0
\]  
(A3.66)

\[
\frac{\partial PW_t^{1}}{\partial w_{tt}^{NF}} y_t = PW_t^{1} \left( \frac{1}{PW_t} \frac{\partial PW_t}{\partial w_{tt}^{NF}} y_t + \frac{\lambda_t}{\lambda_t} \frac{\partial \lambda_t}{\partial w_{tt}^{NF}} y_t \right) = \lambda_t \frac{\partial PW_t}{\partial w_{tt}^{NF}} y_t +
\]  
(A3.67)

\[
PW_t^{1} \lambda_t \left( s_1 \frac{1}{\pi_t} \frac{\partial \pi_t}{\partial w_{tt}^{NF}} y_t - s_3 \frac{1}{w_t^{NF}} \right)
\]

\[
\frac{\partial PW_99_t}{\partial w_{tt}^{NF}} y_t = PW_99_t \left( \frac{1}{PW_t} \frac{\partial PW_t}{\partial w_{tt}^{NF}} y_t + \frac{1}{1 - \lambda_t} \frac{\partial (1 - \lambda_t)}{\partial w_{tt}^{NF}} y_t \right) = (1 - \lambda_t) \frac{\partial PW_t}{\partial w_{tt}^{NF}} y_t -
\]  
(A3.68)

\[
PW_t^{1} \lambda_t \left( s_1 \frac{1}{\pi_t} \frac{\partial \pi_t}{\partial w_{tt}^{NF}} y_t - s_3 \frac{1}{w_t^{NF}} \right)
\]

where

\[
\frac{\partial PW_t}{\partial w_{tt}^{NF}} y_t = PW_t \left( a_F \frac{E_t^{NF}}{WB_t^F} - a_R \frac{E_t^{NF}}{R_t} \right)
\]  
(A3.69)

and

\[
\frac{\partial \pi_t}{\partial w_{tt}^{NF}} y_t = - \frac{\beta_t^N}{T_t^N} < 0
\]  
(A3.70)

\[
\frac{\partial C_t^N}{\partial w_{tt}^{NF}} y_t = C_t^N \left( \frac{c_F}{WB_t^F} - c_R \frac{E_t^{NF}}{R_t} + c_{PW1} \frac{1}{PW_1} \frac{\partial PW_t}{\partial w_{tt}^{NF}} y_t \right)
\]  
(A3.71)

\[
\frac{\partial C_t^H}{\partial w_{tt}^{NF}} y_t = C_t^H \left( \frac{z_F}{WB_t^F} - \frac{z_R}{R_t} + z_{PW1} \frac{1}{PW_1} \frac{\partial PW_t}{\partial w_{tt}^{NF}} y_t \right)
\]  
(A3.72)
A3.2.2 The effect of closing the gender pay gap with rising female wages in N on aggregate output in the next period

As the male wages in N are constant, the rising female wages will reduce the gender pay gap in N in previous period as in equation (A3.77).

\[
\left| \frac{\partial I_t}{\partial w_{t}^{NF}} \right|_{Y_{t}} = I_t \left( i_2 \frac{\partial \pi_t}{\partial w_{t}^{NF}} \left|_{Y_{t}} \right. + i_3 \frac{\partial \pi_t}{\partial w_{t}^{NF}} \left|_{Y_{t}} \right. + \frac{1}{\pi_t} \frac{\partial PW1_t}{\partial w_{t}^{NF}} \left|_{Y_{t}} \right. + i_5 \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial w_{t}^{NF}} \left|_{Y_{t}} \right. \right) 
\]

\[
d_{t}^{\alpha} = \left| \frac{\partial (D \bar{Y})_t}{\partial w_{t}^{NF}} \right|_{Y_{t}} = \frac{1}{Y_t} \left( t_t^B - t_t^W \right) (E_t^{NF}) - t_t^C \left( \frac{\partial C_t^N}{\partial w_{t}^{NF}} \left|_{Y_{t}} \right. + \frac{\partial C_t^H}{\partial w_{t}^{NF}} \left|_{Y_{t}} \right. \right) 
\]

\[
\left| \frac{\partial X_t}{\partial w_{t}^{NF}} \right|_{Y_{t}} = X_t \left( x_2 \frac{\partial \pi_t}{\partial w_{t}^{NF}} \left|_{Y_{t}} \right. \right) < 0 
\]

\[
\left| \frac{\partial M_t}{\partial w_{t}^{NF}} \right|_{Y_{t}} = M_t \left( n_2 \frac{\partial \pi_t}{\partial w_{t}^{NF}} \left|_{Y_{t}} \right. \right) > 0 
\]

A3.2.2 The effect of closing the gender pay gap with rising female wages in N on aggregate output in the next period

As the male wages in N are constant, the rising female wages will reduce the gender pay gap in N in previous period as in equation (A3.77).

\[
\frac{\partial a_{t-1}^N}{\partial w_{t-1}^{NF}} = - \frac{a_{t-1}^N}{w_{t-1}^{NF}} 
\]

\[
\frac{\partial E_{t}^{NF}}{\partial w_{t-1}^{NF}} = e_t^{NF} (1 - \kappa_t^H) Y_t \left( \frac{T_t^N}{(T_t^N)^2} \right) \frac{\partial T_t^N}{\partial w_{t-1}^{NF}} \left|_{Y_{t}} \right. 
\]

\[
\frac{\partial E_{t}^{NM}}{\partial w_{t-1}^{NF}} = e_t^{NM} (1 - \beta_t^H) Y_t \left( \frac{T_t^N}{(T_t^N)^2} \right) \frac{\partial T_t^N}{\partial w_{t-1}^{NF}} \left|_{Y_{t}} \right. 
\]
where

\[ \psi_{\alpha(t-1)} = \frac{dY_{t-1}}{dw_{t-1}^{N\alpha}} \]  

(A3.84)

and

\[ \frac{\partial C_{t-1}^H}{\partial w_{t-1}^{N\alpha} Y_t} = \frac{\partial C_{t-1}^H}{\partial w_{t-1}^{N\alpha} Y_{t-1}} + \frac{\partial C_{t-1}^H}{\partial Y_{t-1}} \frac{\partial Y_{t-1}}{\partial w_{t-1}^{N\alpha} Y_t} = \frac{\partial C_{t-1}^H}{\partial w_{t-1}^{N\alpha} Y_{t-1}} + \frac{\partial C_{t-1}^H}{\partial Y_{t-1}} \psi_{\alpha(t-1)(t-1)} \]  

(A3.85)

\[ \frac{\partial c_t^N}{\partial w_{t-1}^{N\alpha} Y_t} = c_t^N \left( c_F \left( e_{\alpha(t-1)}^{N\alpha} w_{t-1}^{N\alpha} \right) \right) + c_M \alpha_t^N \left( e_{\alpha(t-1)}^{N\alpha} w_{t-1}^{N\alpha} \right) \] 

\[ - c_R \frac{\left( e_{\alpha(t-1)}^{N\alpha} + e_{\alpha(t)}^{N\alpha} \right) w_{t-1}^{N\alpha}}{R_t} + c_{PW1} \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial w_{t-1}^{N\alpha} Y_t} \] 

\[ + c_{PW99} \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial w_{t-1}^{N\alpha} Y_t} \]  

(A3.86)
\[ \frac{\partial PW_{1,t}}{\partial w_{t-1}^{NF}} Y_t = PW_t \left( c_F \left( e_{\alpha(t-1)}^{NF} w_{t}^{NF} \right) \right) + c_M \left( \frac{\lambda_t \alpha_t^{N} \left( e_{\alpha(t-1)}^{NM} w_{t}^{NF} \right)}{WB_t^{F}} \right) - c_R \left( \frac{e_{\alpha(a(t-1))}^{NM} \alpha_t^{N} + e_{\alpha(a(t-1))}^{NF} }{R_t} \right) + \frac{\lambda_t}{PW_{t-1}^{1}} \left( \frac{\partial PW_{t-1}}{\partial w_{t-1}^{NF}} Y_t \right) \]

\[ + s_1 \left( \frac{\lambda_t}{\pi_t} \frac{\partial \pi_t}{\partial w_{t-1}^{NF}} Y_t \right) + s_1 s_5 \left( \frac{\lambda_t}{\pi_t - 1} \frac{\partial \pi_t}{\partial w_{t-1}^{NF}} Y_t \right) \]  

(A3.87)

\[ \frac{\partial PW_{99,t}}{\partial w_{t-1}^{NF}} Y_t = PW_t \left( c_F \left( 1 - \lambda_t \right) \left( e_{\alpha(t-1)}^{NF} w_{t}^{NF} \right) \right) + c_M \left( 1 - \lambda_t \right) \alpha_t^{N} \left( e_{\alpha(a(t-1))}^{NM} w_{t}^{NF} \right) \]

\[ - c_R \left( e_{\alpha(a(t-1))}^{NM} \alpha_t^{N} + e_{\alpha(a(t-1))}^{NF} \right) \left( 1 - \lambda_t \right) w_{t}^{NF} + \frac{(1 - \lambda_t)}{PW_{t-1}^{1}} \left( \frac{\partial PW_{t-1}}{\partial w_{t-1}^{NF}} Y_t \right) \]

\[- s_1 \left( \frac{\lambda_t}{\pi_t} \frac{\partial \pi_t}{\partial w_{t-1}^{NF}} Y_t \right) \]

\[- s_1 s_5 \left( \frac{\lambda_t}{\pi_t - 1} \frac{\partial \pi_t}{\partial w_{t-1}^{NF}} Y_t \right) \]

\[ \frac{\partial \pi_t}{\partial w_{t-1}^{NF}} Y_t = - \frac{\beta_{t-1}^{N}}{T_{t-1}^{N}} < 0 \]  

(A3.88)

and

\[ \frac{\partial PW_{t-1}}{\partial w_{t-1}^{NF}} Y_t = PW_{t-1} \left( a_F \left( e_{F(t-1)}^{N} + e_{Y(t-1)}^{NF} w_{t-1}^{NF} + e_{Y(t-1)}^{HF} w_{t-1}^{HF} \right) \psi^{\alpha}_{(t-1)(t-1)} \right) \]

\[ + a_M \left( e_{Y(t-1)}^{NM} w_{t-1}^{NF} \alpha_t^{N} + e_{Y(t-1)}^{HM} w_{t-1}^{HF} \alpha_t^{H} \right) \psi^{\alpha}_{(t-1)(t-1)} \]

\[ + a_R \left( \frac{1 - \kappa_{t-1}^{N}}{R_{t-1}} \psi^{\alpha}_{(t-1)(t-1)} \right) \]

\[ - a_R \left( e_{F(t-1)}^{N} + e_{Y(t-1)}^{NF} w_{t-1}^{NF} \psi^{\alpha}_{(t-1)(t-1)} \right) + \frac{\alpha_{t-1}^{N} e_{Y(t-1)}^{NM} w_{t-1}^{NF} \psi^{\alpha}_{(t-1)(t-1)}}{R_{t-1}} \]  

(A3.90)

where \( e_{Y(t-1)}^{NF}, e_{Y(t-1)}^{HF}, e_{Y(t-1)}^{NM}, e_{Y(t-1)}^{HM} \) are effects of previous period’s output on previous period’s employment.
\[
\frac{\partial c_H^t}{\partial w_{t-1}^{NF,Y_t}} = c_H^t \left( \frac{\partial \pi_t}{\partial w_{t-1}^{NF,Y_t}} \right) = \left( \frac{\beta_N w_{t-1}^{NF}}{w_t^{NF}} \right) \left( \frac{\partial T_t^N}{\partial w_{t-1}^{NF}} \right) \left( \frac{\partial \pi_t}{\partial w_{t-1}^{NF}} \right) \tag{A3.91}
\]

\[
= z_F \left( e_{\alpha(t-1)}^{M} w_t^{NF} \right) + z_M \left( e_{\alpha(t-1)}^{N} w_t^{NF} \right) - z_R \left( e_{\alpha(t-1)}^{N} + e_{\alpha(t-1)}^{M} w_t^{NF} \right) / R_t + z_{PW1} \left( \frac{1}{PW1_t} \right) \frac{\partial PW1_t}{\partial w_{t-1}^{NF,Y_t}} + z_{PW99} \left( \frac{1}{PW99_t} \right) \frac{\partial PW99_t}{\partial w_{t-1}^{NF,Y_t}} \tag{A3.92}
\]

\[
\frac{\partial l_t}{\partial w_{t-1}^{NF,Y_t}} = l_t \left( i_2 \frac{\partial \pi_t}{\partial w_{t-1}^{NF,Y_t}} + i_3 \frac{D_t^{(Y)}}{\pi_t} + i_4 \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial w_{t-1}^{NF,Y_t}} \right) + i_5 \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial w_{t-1}^{NF,Y_t}} \tag{A3.93}
\]

\[
d_{t(t-1)}^F = \left| \frac{\partial (D_t^{(Y)})}{\partial w_{t-1}^{NF,Y_t}} \right| = \left| \frac{\partial D_t}{\partial w_{t-1}^{NF,Y_t}} \right| \frac{1}{Y_t} = \left( \frac{\partial D_{t-1}^{(Y)}}{\partial w_{t-1}^{NF,Y_t}} \right) + t_t^W \left( e_{\alpha(t-1)}^{NM} + e_{\alpha(t-1)}^{NF} \right) w_t^{NF} - t_t^{PW} \frac{\partial PW_t}{\partial w_{t-1}^{NF,Y_t}} \tag{A3.94}
\]

where

\[
\frac{\partial D_{t-1}^{(Y)}}{\partial w_{t-1}^{NF,Y_t}} = Y_{t-1} \frac{d (D_t^{(Y)})_{t-1}}{dw_{t-1}^{NF,Y_t}} + \psi_{(t-1)(t-1)} D_{t-1} \frac{D_{t-1}}{Y_{t-1}} \tag{A3.95}
\]

\[
\frac{\partial X_t}{\partial w_{t-1}^{NF,Y_t}} = X_t \left( \frac{\partial \pi_t}{\partial w_{t-1}^{NF,Y_t}} \right) \tag{A3.96}
\]

\[
\frac{\partial M_t}{\partial w_{t-1}^{NF,Y_t}} = M_t \left( \frac{\partial \pi_t}{\partial w_{t-1}^{NF,Y_t}} \right) \tag{A3.97}
\]

82
A3.2.3 The effect of closing the gender pay gap with rising female wages in N on employment

\[
\frac{dE^t_t}{d\omega_t^t} = \beta^N_t \frac{(1 - \kappa^H_t)}{T^N_t} \psi^q_t + \beta^M_t \kappa^H_t \frac{w^H_t}{w^H_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)} \psi^q_t
\]  
(A3.98)

\[
\frac{dE^M_t}{d\omega_t^t} = (1 - \beta^N_t) \frac{(1 - \kappa^H_t)}{T^N_t} \psi^q_t + \beta^M_t \kappa^H_t \frac{w^H_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)}{w^H_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)} \psi^q_t
\]  
(A3.99)

\[
\frac{dE^F_t}{d\omega_t^t} = \frac{(1 - \kappa^H_t)}{T^N_t} + \frac{w^H_t}{w^H_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)} \psi^q_t
\]  
(A3.100)

\[
\frac{dE^M_t}{d\omega_t^t} = e^{NF}_{\alpha(t-1)} + \beta^N_t \frac{(1 - \kappa^H_t)}{T^N_t} \psi^q_t + e^{NM}_{\alpha(t-1)} + (1 - \beta^N_t) \frac{(1 - \kappa^H_t)}{T^N_t} \psi^q_t + \frac{w^H_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)}{w^H_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)} \psi^q_t
\]  
(A3.101)

\[
\frac{dE^F_t}{d\omega_t^t} = e^{NF}_{\alpha(t-1)} + e^{NM}_{\alpha(t-1)} + \frac{(1 - \kappa^H_t)}{T^N_t} \psi^q_t + \frac{w^H_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)}{w^H_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)} \psi^q_t
\]  
(A3.102)

A3.2.4 The effect of closing the gender pay gap with rising female wages in N on public debt

\[
\frac{d(D^F)}{d\omega_t^t} = d_t^a + a_t^r \psi^q_t
\]  
(A3.104)

\[
\frac{d(D^F)}{d\omega_t^t} = d_t^a + a_t^r \psi^q_t
\]  
(A3.105)
Appendix 4 The effects of fiscal policy

A4.1 The effects of public social infrastructure investment

In this section, we analyse the case where social expenditure increase solely through new public sector employment in the social sector rather than rising wages in this sector \((w_t^{HM} = w_t^{HM*}, w_t^{HF} = w_t^{HF*})\).

A4.1.1 The short-run effect of a change in public social infrastructure investment/GDP on aggregate output

\[
\psi^k_{tt} = \frac{dY_t}{d\kappa_t^H} = \frac{dY_t}{d\kappa_t^N} \frac{dY_t^N}{d\kappa_t^H} + \left( \frac{dY_t}{d\kappa_t^H} \right)_{Y_t^N} \frac{1}{1 - \varphi_k}
\]

\[
= \left( \frac{\partial C_t^N}{\partial \kappa_t^H} + \frac{\partial C_t^H}{\partial \kappa_t^H} + \frac{\partial I_t}{\partial \kappa_t^H} + \frac{\partial X_t}{\partial \kappa_t^H} - \frac{\partial M_t}{\partial \kappa_t^H} + \frac{\partial G_t^C}{\partial \kappa_t^H} + \frac{\partial I_t^C}{\partial \kappa_t^H} \right)_{Y_t^N} \frac{1}{1 - \varphi_k}
\]  
\tag{A4.1}

where

\[
dY_t^N = \frac{\partial C_t^N}{\partial \kappa_t^H} + \frac{\partial C_t^H}{\partial \kappa_t^H} + \frac{\partial I_t}{\partial \kappa_t^H} + \frac{\partial X_t}{\partial \kappa_t^H} - \frac{\partial M_t}{\partial \kappa_t^H} + \frac{\partial G_t^C}{\partial \kappa_t^H} + \frac{\partial I_t^C}{\partial \kappa_t^H} \right)_{Y_t^N} \frac{1}{1 - \varphi_k}
\]  
\tag{A4.2}

and

\[
\varphi_k = \left( \frac{\partial C_t^N}{\partial Y_t^N} \frac{\partial C_t^H}{\partial Y_t^N} + \frac{\partial I_t}{\partial Y_t^N} + \frac{\partial X_t}{\partial Y_t^N} - \frac{\partial M_t}{\partial Y_t^N} + \frac{\partial G_t^C}{\partial Y_t^N} + \frac{\partial I_t^C}{\partial Y_t^N} \right)_{\kappa_t^H} \frac{1}{1 - \varphi_k}
\]  
\tag{A4.3}

and the multiplier term is \((\frac{1}{1 - \varphi_k})(\frac{1}{1 - \varphi_k})\) which will be shown in detail in A4.1.5.

\[
e_t^{HF} = \frac{\partial E_t^{HF}}{\partial \kappa_t^H} = \frac{\beta_t^{HF} Y_t^N}{w_t^{HF}(\beta_t^{HF} + \alpha_t^{HF} - \beta_t^{HF} \alpha_t^{HF})} \frac{1}{(1 - \kappa_t^H)^2} > 0
\]  
\tag{A4.4}

\[
e_t^{HM} = \frac{\partial E_t^{HM}}{\partial \kappa_t^H} = \frac{(1 - \beta_t^{HF}) Y_t^N}{w_t^{HF}(\beta_t^{HF} + \alpha_t^{HF} - \beta_t^{HF} \alpha_t^{HF})} \frac{1}{(1 - \kappa_t^H)^2} > 0
\]  
\tag{A4.5}

\[
e_t^{NF} = \frac{\partial E_t^{NF}}{\partial \kappa_t^H} = 0
\]  
\tag{A4.6}

\[
e_t^{NM} = \frac{\partial E_t^{NM}}{\partial \kappa_t^H} = 0
\]  
\tag{A4.7}

\[
\left( \frac{\partial \pi_t}{\partial \kappa_t^H} \right)_{Y_t^N} = 0
\]  
\tag{A4.8}
\[
\frac{\partial c_t^N}{\partial \kappa_t^H} = c_t^N \left( \frac{c_{e_{tt}} w_t^{HF}}{WB_t^F} + c_{e_k} w_t^{HF} w_t^H + c_{w_t} \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial \kappa_t^H} \right) \\
+ c_{PW99} \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} \\
+ \lambda_t \frac{\partial PW_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} \\
+ (1 - \lambda_t) \frac{\partial PW_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} \\
\frac{\partial PW_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} = PW_t \left( a_F \frac{w_t^{HF}}{WB_t^F} + a_{e_{kt}} w_t^{HF} w_t^H \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial \kappa_t^H} \right) \\
+ z_{PW99} \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} \\
\frac{\partial l_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} = l_t \left( i_1 \frac{1}{1 - \kappa_t^H} + i_3 \frac{d_{tt}}{(D/Y)_t} \frac{1}{Y_t^N} - i_4 \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial \kappa_t^H} \right) \\
+ i_5 \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} \\
\right)
\]

where

\[
d_{tt} = \frac{\partial (D/Y)_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} = \frac{\partial D_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} \frac{1}{Y_t^N} - \frac{1}{(1 - \kappa_t^H) Y_t^N} \\
\]

and

\[
\frac{\partial D_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} = \frac{Y_t (1 + \kappa_t^H + \kappa_t^C)}{(1 - \kappa_t^H)} - t_t^w w_t^{HF} (e_t^H e_t^{HM} + e_{kt}^H) - t_t^{PW} \frac{\partial PW_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} \\
- t_t^C \left( \frac{\partial c_t^N}{\partial \kappa_t^H} \Bigg|_{Y_t^N} + \frac{\partial c_t^H}{\partial \kappa_t^H} \Bigg|_{Y_t^N} \right) \\
\frac{\partial X_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} = 0 \\
\frac{\partial M_t}{\partial \kappa_t^H} \Bigg|_{y_t^N} = 0 \\
\frac{\partial G_t^H}{\partial \kappa_t^H} \Bigg|_{y_t^N} = \frac{Y_t^N}{(1 - \kappa_t^H)^2} > 0 \\
\frac{\partial G_t^C}{\partial \kappa_t^H} \Bigg|_{y_t^N} = \frac{\kappa_t^H Y_t^N}{(1 - \kappa_t^H)^2} > 0 \\
\]
A4.1.2 The effect of a change in public social infrastructure investment/GDP on aggregate output in the next period

\[
\psi_{t(t-1)}^k = \frac{dY_t}{d\kappa_t^h} = \frac{dY_t}{d\kappa_t^h} N \frac{dY_t^N}{d\kappa_t^H} \]

\[
= \frac{\left| \frac{\partial C_t^N}{\partial \kappa_t^H} \right|_t^N + \left| \frac{\partial C_t^H}{\partial \kappa_t^H} \right|_t^H + \left| \frac{\partial l_t^H}{\partial \kappa_t^H} \right|_t^H + \left| \frac{\partial X_t^H}{\partial \kappa_t^H} \right|_t^H - \left| \frac{\partial M_t^H}{\partial \kappa_t^H} \right|_t^H}{(1 - \varphi_h)(1 - \kappa_t^H)} \]

\[
+ \frac{\left| \frac{\partial G_t^C}{\partial \kappa_t^H} \right|_t^C + \left| \frac{\partial l_t^G}{\partial \kappa_t^H} \right|_t^G}{(1 - \varphi_h)(1 - \kappa_t^H)} \]

\[
\frac{dY_t^N}{d\kappa_t^H} = \frac{\left| \frac{\partial C_t^N}{\partial \kappa_t^H} \right|_t^N + \left| \frac{\partial C_t^H}{\partial \kappa_t^H} \right|_t^H + \left| \frac{\partial l_t^H}{\partial \kappa_t^H} \right|_t^H + \left| \frac{\partial X_t^H}{\partial \kappa_t^H} \right|_t^H - \left| \frac{\partial M_t^H}{\partial \kappa_t^H} \right|_t^H}{(1 - \varphi_h)} \]

\[
+ \frac{\left| \frac{\partial G_t^C}{\partial \kappa_t^H} \right|_t^C + \left| \frac{\partial l_t^G}{\partial \kappa_t^H} \right|_t^G}{(1 - \varphi_h)} \]

\[
\frac{\partial T_t^N}{\partial \kappa_t^H} = T_t^N \left( \frac{\left| \frac{\partial C_t^N}{\partial \kappa_t^H} \right|_t^N + \kappa_t^H \psi_{t(t-1)}^k + Y_t^N}{C_t^H + G_t^H} \right) \]

\[
+ h_2 \left( \psi_{t(t-1)}^k \kappa_t^H + Y_t^N - Y_t - 1 \right) \]

\[
\frac{\partial C_t^H}{\partial \kappa_t^H} = \frac{\partial C_t^H}{\partial \kappa_t^H} - \frac{\partial C_t^H}{\partial Y_t^N} \frac{\partial Y_t}{\partial \kappa_t^H} \]

\[
e_{k(t-1)} = \frac{\partial E_{t}^{NF}}{\partial \kappa_t^H} = -\beta_t^N Y_t^N \frac{\partial T_t^N}{\partial \kappa_t^H} \]

\[
e_{k(t-1)} = \frac{\partial E_{t}^{NM}}{\partial \kappa_t^H} = -\left( 1 - \beta_t^N \right) Y_t^N \frac{\partial T_t^N}{\partial \kappa_t^H} \]

\[
\frac{\partial \pi_t}{\partial \kappa_t^H} = \left( \alpha_t^N - \beta_t^N \beta_t^N + \beta_t^N \right) \frac{\partial T_t^N}{\partial \kappa_t^H} \]

(A4.21)

(A4.22)

(A4.23)

(A4.24)

(A4.25)

(A4.26)

(A4.27)

(A4.28)

(A4.29)

(A4.30)
\[
\left. \frac{\partial C_t^N}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| = c_t^N \left( c_F \frac{e_k^{t-1} W_t^{N_F}}{WB_t^F} + c_M \frac{e_k^{N_M} W_t^{N_F} \alpha_t^N}{WB_t^M} \right) \\
- c_R \left( \frac{e_k^{N_M}}{R_t} \frac{\alpha_t^N}{\kappa_t^{-1}} \right) + c_{PW1} \left. \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| \\
+ c_{PW99} \left. \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| \\
\left. \frac{\partial PW1_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| = \lambda_t \left. \frac{\partial PW_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| + PW_t \left( s_1 \frac{\alpha_t}{\eta_t} \frac{\partial \pi_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right) \\
\left. \frac{\partial PW99_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| = (1 - \lambda_t) \left. \frac{\partial PW_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| - PW_t \left( s_1 \frac{\alpha_t}{\eta_t} \frac{\partial \pi_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right)
\]

where

\[
\left. \frac{\partial PW_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| = PW_t \left( a_F \frac{e_k^{t-1} W_t^{N_F}}{WB_t^F} + a_M \frac{e_k^{N_M} W_t^{N_F} \alpha_t^N}{WB_t^M} \right) \\
- a_R \left( \frac{\alpha_t^N}{\kappa_t^{-1}} \right) + a_c \left. \frac{\partial PW_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| - \frac{1}{PW_{t-1}}
\]

and

\[
\left. \frac{\partial PW_{t-1}}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| = PW_{t-1} \left( a_F \frac{e_k^{t-1} W_{t-1}^{N_F}}{WB_{t-1}^F} + a_M \frac{e_k^{N_M} W_{t-1}^{N_F} \alpha_{t-1}^N}{WB_{t-1}^M} \right) \\
+ \left. \frac{\partial PW_{t-1}}{\partial \gamma_t^N} \frac{\partial \gamma_t^N}{\partial \kappa_t^{H_{-1}}} \right| + \left. \frac{\partial PW_{t-1}}{\partial \gamma_t^N} \right|
\]

\[
\left. \frac{\partial C_t^{H_{-1}}}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| = c_t^{H_{-1}} \left( z_F \frac{e_k^{t-1} W_t^{N_F}}{WB_t^F} + z_M \frac{e_k^{N_M} W_t^{N_F} \alpha_t^N}{WB_t^M} \right) \\
- z_R \left( \frac{\alpha_t^N}{\kappa_t^{-1}} \right) + z_{PW1} \left. \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| \\
+ z_{PW99} \left. \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| \\
\left. \frac{\partial l_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right| = l_t \left( i_2 \frac{\partial \pi_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} + i_3 \frac{\partial \pi_t}{\partial \gamma_t} \left( D_t \right)_t + i_4 \frac{\partial PW1_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N} \right) \\
+ i_5 \frac{\partial PW99_t}{\partial \kappa_t^{H_{-1}}}_{\gamma_t^N}
\]

(A4.31)

(A4.32)

(A4.33)

(A4.34)

(A4.35)

(A4.36)

(A4.37)
\[
\frac{d^{k}_{t(t-1)}}{d \kappa^{H}_{t-1} Y^{N}_{t}} = \left( \frac{\partial D_{t}}{\partial \kappa^{H}_{t-1} Y^{N}_{t}} \right) + \frac{1}{Y^{N}_{t}} \frac{\partial D_{t}}{\partial \kappa^{H}_{t-1} Y^{N}_{t}} \\
= \left( \frac{\partial D_{t-1}}{\partial \kappa^{H}_{t-1}} (1 + r_{t-1}) - t^{W}_{t} (e^{NM}_{k(t-1)} \alpha^{N}_{t} + e^{NF}_{k(t-1)}) w^{NF}_{t} \\
+ t^{R}_{t} (e^{NM}_{k(t-1)} \alpha^{N}_{t} + e^{NF}_{k(t-1)}) w^{NF}_{t} - t^{PW}_{t} \left( \frac{\partial P W_{t}}{\partial \kappa^{H}_{t-1} Y^{N}_{t}} \right) \right) \frac{1}{Y^{N}_{t}} \\
- t^{C}_{t} \left( \left( \frac{\partial c^{N}_{t}}{\partial \kappa^{H}_{t-1} Y^{N}_{t}} + \frac{\partial c^{H}_{t}}{\partial \kappa^{H}_{t-1} Y^{N}_{t}} \right) \frac{1}{Y^{N}_{t}} \right) \right) \\
\frac{\partial D_{t-1}}{\partial \kappa^{H}_{t-1}} = Y_{t-1} d \left( \frac{D_{t-1}}{Y^{N}_{t-1}} \right) + \psi^{k}_{(t-1)(t-1)} \frac{D_{t-1}}{Y_{t-1}} \\
(A4.39)
\]

\[
\left| \frac{\partial X_{t}}{\partial \kappa^{H}_{t-1} Y^{N}_{t}} \right| = X_{t} \left( \frac{x_{2}}{\pi_{t}} \right) \\
(A4.40)
\]

\[
\left| \frac{\partial M_{t}}{\partial \kappa^{H}_{t-1} Y^{N}_{t}} \right| = M_{t} \left( \frac{n_{2}}{\pi_{t}} \right) \\
(A4.41)
\]

\[
\left| \frac{\partial G^{C}_{t}}{\partial \kappa^{H}_{t-1} Y^{N}_{t}} \right| = 0 \\
(A4.42)
\]

\[
\left| \frac{\partial I^{C}_{t}}{\partial \kappa^{H}_{t-1} Y^{N}_{t}} \right| = 0 \\
(A4.43)
\]

### A4.1.3 The effect of a change in public social infrastructure investment/GDP on employment

\[
\frac{d E^{E}_{t}}{d \kappa^{H}_{t}} = \left( \beta^{N}_{t} \left( 1 - \kappa^{H}_{t} \right) + \beta^{H}_{t} \frac{\kappa^{H}_{t}}{T^{N}_{t}} \right) + \frac{w^{HF}_{t} \left( \beta^{H}_{t} + \alpha^{H}_{t} - \beta^{H}_{t} \alpha^{H}_{t} \right)}{w^{HF}_{t} \left( \beta^{H}_{t} + \alpha^{H}_{t} - \beta^{H}_{t} \alpha^{H}_{t} \right) (1 - \kappa^{H}_{t})^{2}} \psi^{k}_{t} \\
(A4.44)
\]

\[
\frac{d E^{M}_{t}}{d \kappa^{H}_{t}} = \left( \beta^{N}_{t} \left( 1 - \kappa^{H}_{t} \right) + \beta^{H}_{t} \frac{\kappa^{H}_{t}}{T^{N}_{t}} \right) \frac{w^{HF}_{t} \left( \beta^{H}_{t} + \alpha^{H}_{t} - \beta^{H}_{t} \alpha^{H}_{t} \right) (1 - \kappa^{H}_{t})^{2}}{w^{HF}_{t} \left( \beta^{H}_{t} + \alpha^{H}_{t} - \beta^{H}_{t} \alpha^{H}_{t} \right) (1 - \kappa^{H}_{t})^{2}} \psi^{k}_{t} \\
(A4.45)
\]

\[
\frac{d E^{H}_{t}}{d \kappa^{H}_{t}} = \left( \beta^{N}_{t} \left( 1 - \kappa^{H}_{t} \right) + \beta^{H}_{t} \frac{\kappa^{H}_{t}}{T^{N}_{t}} \right) + \frac{w^{HF}_{t} \left( \beta^{H}_{t} + \alpha^{H}_{t} - \beta^{H}_{t} \alpha^{H}_{t} \right) (1 - \kappa^{H}_{t})^{2}}{w^{HF}_{t} \left( \beta^{H}_{t} + \alpha^{H}_{t} - \beta^{H}_{t} \alpha^{H}_{t} \right) (1 - \kappa^{H}_{t})^{2}} \psi^{k}_{t} \\
(A4.46)
\]
\[
\begin{align*}
\frac{dE_t^F}{d\kappa_{t-1}^H} &= e_{k(t-1)}^{NF} + \frac{\beta_t^N}{T_t^N} \frac{dY_{t-1}^N}{d\kappa_{t-1}^H} + \left( \frac{\beta_t^H \kappa_t^H}{w_t^{HF}(\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \Psi_{k(t-1)}^k \tag{A4.47} \\
\frac{dE_t^M}{d\kappa_{t-1}^H} &= e_{k(t-1)}^{NM} + \frac{(1 - \beta_t^N)}{T_t^N} \frac{dY_{t-1}^N}{d\kappa_{t-1}^H} + \left( \frac{\beta_t^H \kappa_t^H}{w_t^{MF}(\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \Psi_{k(t-1)}^k \tag{A4.48} \\
\frac{dE_t}{d\kappa_{t-1}^H} &= e_{k(t-1)}^{NF} + e_{k(t-1)}^{NM} + \frac{1}{T_t^N} \frac{dY_{t-1}^N}{d\kappa_{t-1}^H} + \left( \frac{\beta_t^H \kappa_t^H}{w_t^{HF}(\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \Psi_{k(t-1)}^k \tag{A4.49}
\end{align*}
\]

A4.1.4 The effect of a change in public social infrastructure investment/GDP on public debt

\[
\frac{d(D/Y)_t}{d\kappa_{t-1}^H} = d_\kappa^k_t + \frac{\partial (D/Y)_t}{\partial Y_t} \frac{\partial Y_t}{\partial \kappa_{t-1}^H} \tag{A4.50}
\]

\[
\frac{d(D/Y)_t}{d\kappa_{t-1}^H} = d_\kappa^k_{t-1} + \frac{\partial (D/Y)_t}{\partial Y_t} \frac{\partial Y_t}{\partial \kappa_{t-1}^H} \tag{A4.51}
\]

A4.1.5 Multiplier (with respect to \(Y_n\))

\[
\varphi_k = \left| \frac{\partial C_t^N}{\partial Y_t^N} \right|_{\kappa_t^H} + \left| \frac{\partial C_t^H}{\partial Y_t^N} \right|_{\kappa_t^H} + \frac{\partial l_t}{\partial Y_t^N} \left| \frac{\partial Y_t^N}{\partial \kappa_t^H} \right| - \left| \frac{\partial M_t}{\partial Y_t^N} \right| + \left| \frac{\partial G_t^C}{\partial Y_t^N} \right|_{\kappa_t^H} \tag{A4.52}
\]

\[
\frac{\partial E_t^{NF}}{\partial Y_t^N} = e_{Yt}^{NF} = \frac{\beta_t^N}{T_t^N} > 0 \tag{A4.53}
\]

\[
\frac{\partial E_t^{NM}}{\partial Y_t^N} = e_{Yt}^{NM} = \frac{(1 - \beta_t^N)}{T_t^N} > 0 \tag{A4.54}
\]

\[
\frac{\partial E_t^{HF}}{\partial Y_t^N} = e_{Yt}^{HF} = \frac{\beta_t^H \kappa_t^H}{w_t^{HF}(\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)(1 - \kappa_t^H)} > 0 \tag{A4.55}
\]

\[
\frac{\partial E_t^{HM}}{\partial Y_t^N} = e_{Yt}^{HM} = \frac{(1 - \beta_t^H \kappa_t^H)}{w_t^{MF}(\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)(1 - \kappa_t^H)} > 0 \tag{A4.56}
\]

\[
\frac{\partial C_t^N}{\partial Y_t^N} = C_t^N \left( \frac{W_{t}^{FE} e_{YN}^{Y_N} + W_{t}^{HE} e_{YN}^{H_N} + W_{t}^{AE} e_{YN}^{E_N} + W_{t}^{AH} e_{YN}^{H_N}}{w_{t}^{AE} e_{YN}^{A_N} e_{YN}^{E_N} + w_{t}^{AH} e_{YN}^{H_N}} + c_{R} \frac{1 - \alpha_t^N w_{t}^{NF} e_{YN}^{N_F} - w_{t}^{NF} e_{YN}^{N_F} + c_{PW} \frac{1}{PW_{W99} \frac{dPW_{W99}^t}{dY_t^N}}}{R_t} \right) \tag{A4.57}
\]

where

\[
\frac{\partial PW_{W99}^t}{dY_t^N} = PW_{W99}^t \left( \frac{1}{PW_{W99} \frac{dPW_{W99}^t}{dY_t^N}} + \frac{1}{\lambda_t \frac{dY_t^N}{dY_t^N}} \right) = \lambda_t \frac{dPW_{W99}^t}{dY_t^N} \tag{A4.58}
\]

and
\[
\frac{\partial PW_{99_t}}{\partial Y_t^{N}} = PW_{99_t}\left(\frac{1}{PW_t} \frac{\partial PW_t}{\partial Y_t^{N}} + \frac{1}{(1 - \lambda_t)} \frac{\partial (1 - \lambda_t)}{\partial Y_t^{N}}\right) = (1 - \lambda_t) \frac{\partial PW_t}{\partial Y_t^{N}} \tag{A4.59}
\]

where

\[
\frac{\partial PW_t}{\partial Y_t^{N}} = PW_t \left(a_F \left(\frac{e_t^{NF} w_t^{NF} + e_t^{HF} w_t^{HF}}{W_{B_t}^{F}}\right)
+ a_M \left(\frac{e_t^{NM} w_t^{NM} \alpha_t^N + e_t^{HM} w_t^{HF} \alpha_t^H}{W_{B_t}^{M}}\right)
+ a_R \left(1 - \alpha_t^N w_t^{NF} e_t^{NM} - w_t^{NM} e_t^{NF}\right)\right) \tag{A4.60}
\]

\[
\frac{\partial C_t^H}{\partial Y_t^{N}} = C_t^H \left(z_F \frac{w_t^{NF} e_t^{NF} + w_t^{HF} e_t^{HF}}{W_{B_t}^{F}} + z_M \frac{w_t^{NF} \alpha_t^N e_t^{NM} + w_t^{HF} \alpha_t^H e_t^{HM}}{W_{B_t}^{M}}
+ z_R \frac{1 - \alpha_t^N w_t^{NF} e_t^{NM} - w_t^{NM} e_t^{NF}}{R_t}\right)
+ z_{PW_{99}} \frac{1}{PW_{99_t}} \frac{\partial PW_{99_t}}{\partial Y_t^{N}} \tag{A4.61}
\]

\[
\frac{\partial I_t}{\partial Y_t^{N}} = I_t \left(i_1 \frac{1}{Y_t^{N}} + i_3 \frac{\left(\frac{\partial (D_t)}{\partial Y_t^{N}}\right)}{Y_t} + i_4 \frac{1}{PW_t} \frac{\partial PW_t}{\partial Y_t^{N}}
+ i_5 \frac{1}{PW_{99_t}} \frac{\partial PW_{99_t}}{\partial Y_t^{N}}\right) \tag{A4.62}
\]

\[
\frac{\partial (\frac{D_t}{Y_t})}{\partial Y_t^{N}} = \frac{\partial D_t}{\partial Y_t^{N}} \frac{Y_t}{Y_t^2} - \frac{D_t}{Y_t^2 (1 - \kappa_t^H)} = \frac{\partial D_t}{\partial Y_t^{N}} \frac{Y_t}{Y_t^2 (1 - \kappa_t^H)} \tag{A4.63}
\]

\[
\frac{\partial (\frac{D_t}{Y_t})}{\partial Y_t^{N}} = \left(\frac{\kappa_t^H + \kappa_t^C + \kappa_t^L}{1 - \kappa_t^H} - t_t^w - (t_t^w - t_t^R)w_t^{NF} (e_t^{NF} + \alpha_t^{NM})
- t_t^w w_t^{HF} (e_t^{NF} + \alpha_t^H e_t^{NM}) - t_t^C \left(\frac{\partial C_t^N}{\partial Y_t} + \frac{\partial C_t^H}{\partial Y_t}\right)\right)
- \frac{D_t}{Y_t (1 - \kappa_t^H)} \frac{1}{Y_t} \tag{A4.63'}
\]
The effects of a change in female and male wages in H

A4.2.1 The short-run effect of in a change in female and male wages in H on aggregate output

The impact of rising wages in H on public social expenditures/GDP is

\[
\frac{dY_t}{dw_{t}^{HF}} = \frac{dY_t}{dY_t^N} + \left( \frac{dY_t}{dw_{t}^{HF}} \right)_{t}^N (1 - \varphi_k) + \frac{dY_t}{dY_t^N} \left( \frac{dI_t}{dw_{t}^{HF}} \right)_{t}^N (1 - \varphi_k)
\]

where

\[
\varphi_k = \left( \frac{1 - \varphi_k}{1 - \kappa^H} \right)
\]

and the multiplier term is \( \left( \frac{1}{1 - \kappa^H} \right) \).

The partial derivatives on employment are zero

\[
\frac{\partial E_{t}^{HF}}{\partial w_{t}^{HF}} = \frac{\partial E_{t}^{HM}}{\partial w_{t}^{HF}} = \frac{\partial E_{t}^{NF}}{\partial w_{t}^{HF}} = \frac{\partial E_{t}^{NM}}{\partial w_{t}^{HF}} = 0
\]
\[
\frac{\partial C_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} = C_t \left( c_F \frac{E_t^{HF}}{W_{BF}^{HF}} + c_M \frac{E_t^{HM} a_t^H}{W_{BM}^{HF}} + c_{PW} \frac{1}{PW_{1t}} \frac{\partial PW_{1t}}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} \right)
\]
\[
+ c_{PW99} \frac{1}{PW_{99t}} \frac{\partial PW_{99t}}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}}
\]
\[
= \lambda_t \frac{\partial PW_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} - (1 - \lambda_t) \frac{\partial PW_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}}
\]
\[
= \left( \alpha_F \frac{E_t^{HF}}{W_{BF}^{HF}} + \alpha_M \frac{E_t^{HM} a_t^H}{W_{BM}^{HF}} \right) PW_t
\]
\[
\frac{\partial C_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} = \frac{1}{PW_{99t}} \frac{\partial PW_{99t}}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}}
\]
\[
\frac{\partial d_{t}}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} = I_t \left( \frac{1}{1 - \kappa_t^H} \frac{d\kappa_t^H}{dw_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} + i_3 \frac{d w_{HF}^{WH}}{D_{Y_t}} + i_4 \frac{1}{PW_{1t}} \frac{\partial PW_{1t}}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} \right)
\]
\[
+ i_5 \frac{1}{PW_{99t}} \frac{\partial PW_{99t}}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}}
\]
\]

where
\[
d_{t}^{WH} = \frac{\partial (D/Y)_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} = \frac{\partial D_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} \frac{1}{Y_t^N} - \frac{1}{1 - \kappa_t^H} \frac{D_t}{Y_t} \frac{d\kappa_t^H}{dw_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}}
\]
\[
\frac{\partial D_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} = Y_t (1 + \kappa_t^C + \kappa_t^L) \frac{d\kappa_t^H}{dw_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} - t_t^L \left( \alpha_t^H E_t^{HM} + E_t^{HF} \right)
\]
\[
- t_t^{PW} \frac{\partial PW_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} - t_t^C \left( \frac{\partial C_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} + \frac{\partial C_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} \right)
\]
\[
\frac{\partial X_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} = 0
\]
\[
\frac{\partial M_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} = 0
\]
\[
\frac{\partial G_t}{\partial w_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} = \frac{\kappa_t^C Y_t}{(1 - \kappa_t^H)^2} \frac{d\kappa_t^H}{dw_{HF}}_{Y_t^N, E_t^{HF}, a_t^{HF}} > 0
\]
A4.2.2 The effect of a change in female and male wages in $H$ on aggregate output in the next period

\[
\psi^H_{t(t-1)} = \frac{dY_t}{dw_{t-1}^H} = \frac{dY_t}{dY_t^N} \frac{dY_t^N}{dw_{t-1}^H} \frac{dC_t^N}{dw_{t-1}^H \gamma_t^N} + \frac{dC_t^H}{dw_{t-1}^H \gamma_t^N} + \frac{dI_t^L}{dw_{t-1}^H \gamma_t^N} + \frac{dX_t}{dw_{t-1}^H \gamma_t^N} - \frac{dM_t}{dw_{t-1}^H \gamma_t^N}
\]

\[(1 - \varphi_k)(1 - \kappa_t^H)\]

\[
\frac{\partial G_{t-1}^H}{\partial w_{t-1}^H \gamma_t^N} + \frac{\partial l_{t-1}^L}{\partial w_{t-1}^H \gamma_t^N}
\]

where

\[
\frac{dY_t^N}{dw_{t-1}^H \gamma_t^N} = \frac{\partial C_t^N}{\partial w_{t-1}^H \gamma_t^N} + \frac{\partial C_t^H}{\partial w_{t-1}^H \gamma_t^N} + \frac{\partial I_t^L}{\partial w_{t-1}^H \gamma_t^N} + \frac{\partial X_t}{\partial w_{t-1}^H \gamma_t^N} - \frac{\partial M_t}{\partial w_{t-1}^H \gamma_t^N}
\]

\[(1 - \varphi_k)\]

\[
+ \frac{\partial G_{t-1}^H}{\partial w_{t-1}^H \gamma_t^N} + \frac{\partial l_{t-1}^L}{\partial w_{t-1}^H \gamma_t^N}
\]

The rising wage in $H$ also increases the public social expenditures/GDP in the previous period by the following amount:

\[
\frac{d \kappa^H_{t-1}}{dw_{t-1}^H} = \frac{E_{t-1}^H + \alpha_{t-1}^H + w_{t-1}^H (e_{Y(t-1)}^H + e_{Y(t-1)}^H \alpha_{t-1}^H)}{(1 - \kappa_t^H)} + \psi^H_{(t-1)(t-1)} \kappa^H_{t-1}
\]

\[
\frac{\partial T_t^N}{\partial w_{t-1}^H \gamma_t^N} = T_t^N \left( h_1 \frac{\partial C_{t-1}^H}{\partial \kappa_{t-1}^H \gamma_t^N} + \kappa_{t-1}^H \psi^H_{(t-1)(t-1)} + Y_{t-1} \frac{d \kappa^H_{t-1}}{dw_{t-1}^H} \right) + h_2 \left( \psi^H_{(t-1)(t-1)} \kappa_{t-1}^H + Y_{t-1} \frac{d \kappa^H_{t-1}}{dw_{t-1}^H} \right)
\]

\[(1 - \kappa_{t-1}^H)\]

\[
+ h_3 \left( \psi^H_{(t-1)(t-1)} \gamma_t^N \frac{d \kappa^H_{t-1}}{dw_{t-1}^H} - \frac{1}{(1 - \kappa_{t-1}^H) \frac{d \kappa^H_{t-1}}{dw_{t-1}^H}} \right)
\]
\[
\frac{\partial C_i^{H+t-1}}{\partial w_{t-1}^{HF}} = \frac{\partial C_i^{H+t-1}}{\partial w_{t-1}^{HF}} + \left(\frac{\partial C_i^{H+t-1}}{\partial w_{t-1}^{HF}}\right)_{Y_i^{N}} \frac{\partial Y_i^{N}}{\partial w_{t-1}^{HF}}_{Y_i^{N}} \quad (A4.89)
\]
\[
e^n_{H(t-1)} = \left(\frac{\partial e^n}{\partial w_{t-1}^{HF}}\right)_{Y_i^{N}} = -\beta_t^{N} Y_t^{N} \quad (A4.90)
\]
\[
e^{NM}_{H(t-1)} = \left(\frac{\partial e^{NM}}{\partial w_{t-1}^{HF}}\right)_{Y_i^{N}} = -\frac{(1 - \beta_t^{N}) Y_t^{N}}{(T_t^{N})^2} \quad (A4.91)
\]
\[
e^{HF}_{H(t-1)} = \left(\frac{\partial e^{HF}}{\partial w_{t-1}^{HF}}\right)_{Y_i^{N}} = 0 \quad (A4.92)
\]
\[
e^{HM}_{H(t-1)} = \left(\frac{\partial e^{HM}}{\partial w_{t-1}^{HF}}\right)_{Y_i^{N}} = 0 \quad (A4.93)
\]
\[
\frac{\partial \pi_t}{\partial w_{t-1}^{HF}} = \left(\frac{\alpha_t^{N} - \alpha_t^{N} \beta_t^{N} + \beta_t^{N}}{(T_t^{N})^2}\right) \frac{\partial T_t^{N}}{\partial w_{t-1}^{HF}}_{Y_i^{N}} \quad (A4.94)
\]
\[
\frac{\partial C_i^{t}}{\partial w_{t-1}^{HF}} = C_i^{t} \left(\frac{c_F}{W B_t^{F}} \frac{e_{H(t-1)}^{NF} w_{t-1}^{NF}}{R_t} + \frac{c_M}{W B_t^{M}} \frac{e_{H(t-1)}^{NM} w_{t-1}^{NM}}{R_t} - c_R \frac{(e_{H(t-1)}^{NM} w_{t-1}^{NM})}{W B_t^{F}} + c_{PW1} \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial w_{t-1}^{HF}}_{Y_i^{N}} \right) + c_{PW99} \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial w_{t-1}^{HF}}_{Y_i^{N}} \quad (A4.95)
\]
\[
\frac{\partial PW1_t}{\partial w_{t-1}^{HF}}_{Y_i^{N}} = \lambda_t \frac{\partial PW_t}{\partial w_{t-1}^{HF}}_{Y_i^{N}} + PW_t \left(\frac{\lambda_t}{\pi_t} \frac{\partial \pi_t}{\partial w_{t-1}^{HF}}_{Y_i^{N}}\right) \quad (A4.96)
\]
\[
\frac{\partial PW99_t}{\partial w_{t-1}^{HF}}_{Y_i^{N}} = \left(1 - \lambda_t\right) \frac{\partial PW_t}{\partial w_{t-1}^{HF}}_{Y_i^{N}} - PW_t \left(\frac{\lambda_t}{\pi_t} \frac{\partial \pi_t}{\partial w_{t-1}^{HF}}_{Y_i^{N}}\right) \quad (A4.97)
\]

where
\[
\frac{\partial PW_t}{\partial w_{t-1}^{HF}}_{Y_i^{N}} = PW_t \left(\frac{\alpha_t^{N}}{W B_t^{F}} \frac{e_{H(t-1)}^{NF} w_{t-1}^{NF}}{R_t} + \alpha_t^{N} \frac{e_{H(t-1)}^{NM} w_{t-1}^{NM}}{W B_t^{M}} - a_R \frac{(e_{H(t-1)}^{NM} w_{t-1}^{NM})}{W B_t^{F}} + a_c \frac{\partial PW_t}{\partial w_{t-1}^{HF}}_{Y_i^{N}} \frac{1}{PW_t}\right) \quad (A4.98)
\]
and
\[
\frac{\partial PW_t}{\partial w_{t-1}^{HF}}_{Y_i^{N}} = PW_t \left(\frac{\alpha_t^{N}}{W B_t^{F}} \frac{e_{H(t-1)}^{NF} w_{t-1}^{NF}}{R_t} + \alpha_t^{N} \frac{e_{H(t-1)}^{NM} w_{t-1}^{NM}}{W B_t^{M}} \right) \quad (A4.99)
\]
\[
\begin{align*}
\frac{\partial C_t^N}{\partial w_{t-1}^{HF}} = & c_t^H \left( z_F \frac{e_H^{NF}(t-1)w_t^{NF}}{WB_t^F} + z_M \frac{e_H^{NM}(t-1)w_t^{NF} \alpha_t^N}{WB_t^M} \right. \\
& \left. - z_R \frac{e_H^{NM}(t-1) \alpha_t^N + e_H^{NF}(t-1)}{R_t} w_t^{NF} \right) + z_{PW1} \frac{1}{PW1_t} \left| \frac{\partial PW1_t}{\partial w_{t-1}^{HF}} \right|_{Y_t^N} \\
& + z_{PW99} \frac{1}{PW99_t} \left| \frac{\partial PW99_t}{\partial w_{t-1}^{HF}} \right|_{Y_t^N} \\
\frac{\partial l_t}{\partial w_{t-1}^{HF}} = & l_t \left( i_2 \left| \frac{\partial \pi_t}{\partial w_{t-1}^{HF}} \right|_{Y_t^N} \frac{\pi_t}{\pi_t} + i_3 \frac{a_{t(t-1)}}{Y_t} + i_4 \frac{1}{PW1_t} \left| \frac{\partial PW1_t}{\partial w_{t-1}^{HF}} \right|_{Y_t^N} \\
& + i_5 \frac{1}{PW99_t} \left| \frac{\partial PW99_t}{\partial w_{t-1}^{HF}} \right|_{Y_t^N} \right) \\
d_{t(t-1)}^H = & \left| \frac{\partial (D/Y)_t}{\partial w_{t-1}^{HF}} \right|_{Y_t^N} = \left| \frac{\partial D_t}{\partial w_{t-1}^{HF}} \right|_{Y_t} \frac{1}{Y_t} \\
& = \left( \frac{\partial D_{t-1}}{\partial w_{t-1}^{HF}} (1 + r_{t-1}) - t_t^W (e_H^{NM}(t-1) \alpha_t^N + e_H^{NF}(t-1)) w_t^{NF} \right) \\
& + t_t^R (e_H^{NM}(t-1) \alpha_t^N + e_H^{NF}(t-1)) w_t^{NF} - t_{PW} \left| \frac{\partial PW_t}{\partial w_{t-1}^{HF}} \right|_{Y_t^N} \\
& - t_t^C \left( \left| \frac{\partial C_t^N}{\partial w_{t-1}^{HF}} \right|_{Y_t^N} + \left| \frac{\partial C_t^H}{\partial w_{t-1}^{HF}} \right|_{Y_t^N} \right) \frac{1}{Y_t} \\
\frac{\partial D_{t-1}}{\partial w_{t-1}^{HF}} = & Y_{t-1} \frac{d (D/Y)_{t-1}}{dw_{t-1}^{HF}} + \Psi_{t(t-1)} \frac{D_{t-1}}{Y_{t-1}} \\
\frac{\partial X_t}{\partial w_{t-1}^{HF}} = & X_t \left( x_2 \frac{\left| \frac{\partial \pi_t}{\partial w_{t-1}^{HF}} \right|_{Y_t^N}}{\pi_t} \right) \\
\frac{\partial M_t}{\partial w_{t-1}^{HF}} = & M_t \left( n_2 \frac{\left| \frac{\partial \pi_t}{\partial w_{t-1}^{HF}} \right|_{Y_t^N}}{\pi_t} \right) \\
\frac{\partial G_t^C}{\partial w_{t-1}^{HF}} = & 0
\end{align*}
\]
A4.2.3 The effect of in a change in female and male wages on employment

\[
\frac{dE_t^F}{dw_{t}^{HF}} = \left( \beta_t^N \left( 1 - \kappa_t^H \right) \frac{T_t^N}{T_t^N} + \beta_t^H \kappa_t^H \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right) \right) \Psi_t^H
\]  
(A4.108)

\[
\frac{dE_t^M}{dw_{t}^{HF}} = \left( \left( 1 - \beta_t^N \right) \left( 1 - \kappa_t^H \right) \frac{T_t^N}{T_t^N} + \left( 1 - \beta_t^H \right) \frac{\kappa_t^H}{w_t^H \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right)} \right) \Psi_t^H
\]  
(A4.109)

\[
\frac{dE_t}{d\kappa_t^H} = \left( \frac{1 - \kappa_t^H}{T_t^N} + \frac{\kappa_t^H}{w_t^H \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right)} \right) \Psi_t^H
\]  
(A4.110)

\[
\frac{dE_t^F}{dw_{t-1}^{HF}} = e_{H(t-1)}^{NF} + \beta_t^N \frac{1}{T_t^N} \left| \frac{dY_t^N}{dY_{t-1}^N} \right|_{\xi_t^N,\alpha_t^L} + \left( \frac{\beta_t^H \kappa_t^H}{w_t^H \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right)} \right) \Psi_{t(t-1)}^H
\]  
(A4.111)

\[
\frac{dE_t^M}{dw_{t-1}^{HF}} = e_{H(t-1)}^{NM} + \left( 1 - \beta_t^N \right) \frac{1}{T_t^N} \left| \frac{dY_t^N}{dY_{t-1}^N} \right|_{\xi_t^N,\alpha_t^L} + \left( \frac{\left( 1 - \beta_t^H \right) \kappa_t^H}{w_t^H \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right)} \right) \Psi_{t(t-1)}^H
\]  
(A4.112)

\[
\frac{dE_t}{dw_{t-1}^{HF}} = e_{H(t-1)}^{NF} + e_{H(t-1)}^{NM} + \frac{1}{T_t^N} \left| \frac{dY_t^N}{dY_{t-1}^N} \right|_{\xi_t^N,\alpha_t^L} + \left( \frac{\kappa_t^H}{w_t^H \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right)} \right) \Psi_{t(t-1)}^H
\]  
(A4.113)

A4.2.4 The effect of in a change in female and male wages on public debt

\[
\frac{d(D_t)}{dw_{t}^{HF}} = \frac{dH_t}{dt} + \frac{\partial (D_t)}{\partial Y_t^N} \left| \frac{dY_t^N}{dY_{t-1}^N} \right|_{\xi_t^N,\alpha_t^L}
\]  
(A4.114)

\[
\frac{d(D_t)}{dw_{t}^{HF}} = \frac{dH_{t(t-1)}}{dt} + \frac{\partial (D_t)}{\partial Y_t^N} \left| \frac{dY_t^N}{dY_{t-1}^N} \right|_{\xi_t^N,\alpha_t^L}
\]  
(A4.115)

A4.3 The effects of a change in the gender wage gap in H

A4.3.1 The short-run effect of in a change in gender wage gap in H on aggregate output

The impact of rising social expenditures through closing the gender wage gap in H, i.e. increasing female wages with a constant male wage on public social expenditures/GDP is
\[
\frac{d\kappa^H_t}{dw^H_{t | Y^N_{t, E^H_t}}} = \frac{E^H_t (1 - \kappa_t^H)}{Y_t}
\]  
(A4.116)

\[
\psi^{aH}_{tt} = \frac{dY_t}{dw^H_{t | Y^N_{t, E^H_t}}} = \frac{dY_t}{dw^H_{t | Y^N_{t, E^H_t}}} + \left( \frac{\partial C^N_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} + \frac{\partial C^H_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} + \frac{\partial I_t^c}{\partial w^H_{t | Y^N_{t, E^H_t}}} + \frac{\partial X_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} - \frac{\partial M_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} \right) \left( 1 - \varphi_k \right)
\]

\[
\varphi_k = \frac{\partial C^N_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} + \frac{\partial C^H_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} + \frac{\partial I_t^c}{\partial w^H_{t | Y^N_{t, E^H_t}}} + \frac{\partial X_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} - \frac{\partial M_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} + \frac{\partial G^C_t}{\partial w^H_{t | Y^N_{t, E^H_t}}}
\]

The partial derivatives on employment are zero
\[
\frac{\partial E^H_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} = 0
\]  
(A4.120)

\[
\frac{\partial \pi_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} = 0
\]  
(A4.121)

\[
\frac{\partial C^N_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} = C^N_t \left( c_F E^H_t \frac{w^{BH}_t}{WB^t} + c_{PW1} \frac{1}{PW1_t} \frac{\partial PW1_t}{\partial w^H_{t | Y^N_{t, E^H_t}}} \right)
\]

\[
+ c_{PW99} \frac{1}{PW99_t} \frac{\partial PW99_t}{\partial w^H_{t | Y^N_{t, E^H_t}}}
\]

\[
\frac{\partial PW1_t}{w^H_{t | Y^N_{t, E^H_t}}} = \lambda_t \frac{\partial PW_t}{\partial w^H_{t | Y^N_{t, E^H_t}}}
\]  
(A4.122)
\[
\frac{\partial PW_{99t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} = (1 - \lambda_{t}) \left| \frac{\partial PW_{t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right|
\]
(A4.124)
\[
\frac{\partial PW_{t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} = PW_{t} \left( a_{F} \frac{E_{t}^{HF}}{W_{t}^{HF}} \right)
\]
(A4.125)
\[
\left| \frac{\partial C^{H}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right| = C^{H} \left( z_{F} \frac{E_{t}^{HF}}{W_{t}^{HF}} + z_{PW_{1}} \frac{1}{PW_{1t}} \left| \frac{\partial PW_{1t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right| + \frac{1}{PW_{99t}} \left| \frac{\partial PW_{99t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right| \right)
\]
(A4.126)
\[
\left| \frac{\partial I_{t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right| = I_{t} \left( i_{1} \frac{1}{1 - \kappa_{t}^{H}} \left| \frac{d\kappa_{t}^{H}}{d w_{t}^{HF}}_{t_{t},E_{t}} \right| + i_{3} \frac{d_{t}^{H}}{D_{t}^{H}} + i_{4} \frac{1}{PW_{1t}} \left| \frac{\partial PW_{1t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right| + i_{5} \frac{1}{PW_{99t}} \left| \frac{\partial PW_{99t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right| \right)
\]
(A4.127)

where
\[
d_{t_{tt}}^{HF} = \left| \frac{\partial (D/Y)_{t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right|_{t_{t},E_{t}} = \left| \frac{\partial D_{t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right|_{t_{t},E_{t}} \left( 1 - \kappa_{t}^{H} \right) Y_{t} + \left| \frac{\partial \kappa_{t}^{H}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right|_{t_{t},E_{t}}
\]
(A4.128)

and
\[
\left| \frac{\partial D_{t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right|_{t_{t},E_{t}} = Y_{t} \left( 1 + \kappa_{t}^{C} + \kappa_{t}^{C} \right) \left| \frac{d\kappa_{t}^{H}}{d w_{t}^{HF}}_{t_{t},E_{t}} \right|_{t_{t},E_{t}} - t_{t}^{w} \left( E_{t}^{HF} \right) - t_{t}^{pw} \left| \frac{\partial PW_{t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right|_{t_{t},E_{t}}
\]
(A4.129)
\[
\left| \frac{\partial X_{t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right|_{t_{t},E_{t}} = 0
\]
(A4.130)
\[
\left| \frac{\partial M_{t}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right|_{t_{t},E_{t}} = 0
\]
(A4.131)
\[
\left| \frac{\partial G_{t}^{H}}{\partial w_{t}^{HF}}_{t_{t},E_{t}} \right|_{t_{t},E_{t}} = \frac{Y_{t}^{N}}{(1 - \kappa_{t}^{H})^{2}} \left| \frac{d\kappa_{t}^{H}}{d w_{t}^{HF}}_{t_{t},E_{t}} \right|_{t_{t},E_{t}} > 0
\]
(A4.132)
A4.3.2 The effect of in a change in the gender wage gap in H on aggregate output in the next period

Closing the gender wage gap in H with increasing female wages in H also increases the public social expenditures/GDP in the previous period by the following amount:

\[
\frac{d \kappa_t^H}{dw_{t-1}^H} = \frac{E_{t-1}^{HF} + w_{t-1}^{HF}(\gamma_{t-1}^{H} + \eta_{t-1}^{HF})}{1 - \kappa_t^H} + \Psi_{t-1}^{H}(1 - \kappa_t^H)\kappa_t^{H-1}
\]

(A4.135)

and

\[
\frac{d \alpha_{t-1}^{H}}{dw_{t-1}^H} = -\alpha_{t-1}^{H} \frac{w_{t-1}^{HF}}{w_{t-1}^{HF}}
\]

(A4.136)

\[
\Psi_{t-1}^{H}(1 - \kappa_t^H) = \frac{dY_t}{dw_{t-1}^H} = \frac{dY_t}{dY_t^H} \frac{dY_t^H}{dw_{t-1}^H}
\]

\[
\frac{\partial C_t^N}{\partial \kappa_{t-1}^H} + \frac{\partial C_t^H}{\partial \kappa_{t-1}^H} + \frac{\partial I_t^G}{\partial \kappa_{t-1}^H} + \frac{\partial X_t}{\partial \kappa_{t-1}^H} - \frac{\partial M_t}{\partial \kappa_{t-1}^H}
\]

(A4.137)

\[
\frac{\partial G_t^C}{\partial \kappa_{t-1}^H} + \frac{\partial I_t^C}{\partial \kappa_{t-1}^H} + \frac{\partial I_t^G}{\partial \kappa_{t-1}^H} + \frac{\partial X_t}{\partial \kappa_{t-1}^H} - \frac{\partial M_t}{\partial \kappa_{t-1}^H}
\]

(A4.138)
\[
\left| \frac{\partial T^N_{t}}{\partial w_{t-1}^{HF}} \right|_{y_t^{N,E_t^{11}}} = T^N_t \left( h_1 \frac{\left| \frac{\partial C^H_{t-1}}{\partial \kappa^H_{t-1}} \right|_{y_t^{N,E_t^{11}}} + \kappa^H_{t-1} \Psi^H_{(t-1)(t-1)} + Y_{t-1} \frac{d \kappa^H_{t-1}}{d w_{t-1}^{HF}}}{C^H_{t-1} + G^H_{t-1}} \right) \\
+ h_2 \frac{\Psi^H_{(t-1)(t-1)} \kappa^H_{t-1}}{I^G_{t-1}} + h_3 \left( \frac{\Psi^H_{(t-1)(t-1)}}{Y_{t-1}} - \frac{1}{(1 - \kappa^H_{t-1})} \frac{d \kappa^H_{t-1}}{d w_{t-1}^{HF}} \right) 
\]
(A4.139)

\[
\left| \frac{\partial C^H_{t-1}}{\partial w_{t-1}^{HF}} \right|_{y_t^{N,E_t^{11}}} = \frac{\partial C^H_{t-1}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} + \frac{\partial C^H_{t-1}}{\partial Y_{t-1}^{N,E_t^{11}}} \frac{\partial Y_{t-1}^{N,E_t^{11}}}{\partial w_{t-1}^{HF}} \right|_{y_t^{N,E_t^{11}}} 
\]
(A4.140)

\[
\begin{align*}
\varepsilon^{NF}_{aH(t-1)} &= \frac{\partial E^{NF}_{t}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} = - \frac{\beta^N_{t} Y_{t}^{N}}{(T_{t}^{N})^2} \frac{\partial T_{t}^{N}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}}} 
\varepsilon^{NM}_{aH(t-1)} &= \frac{\partial E^{NM}_{t}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} = \frac{1 - \beta^N_{t} Y_{t}^{N}}{(T_{t}^{N})^2} \frac{\partial T_{t}^{N}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} \right. 
\varepsilon^{HF}_{aH(t-1)} &= \frac{\partial E^{HF}_{t}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} = 0 
\varepsilon^{HM}_{aH(t-1)} &= \frac{\partial E^{HM}_{t}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} = 0 \end{align*} 
\]
(A4.141-4.144)

\[
\frac{\partial \pi_{t}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} = \left( \frac{\alpha^N_{t} - \alpha^N_{t} \beta^N_{t} + \beta^N_{t} \beta^N_{t}}{(T_{t}^{N})^2} \right) \frac{\partial T_{t}^{N}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} \right. 
\]
(A4.145)

\[
\begin{align*}
\frac{\partial C^N_{t}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} &= c^N_t \left( c_F \frac{e^{NF}_{aH(t-1)}}{W_{t}^{F}} + c_M \frac{e^{NM}_{aH(t-1)}}{W_{t}^{M}} \right) \\
&- c_R \left( \frac{e^{NM}_{aH(t-1)} \alpha^N_{t} + e^{NF}_{aH(t-1)}}{R_{t}} \right) + c_{PW1} \left( \frac{1}{P_{W99}} \right) \frac{\partial P_{W99}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}}} 
\frac{\partial P_{W1}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} &= \lambda_t \frac{\partial P_{W1}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} + P_{W1} \left( \frac{\lambda_t}{\pi_t} \frac{\partial \pi_{t}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}}} \right. 
\frac{\partial P_{W99}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} &= \left( 1 - \lambda_t \right) \frac{\partial P_{W1}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} - P_{W1} \left( \frac{\lambda_t}{\pi_t} \frac{\partial \pi_{t}}{\partial w_{t-1}^{HF}} \left|_{y_t^{N,E_t^{11}}} \right. \right) 
\end{align*} 
\]
(A4.146-4.148)

100
where

\[
\left. \frac{\partial PW_t}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} = PW_t \left( a_F \frac{e_{ah(t-1)}^{NF} w_t^{NF}}{WB_t^F} + a_M \frac{e_{ah(t-1)}^{NM} w_t^{NF}}{WB_t^{NM}} - a_R \frac{(e_{ah(t-1)}^{NF} + e_{ah(t-1)}^{NM}) w_t^{NF}}{R_t} + a_c \left| \frac{\partial PW_t}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} \right|_{Y_t^N, e^H_t} \frac{1}{PW_t} \right) \tag{A4.149}
\]

and

\[
\left. \frac{\partial PW_{t-1}}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} = PW_{t-1} \left( a_F \frac{e_{ah(t-1)}^{NF} w_{t-1}^{NF}}{WB_{t-1}^F} + a_M \frac{e_{ah(t-1)}^{NM} w_{t-1}^{NF} \alpha_t^N}{WB_{t-1}^{NM}} \right) + \left| \frac{\partial PW_t}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} \frac{\partial Y_t^N}{\partial w^{HF}_{t-1}} \tag{A4.150}
\]

\[
\left. \frac{\partial C^H_t}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} = C^H_t \left( z_F \frac{e_{ah(t-1)}^{NF} w_t^{NF}}{WB_t^F} + z_M \frac{e_{ah(t-1)}^{NM} w_t^{NF} \alpha_t^N}{WB_t^{NM}} \right) - z_R \frac{(e_{ah(t-1)}^{NF} + e_{ah(t-1)}^{NM}) w_t^{NF}}{R_t} + z_{PW99} \left| \frac{\partial PW_{t-1}}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} \right|_{Y_t^N, e^H_t} \frac{1}{PW99_t} \left| \frac{\partial PW_{t-1}}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} \tag{A4.151}
\]

\[
\left. \frac{\partial I_t}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} = I_t \left( i_2 \frac{\partial \pi_t}{\partial w^{HF}_{t-1}} \left|_{Y_t^N, e^H_t} \right|_{Y_t^N, e^H_t} + i_3 \frac{d_{t(t-1)}^{ah}}{D^{Y_t}} + i_4 \left| \frac{\partial PW_t}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} \right|_{Y_t^N, e^H_t} \right|_{Y_t^N, e^H_t} \frac{1}{PW99_t} \left| \frac{\partial PW_{t-1}}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} \tag{A4.152}
\]

\[
d_{t(t-1)}^{ah} = \left. \frac{\partial (D/Y)_t}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} = \left. \frac{\partial D_t}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} \frac{1}{Y_t} = \left( \frac{\partial D_{t-1}}{\partial w^{HF}_{t-1}} (1 + r_{t-1}) - t^{w}_{t-1} (e_{ah(t-1)}^{NM} \alpha_t^N + e_{ah(t-1)}^{NF}) w_t^{NF} \right) + t^{R}_{t} \left( e_{ah(t-1)}^{NM} \alpha_t^N + e_{ah(t-1)}^{NF} \right) w_t^{NF} - t^{PW}_{t} \left| \frac{\partial PW_t}{\partial w^{HF}_{t-1}} \right|_{Y_t^N, e^H_t} \right|_{Y_t^N, e^H_t} \frac{1}{Y_t} \tag{A4.153}
\]
A4.3.3 The effect of a change in the gender wage gap in $H$ on employment

\[
\frac{dE_t^F}{dw_{t-1}^{HF}} = \left( \beta_t^N \frac{(1 - \kappa_t^H)}{T_t^N} + \beta_t^H \frac{\kappa_t^H}{w_t^{HF} (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi_{at}^{\alpha H} \tag{A4.160}
\]

\[
\frac{dE_t^M}{dw_{t-1}^{HF}} = \left( (1 - \beta_t^N) \frac{(1 - \kappa_t^H)}{T_t^N} + (1 - \beta_t^H) \frac{\kappa_t^H}{w_t^{HF} (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi_{at}^{\alpha H} \tag{A4.161}
\]

\[
\frac{dE_t}{d\kappa_t^H} = \left( \frac{(1 - \kappa_t^H)}{T_t^N} + \frac{\kappa_t^H}{w_t^{HF} (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi_{at}^{\alpha H} \tag{A4.162}
\]

\[
\frac{dE_t^F}{dw_{t-1}^{HF}} = e_{at}^{NF(t-1)} + \beta_t^N \frac{1}{T_t^N} \frac{dY_t^N}{dw_{t-1}^{HF}} \left( \frac{\beta_t^H \kappa_t^H}{w_t^{HF} (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi_{t(t-1)}^{\alpha H} \tag{A4.163}
\]

\[
\frac{dE_t^M}{dw_{t-1}^{HF}} = e_{at}^{NM(t-1)} + (1 - \beta_t^N) \frac{1}{T_t^N} \frac{dY_t^N}{dw_{t-1}^{HF}}
+ \left( \frac{(1 - \beta_t^H)}{w_t^{HF} (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi_{t(t-1)}^{\alpha H} \tag{A4.164}
\]

\[
\frac{dE_t}{dw_{t-1}^{HF}} = e_{at}^{NF(t-1)} + e_{at}^{NM(t-1)} + \frac{1}{T_t^N} \frac{dY_t^N}{dw_{t-1}^{HF}}
+ \left( \frac{\kappa_t^H}{w_t^{HF} (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi_{t(t-1)}^{\alpha H} \tag{A4.165}
\]

A4.3.4 The effect of a change in the gender wage gap in $H$ on public debt

\[
\frac{d(D_t^N)}{dw_{t-1}^{HF}} = \frac{d(D_t^N)}{\partial Y_t^N} \left. \frac{dY_t^N}{dw_{t-1}^{HF}} \right|_{\phi_t^H} + \frac{\partial (D_t^N)}{\partial Y_t^N} \left. \frac{dY_t^N}{dw_{t-1}^{HF}} \right|_{\phi_t^H} \tag{A4.166}
\]
A4.4 The effects of public physical infrastructure

A4.4.1 The short-run effect of a change in public physical infrastructure investment/GDP on aggregate output

\[
\psi_t^G = \frac{dY_t}{d\kappa_t^G} = \frac{\left| \frac{\partial C_t^N}{\partial \kappa_t^G} \right|_{Y_t} + \left| \frac{\partial I_t}{\partial \kappa_t^G} \right|_{Y_t} + \left| \frac{\partial X_t}{\partial \kappa_t^G} \right|_{Y_t} - \left| \frac{\partial M_t}{\partial \kappa_t^G} \right|_{Y_t} + Y_t}{1 - \varphi_{NF}} \quad (A4.168)
\]

and the multiplier term is \( \left( \frac{1}{1 - \varphi_{NF}} \right) \).

\[
\left| \frac{\partial E_t^{HF}}{\partial \kappa_t^G} \right|_{Y_t} = \left| \frac{\partial E_t^{HM}}{\partial \kappa_t^G} \right|_{Y_t} = \left| \frac{\partial E_t^{NF}}{\partial \kappa_t^G} \right|_{Y_t} = \left| \frac{\partial E_t^{NM}}{\partial \kappa_t^G} \right|_{Y_t} = 0 \quad (A4.169)
\]

\[
\left| \frac{\partial \pi_t}{\partial \kappa_t^G} \right|_{Y_t} = 0 \quad (A4.170)
\]

\[
\left| \frac{\partial PW_t}{\partial \kappa_t^G} \right|_{Y_t} = 0 \quad (A4.171)
\]

\[
\left| \frac{\partial C_t^N}{\partial \kappa_t^G} \right|_{Y_t} = 0 \quad (A4.172)
\]

\[
\left| \frac{\partial C_t^H}{\partial \kappa_t^G} \right|_{Y_t} = 0 \quad (A4.173)
\]

\[
\left| \frac{\partial I_t}{\partial \kappa_t^G} \right|_{Y_t} = I_t \left( 3 \frac{d\xi_t}{(Y_t)} \right) \quad (A4.174)
\]

where

\[
d_t^G = \left| \frac{\partial (D/Y)_t}{\partial \kappa_t^G} \right|_{Y_t} = \left| \frac{\partial D_t}{\partial \kappa_t^G} \right|_{Y_t} \frac{1}{Y_t} = Y_t \frac{1}{Y_t} = 1 \quad (A4.175)
\]

\[
\left| \frac{\partial X_t}{\partial \kappa_t^G} \right|_{Y_t} = 0 \quad (A4.176)
\]

\[
\left| \frac{\partial M_t}{\partial \kappa_t^G} \right|_{Y_t} = 0 \quad (A4.177)
\]

A4.4.2 The effect of a change in public physical infrastructure investment/GDP on aggregate output in the next period
\[
\psi_t^{G(t-1)} = \frac{dY_t}{d\kappa_t^G} \\
= \frac{\left| \frac{\partial C_t^N}{\partial \kappa_t^G Y_t} \right| + \left| \frac{\partial C_t^H}{\partial \kappa_t^G Y_t} \right| + \left| \frac{\partial I_t^c}{\partial \kappa_t^G Y_t} \right| + \left| \frac{\partial X_t^c}{\partial \kappa_t^G Y_t} \right| - \left| \frac{\partial M_t^c}{\partial \kappa_t^G Y_t} \right|}{\left| \frac{\partial G_t^H}{\partial \kappa_t^G Y_t} \right| + \left| \frac{\partial G_t^C}{\partial \kappa_t^G Y_t} \right| + \left| \frac{\partial I_t^G}{\partial \kappa_t^G Y_t} \right|}
+ \frac{(1 - \varphi_{NF})}{(1 - \varphi_{NF})} \left| \frac{\partial T_t^N}{\partial \kappa_t^G Y_t} \right|
= T_t^N \left( h_1 \left( \frac{\partial C_t^H}{\partial \kappa_t^G Y_t} \right) + \frac{\kappa_t^H}{C_t^H + G_t^H} \psi_t^{G(t-1)(t-1)} \right)
+ h_2 \frac{\psi_t^{G(t-1)(t-1)} \kappa_t^G + Y_t}{l_t^G}
\]

\[
E_{G(t-1)}^{NF} = \frac{\partial E_t^{NF}}{\partial \kappa_t^G Y_t} = -\beta_t^{NF} \left( \frac{T_t^N}{(T_t^N)^2} \right) \left| \frac{\partial T_t^N}{\partial \kappa_t^G Y_t} \right|
\]

\[
e_{G(t-1)}^{NM} = \left| \frac{\partial E_t^{NM}}{\partial \kappa_t^G Y_t} \right| = -\left( 1 - \beta_t^{NM} \right) \left| \frac{\partial T_t^N}{\partial \kappa_t^G Y_t} \right|
\]

\[
e_{G(t-1)}^{HF} = 0
\]

\[
e_{G(t-1)}^{NM} = 0
\]

\[
\left| \frac{\partial \pi_t}{\partial \kappa_t^G Y_t} \right| = C_t \left( c_f \frac{e_{G(t-1)}^{NF} w_t^{NF}}{W B_t^F} + c_m \frac{e_{G(t-1)}^{NM} w_t^{NF}}{W B_t^M} \right) \left( \frac{\alpha_t^N - \alpha_t^{NF} \beta_t^N + \beta_t^{NF}}{(T_t^N)^2} \right) \left| \frac{\partial T_t^N}{\partial \kappa_t^G Y_t} \right|
= C_t \left( c_f \frac{e_{G(t-1)}^{NF} w_t^{NF}}{W B_t^F} + c_m \frac{e_{G(t-1)}^{NM} w_t^{NF}}{W B_t^M} \right) \left( \frac{\alpha_t^N - \alpha_t^{NF} \beta_t^N + \beta_t^{NF}}{(T_t^N)^2} \right) \left| \frac{\partial T_t^N}{\partial \kappa_t^G Y_t} \right|
= C_t \left( \frac{e_{G(t-1)}^{NM} \alpha_t^N}{R_t} \left( \frac{e_{G(t-1)}^{NM} \alpha_t^N}{R_t} \right) + \frac{e_{G(t-1)}^{NF} w_t^{NF}}{W B_t^F} \right) + c_{PW1} \frac{1}{PW1_t} \left| \frac{\partial PW1_t}{\partial \kappa_t^G Y_t} \right|
+ c_{PW99} \frac{1}{PW99_t} \left| \frac{\partial PW99_t}{\partial \kappa_t^G Y_t} \right|
\]

\[
\left| \frac{\partial PW1_t}{\partial \kappa_t^G Y_t} \right| = \lambda_t \left| \frac{\partial PW_t}{\partial \kappa_t^G Y_t} \right| + PW_t \left( s_1 \frac{\lambda_t}{\pi_t} \left| \frac{\partial \pi_t}{\partial \kappa_t^G Y_t} \right| \right)
= \left( 1 - \lambda_t \right) \frac{\partial PW_t}{\partial \kappa_t^G Y_t} - PW_t \left( s_1 \frac{\lambda_t}{\pi_t} \left| \frac{\partial \pi_t}{\partial \kappa_t^G Y_t} \right| \right)
\]

\[
\left| \frac{\partial PW99_t}{\partial \kappa_t^G Y_t} \right| = \lambda_t \left| \frac{\partial PW_t}{\partial \kappa_t^G Y_t} \right| + PW_t \left( s_1 \frac{\lambda_t}{\pi_t} \left| \frac{\partial \pi_t}{\partial \kappa_t^G Y_t} \right| \right)
= \left( 1 - \lambda_t \right) \frac{\partial PW_t}{\partial \kappa_t^G Y_t} - PW_t \left( s_1 \frac{\lambda_t}{\pi_t} \left| \frac{\partial \pi_t}{\partial \kappa_t^G Y_t} \right| \right)
\]

where
\[
\frac{\partial PW_t}{\partial \kappa_t^{G-1} Y_t} = PW_t \left( a_F \frac{e_t^{N F} W_t^{N F}}{W B_t^F}+ a_M \frac{e_t^{N M} W_t^{N F}}{W B_t^{N M}} \right. \\
\left. - a_R \frac{\alpha_t^{N M} w_t^{N F}}{R_t} + a_e \left( \frac{\partial PW_t}{\partial \kappa_t^{G-1} Y_t} \right) \frac{1}{Y_t} \right)
\]  
(A4.188)

and

\[
\frac{\partial PW_t}{\partial \kappa_t^{G-1} Y_t} = PW_t \left( \frac{e_t^{N F} W_t^{N F}}{W B_t^{H F}}+ a_M \frac{e_t^{N M} W_t^{N F}}{W B_t^{H M}} \right. \\
\left. + a_R \left( 1 - \kappa_t^{H} \right) \frac{\alpha_t^{N M} w_t^{N F}}{R_t} + a_e \left( \frac{\partial PW_t}{\partial \kappa_t^{G-1} Y_t} \right) \frac{1}{Y_t} \right)
\]  
(A4.189)

\[
\frac{\partial C_t^H}{\partial \kappa_t^{G-1} Y_t} = C_t^H \left( z_F \frac{e_t^{N F} W_t^{N F}}{W B_t^F}+ z_M \frac{e_t^{N M} W_t^{N F}}{W B_t^{N M}} \right. \\
\left. - z_R \frac{\alpha_t^{N M} w_t^{N F}}{R_t} + z_{PW99} \frac{1}{P W99_t} \left( \frac{\partial PW99_t}{\partial \kappa_t^{G-1} Y_t} \right) \right)
\]  
(A4.190)

\[
\frac{\partial I_t}{\partial \kappa_t^{G-1} Y_t} = I_t \left( \frac{\partial \pi_t}{\partial \kappa_t^{G-1} Y_t} \right) + i_2 \frac{d_t^{G(t-1)}}{\pi_t} + i_3 \frac{1}{P W99_t} \left( \frac{\partial PW99_t}{\partial \kappa_t^{G-1} Y_t} \right) \right)
\]  
(A4.191)

\[
d_t^{G(t-1)} = \frac{\partial (D/Y)_t}{\partial \kappa_t^{G-1} Y_t} = \left( \frac{\partial D_t}{\partial \kappa_t^{G-1} Y_t} \right) \frac{1}{Y_t}
\]  
(A4.192)
A4.4.3 The effect of a change in public physical infrastructure investment/GDP on employment

\[
\frac{dE^f_t}{d\kappa^G_{t-1}} = e^N_{G(t-1)} + \left( \beta^N_t \frac{(1 - \kappa^H_t)}{T^N_t} + \frac{\beta^H_t \kappa^H_t}{T^H_t} \right) \psi^G_{t(t-1)}
\]

\[
\frac{dE^M_t}{d\kappa^G_{t-1}} = e^N_{G(t-1)} + \left( \beta^N_t \frac{(1 - \kappa^H_t)}{T^N_t} + \frac{\beta^H_t \kappa^H_t}{T^H_t} \right) \psi^G_{t(t-1)}
\]

\[
\frac{dE^f_t}{d\kappa^G_{t-1}} = e^N_{G(t-1)} + \left( \beta^N_t \frac{(1 - \kappa^H_t)}{T^N_t} + \frac{\beta^H_t \kappa^H_t}{T^H_t} \right) \psi^G_{t(t-1)}
\]

A4.4.4 The effect of a change in public physical infrastructure investment/GDP on public debt

\[
\frac{d}{d\kappa^G_{t-1}} \left( \frac{D}{Y} \right)_t = d^G_{t(t-1)} + d^Y_{t(t-1)} \psi^G_{t(t-1)}
\]

\[
\frac{d}{d\kappa^G_{t-1}} \left( \frac{D}{Y} \right)_t = d^G_{t(t-1)} + d^Y_{t(t-1)} \psi^G_{t(t-1)}
\]
A4.5 The effects of taxes

A4.5.1 The short-run effect of a change in taxes on profits on aggregate output

\[ \psi_{tt}^{TR} = \frac{dY_t}{dt_t^{R^T}} = \frac{\left| \frac{\partial C_t^H}{\partial t_t^{R}} \right|_{Y_t} + \left| \frac{\partial C_t^H}{\partial t_t^{R}} \right|_{Y_t} + \left| \frac{\partial I_t}{\partial t_t^{R}} \right|_{Y_t} + \left| \frac{\partial X_t}{\partial t_t^{R}} \right|_{Y_t} - \left| \frac{\partial M_t}{\partial t_t^{R}} \right|_{Y_t}}{1 - \varphi_{NF}} \]  

(A4.207)

and the multiplier term is \( \left( \frac{1}{1 - \varphi_{NF}} \right) \).

\[ \left| \frac{\partial E_t^{RF}}{\partial t_t^{R}} \right|_{Y_t} = \left| \frac{\partial E_t^{HM}}{\partial t_t^{R}} \right|_{Y_t} = \left| \frac{\partial E_t^{NF}}{\partial t_t^{R}} \right|_{Y_t} = \left| \frac{\partial E_t^{NM}}{\partial t_t^{R}} \right|_{Y_t} = 0 \]  

(A4.208)

\[ \left| \frac{\partial \pi_t}{\partial t_t^{R}} \right|_{Y_t} = 0 \]  

(A4.209)

\[ \left| \frac{\partial C_t^N}{\partial t_t^{R}} \right|_{Y_t} = c_t^N \left( -c_R \left( \frac{1}{1 - t_t^R} \right) + \frac{1}{PW_t} \right) \frac{\partial PW_t}{\partial t_t^{R}} \right|_{Y_t} \]  

(A4.210)

\[ \left| \frac{\partial PW_t}{\partial t_t^{R}} \right|_{Y_t} = PW_t \left( \frac{1}{PW_t} \frac{\partial PW_t}{\partial t_t^{R}} \right) \right|_{Y_t} + \frac{\lambda_t}{\lambda_t} \frac{\partial \lambda_t}{\partial t_t^{R}} \right|_{Y_t} \]  

(A4.211)

\[ \left| \frac{\partial PW_t}{\partial t_t^{R}} \right|_{Y_t} = PW_t \left( \frac{1}{PW_t} \frac{\partial PW_t}{\partial t_t^{R}} \right) \right|_{Y_t} - \frac{1}{\lambda_t} \frac{\partial \lambda_t}{\partial t_t^{R}} \right|_{Y_t} \]  

(A4.212)

\[ \left| \frac{\partial PW_t}{\partial t_t^{R}} \right|_{Y_t} = PW_t \left( \frac{1}{PW_t} \frac{\partial PW_t}{\partial t_t^{R}} \right) \right|_{Y_t} - \frac{1}{\lambda_t} \frac{\partial \lambda_t}{\partial t_t^{R}} \right|_{Y_t} \]  

(A4.213)

\[ \left| \frac{\partial C_t^H}{\partial t_t^{R}} \right|_{Y_t} = c_t^H \left( -z_R \left( \frac{1}{1 - t_t^R} \right) + \frac{1}{PW_t} \right) \frac{\partial PW_t}{\partial t_t^{R}} \right|_{Y_t} \]  

(A4.214)

\[ \left| \frac{\partial I_t}{\partial t_t^{R}} \right|_{Y_t} = I_t \left( -i_2 \left( \frac{1}{1 - t_t^R} \right) + i_3 \frac{PW_t}{\partial t_t^{R}} \right) + i_4 \frac{1}{PW_t} \frac{\partial PW_t}{\partial t_t^{R}} \right|_{Y_t} \]  

(A4.215)

where
The effect of a change in the taxes on profit in the next period

\[
\Psi_{t(t-1)}^{TR} = \frac{dY_t}{dT_{t-1}^R} = \frac{dY_t}{dT_{t-1}^R} = \frac{\partial C^N_t}{\partial D_t^N} + \frac{\partial C^N_t}{\partial D_t^R} + \frac{\partial I_t^N}{\partial I_t^N} + \frac{\partial X_t}{\partial X_t} + \frac{\partial M_t}{\partial M_t} (1 - \varphi_{NF})
\]

\[
\frac{\partial T_t^N}{\partial D_{t-1}^R} = T_t^N \left( \frac{\kappa_{t-1}^H \psi_{t(t-1)}^{TR} \psi_{t(t-1)}^{R1} + \frac{\partial C^H_{t-1}}{\partial D_{t-1}^N} \psi_{t(t-1)}^{R1}}{C_{t-1}^H + C_{t-1}^H} + (h_2 + h_3) \frac{\psi_{t(t-1)}^{R1}}{Y_{t-1}} \right)
\]

\[
\frac{\partial C^H_{t-1}}{\partial D_{t-1}^N} = \frac{\partial C^H_{t-1}}{\partial D_{t-1}^R} + \frac{\partial C^H_{t-1}}{\partial D_{t-1}^N} \psi_{t(t-1)}^{R1} \psi_{t(t-1)}^{R1}
\]

\[
e^{TR}_{t(t-1)} = \frac{\partial E_{t(t-1)}^{NM}}{\partial D_{t-1}^R} = -\frac{(1 - \beta_t^N Y_t^N)}{(t_t^N)^2} \frac{\partial T_t^N}{\partial D_{t-1}^R} \psi_{t(t-1)}^{R1}
\]

\[
\frac{\partial \pi_t}{\partial D_{t-1}^R} = \left( \frac{\alpha_t^N - \alpha_t^N \beta_t^N + \beta_t^N w_t^N}{(t_t^N)^2} \right) \frac{\partial T_t^N}{\partial D_{t-1}^R}
\]

\[
\frac{\partial C_t^N}{\partial D_{t-1}^R} = c_t^N \left( c_F \frac{e_{t(t-1)}^{NF} w_t^N}{WB_t^N} + c_M \frac{e_{t(t-1)}^{NM} w_t^N \alpha_t^N}{WB_t^M} - c_R \frac{(e_{t(t-1)}^{NM} \alpha_t^N + e_{t(t-1)}^{NF} w_t^N) w_t^N}{R_t} + c_{PW1} \frac{1}{PW_t^1} \frac{\partial PW1_t}{\partial D_{t-1}^N} \psi_{t(t-1)}^{R1} \psi_{t(t-1)}^{R1} \right)
\]
\[
\frac{\partial PW_{1t}}{\partial t_{t-1}^{R}}|_{Y_{t}} = \lambda_{t} \left( \frac{\partial PW_{t}}{\partial t_{t-1}^{R}} \right)_{Y_{t}} + PW_{t} \left( s_{1} \frac{\lambda_{t}}{\pi_{t}} \frac{\partial \pi_{t}}{\partial t_{t-1}^{R}}_{Y_{t}} - s_{1} s_{5} \frac{1}{(1 - t_{t-1}^{R})} \right) - PW_{t} \left( s_{1} \frac{\lambda_{t}}{\pi_{t}} \frac{\partial \pi_{t}}{\partial t_{t-1}^{R}}_{Y_{t}} - s_{1} s_{5} \frac{1}{(1 - t_{t-1}^{R})} \right) \tag{A4.229}
\]

\[
\frac{\partial PW_{99t}}{\partial t_{t-1}^{R}}|_{Y_{t}} = \left( 1 - \lambda_{t} \right) \left( \frac{\partial PW_{t}}{\partial t_{t-1}^{R}} \right)_{Y_{t}} - PW_{t} \left( \frac{\lambda_{t}}{\pi_{t}} \frac{\partial \pi_{t}}{\partial t_{t-1}^{R}}_{Y_{t}} - s_{1} s_{5} \frac{1}{(1 - t_{t-1}^{R})} \right) \tag{A4.230}
\]

where

\[
\frac{\partial PW_{t}}{\partial t_{t-1}^{R}}|_{Y_{t}} = PW_{t} \left( e_{TR(t-1)}^{NF(t-1)} W_{t}^{NF} + \alpha_{t}^{N} e_{TR(t-1)}^{NM(t-1)} W_{t}^{NM} \right) \left( e_{TR(t-1)}^{NF(t-1)} W_{t}^{NF} + \alpha_{t}^{N} e_{TR(t-1)}^{NM(t-1)} W_{t}^{NM} \right) \tag{A4.231}
\]

and

\[
\frac{\partial PW_{t-1}}{\partial t_{t-1}^{R}}|_{Y_{t}} = PW_{t-1} \left( a_{F} e_{Y(t-1)}^{NF(t-1)} W_{t-1}^{NF} e_{Y(t-1)}^{MF(t-1)} W_{t-1}^{MF} \right) \left( e_{Y(t-1)}^{NF(t-1)} W_{t-1}^{NF} e_{Y(t-1)}^{MF(t-1)} W_{t-1}^{MF} \right) \tag{A4.232}
\]

\[
\left| \frac{\partial c^{H}_{t}}{\partial t_{t-1}^{R}} \right|_{Y_{t}} = c^{H}_{t} \left( e_{TR(t-1)}^{NF(t-1)} W_{t}^{NF} + e_{TR(t-1)}^{NM(t-1)} W_{t}^{NM} \alpha_{t}^{N} \right) \left( e_{TR(t-1)}^{NF(t-1)} W_{t}^{NF} + e_{TR(t-1)}^{NM(t-1)} W_{t}^{NM} \alpha_{t}^{N} \right) - z_{R} \left( e_{TR(t-1)}^{NF(t-1)} W_{t}^{NF} + e_{TR(t-1)}^{NM(t-1)} W_{t}^{NM} \alpha_{t}^{N} \right) \left( e_{TR(t-1)}^{NF(t-1)} W_{t}^{NF} + e_{TR(t-1)}^{NM(t-1)} W_{t}^{NM} \alpha_{t}^{N} \right) \tag{A4.233}
\]

\[
\left| \frac{\partial l_{t}}{\partial t_{t-1}^{R}} \right|_{Y_{t}} = l_{t} \left( \frac{\partial \pi_{t}}{\partial t_{t-1}^{R}} \right)_{Y_{t}} + i_{2} \frac{d_{t}^{TR}}{\pi_{t}} + i_{3} \frac{d_{t}^{TR}}{\pi_{t}} \left( \frac{D}{\pi_{t}} \right)_{Y_{t}} + i_{4} \frac{1}{PW_{t}} \left( \frac{\partial PW_{1t}}{\partial t_{t-1}^{R}} \right)_{Y_{t}} \tag{A4.234}
\]

\[
\left| \frac{\partial PW_{99t}}{\partial t_{t-1}^{R}} \right|_{Y_{t}} = l_{t} \left( \frac{\partial \pi_{t}}{\partial t_{t-1}^{R}} \right)_{Y_{t}} + i_{2} \frac{d_{t}^{TR}}{\pi_{t}} + i_{3} \frac{d_{t}^{TR}}{\pi_{t}} \left( \frac{D}{\pi_{t}} \right)_{Y_{t}} + i_{4} \frac{1}{PW_{t}} \left( \frac{\partial PW_{1t}}{\partial t_{t-1}^{R}} \right)_{Y_{t}} \tag{A4.234}
\]
\[ \frac{d G^G}{dt_{t-1}} = \left| \frac{\partial (D/Y)_t}{\partial t_{t-1}} \right|_{Y_t} = \left| \frac{\partial D_t}{\partial t_{t-1}} \right|_{Y_t} \frac{1}{Y_t} \]

\[ = \left( \frac{\partial D_{t-1}}{\partial t_{t-1}} (1 + r_{t-1}) - t^W_t (e^{NM}_{TR(t-1)} \alpha^N_t + e^{NF}_{TR(t-1)} w^{NF}_t) \right) \frac{1}{Y_t} \]

\[ + t^R_t (e^{NM}_{TR(t-1)} \alpha^N_t + e^{NF}_{TR(t-1)} w^{NF}_t) - t^P_t \right| \frac{\partial P_W}{\partial t_{t-1}} \right|_{Y_t} \]

\[ - t^C_t \left( \left| \frac{\partial c^N_t}{\partial t_{t-1}} \right| + \left| \frac{\partial c^H_t}{\partial t_{t-1}} \right| \right) \frac{1}{Y_t} \]

\[ \frac{\partial D_{t-1}}{\partial t_{t-1}} = Y_{t-1} \frac{d (D/Y)_t}{dt_{t-1}} + \psi^{TR}_{(t-1)(t-1)} \frac{D_{t-1}}{Y_{t-1}} \]  \hspace{1cm} (A4.236)

\[ \frac{\partial X_t}{\partial t_{t-1}} \right|_{Y_t} = X_t \left( \begin{array}{c} \frac{\partial \pi_t}{\partial t_{t-1}} \left|_{Y_t} \\ \pi_t \end{array} \right) \] \hspace{1cm} (A4.237)

\[ \frac{\partial M_t}{\partial t_{t-1}} \right|_{Y_t} = M_t \left( \begin{array}{c} \frac{\partial \pi_t}{\partial t_{t-1}} \left|_{Y_t} \\ \pi_t \end{array} \right) \] \hspace{1cm} (A4.238)

\[ \frac{\partial G^H_t}{\partial t_{t-1}} \right|_{Y_t} = 0 \] \hspace{1cm} (A4.239)

\[ \frac{\partial G^C_t}{\partial t_{t-1}} \right|_{Y_t} = 0 \] \hspace{1cm} (A4.240)

\[ \frac{\partial I^F_t}{\partial t_{t-1}} \right|_{Y_t} = 0 \] \hspace{1cm} (A4.241)

**A4.5.3 The effect of a change in taxes on profits on employment**

\[ \frac{d E^F_t}{dt_{t-1}} = \left( \beta_t^N \frac{(1 - \kappa_t^H)}{T_t^N} + \beta_t^H \frac{T_t^H}{W_t^H (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi^{TR}_{tt} \] \hspace{1cm} (A4.242)

\[ \frac{d E^M_t}{dt_{t-1}} = \left( (1 - \beta_t^N) \frac{(1 - \kappa_t^H)}{T_t^N} + (1 - \beta_t^H) \frac{\kappa_t^H}{W_t^H (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi^{TR}_{tt} \] \hspace{1cm} (A4.243)

\[ \frac{d E_t}{dt_{t-1}} = \left( \frac{(1 - \kappa_t^H)}{T_t^N} + \frac{\kappa_t^H}{W_t^H (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi^{TR}_{tt} \] \hspace{1cm} (A4.244)

\[ \frac{d E^F_t}{dt_{t-1}} = e^{NF}_{TR(t-1)} + \left( \beta_t^N \frac{(1 - \kappa_t^H)}{T_t^N} + \frac{\beta_t^H \kappa_t^H}{W_t^H (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi^{TR}_{tt} \] \hspace{1cm} (A4.245)
\[
\frac{dE_t^M}{dt_t^{R(t-1)}} = e_{TR(t)}^{NM} + \left(1 - \beta^H_t\right) \frac{(1 - \kappa^H_t)}{T^N_t} + \left(1 - \beta^H_t\right) \frac{\kappa^H_t}{w^{HF}_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)} \psi_{t(t-1)}^{TR}
\]  
(A4.246)

\[
\frac{dE_t}{dt_t^{R(t-1)}} = e_{TR(t-1)}^{NM} + e_{TR(t-1)}^{NF} + \left(1 - \frac{\kappa^H_t}{T^N_t} + \frac{\kappa^H_t}{w^{HF}_t (\beta^H_t + \alpha^H_t - \beta^H_t \alpha^H_t)} \psi_{t(t-1)}^{TR}
\]  
(A4.247)

**A4.5.4 The effect of a change in taxes on profits on public debt**

\[
\frac{dD}{dY_t} = d_{tt}^{TR} + d_{tt}^{Y} \psi_{t(t-1)}^{TR}
\]  
(A4.248)

\[
\frac{dD}{dY_t} = d_{t(t-1)}^{TR} + d_{tt}^{Y} \psi_{t(t-1)}^{TR}
\]  
(A4.249)

**A4.6.1 The short-run effect of a change in taxes on wealth on aggregate output**

\[
\psi_{t(t-1)}^{TW} = \frac{dY_t}{dt_t^{PW}} = \frac{\left| \frac{\partial C^N_t}{\partial t_t^{PW}} \right|_{Y_t} + \left| \frac{\partial C^H_t}{\partial t_t^{PW}} \right|_{Y_t} + \left| \frac{\partial I_t}{\partial t_t^{PW}} \right|_{Y_t} + \left| \frac{\partial X_t}{\partial t_t^{PW}} \right|_{Y_t} - \left| \frac{\partial M_t}{\partial t_t^{PW}} \right|_{Y_t}}{1 - \varphi_{NF}}
\]  
(A4.250)

and the multiplier term is \(\left(\frac{1}{1 - \varphi_{NF}}\right)\).

\[
\left| \frac{\partial E^H_t}{\partial t_t^{PW}} \right|_{Y_t} = \left| \frac{\partial E^M_t}{\partial t_t^{PW}} \right|_{Y_t} = \left| \frac{\partial E^N_t}{\partial t_t^{PW}} \right|_{Y_t} = \left| \frac{\partial E^M_t}{\partial t_t^{PW}} \right|_{Y_t} = 0
\]  
(A4.251)

\[
\left| \frac{\partial \pi_t}{\partial t_t^{PW}} \right|_{Y_t} = 0
\]  
(A4.252)

\[
\left| \frac{\partial C^N_t}{\partial t_t^{PW}} \right|_{Y_t} = C^N_t \left( c PW1 \left( \frac{1}{PW1_t} \left| \frac{\partial PW1_t}{\partial t_t^{PW}} \right|_{Y_t} - \frac{1}{1 - t_t^{PW}} \right) + c PW99 \left( \frac{1}{PW99_t} \left| \frac{\partial PW99_t}{\partial t_t^{PW}} \right|_{Y_t} - \frac{1}{1 - t_t^{PW}} \right) \right)
\]  
(A4.253)

\[
\left| \frac{\partial PW1_t}{\partial t_t^{PW}} \right|_{Y_t} = \lambda_t \left| \frac{\partial PW_t}{\partial t_t^{PW}} \right|_{Y_t} + PW_t \lambda_t \left( s_2 \frac{1}{t_t^{PW}} \right)
\]  
(A4.254)

\[
\left| \frac{\partial PW99_t}{\partial t_t^{PW}} \right|_{Y_t} = (1 - \lambda_t) \left| \frac{\partial PW_t}{\partial t_t^{PW}} \right|_{Y_t} - PW_t \lambda_t \left( s_2 \frac{1}{t_t^{PW}} \right)
\]  
(A4.255)

\[
\left| \frac{\partial PW_t}{\partial t_t^{PW}} \right|_{Y_t} = PW_t \left( -\frac{1}{1 - t_t^{PW}} \right)
\]  
(A4.256)
\[\left| \frac{\partial C^H_t}{\partial t_t^{PW}} \right|_{Y_t} = C^H_t \left( z_{PW1} \left( \frac{1}{PW1_t} \left| \frac{\partial PW1_t}{\partial t_t^{PW}} \right|_{Y_t} - \frac{1}{1-t_t^{PW}} \right) + z_{PW99} \left( \frac{1}{PW99_t} \left| \frac{\partial PW99_t}{\partial t_t^{PW}} \right|_{Y_t} - \frac{1}{1-t_t^{PW}} \right) \right) \]  

(A4.257)

\[\left| \frac{\partial I_t}{\partial t_t^{PW}} \right|_{Y_t} = I_t \left( i_3 \frac{d^W_t}{(D/Y_t)} + i_4 \frac{1}{PW1_t} \left| \frac{\partial PW1_t}{\partial t_t^{PW}} \right|_{Y_t} + i_5 \frac{1}{PW99_t} \left| \frac{\partial PW99_t}{\partial t_t^{PW}} \right|_{Y_t} \right) \]  

(A4.258)

where

\[d^W_t = \left| \frac{\partial (D/Y)_t}{\partial t_t^{PW}} \right|_{Y_t} = \left| \frac{\partial D_t}{\partial t_t^{PW}} \right|_{Y_t} \frac{1}{Y_t} \]  

(A4.259)

\[\left| \frac{\partial D_t}{\partial t_t^{PW}} \right|_{Y_t} = -PW_t - t_t^C \left( \left| \frac{\partial C^N_t}{\partial t_t^{PW}} \right|_{Y_t} + \left| \frac{\partial C^H_t}{\partial t_t^{PW}} \right|_{Y_t} \right) \]  

(A4.260)

\[\left| \frac{\partial X_t}{\partial t_t^{PW}} \right|_{Y_t} = 0 \]  

(A4.261)

\[\left| \frac{\partial M_t}{\partial t_t^{PW}} \right|_{Y_t} = 0 \]  

(A4.262)

A4.6.2 The effect of a change in the wealth tax on aggregate output in the next period

\[\psi_{tw}^{TW} |_{t(t-1)} = \frac{dY_t}{dt_t^{PW}} = \frac{\left| \frac{\partial C^N_t}{\partial t_t^{PW}} \right|_{Y_t} + \left| \frac{\partial C^H_t}{\partial t_t^{PW}} \right|_{Y_t} + \left| \frac{\partial I_t}{\partial t_t^{PW}} \right|_{Y_t} + \left| \frac{\partial X_t}{\partial t_t^{PW}} \right|_{Y_t} - \left| \frac{\partial M_t}{\partial t_t^{PW}} \right|_{Y_t}}{(1 - \varphi_{NF})} \]  

(A4.263)

\[\left| \frac{\partial T^N_t}{\partial t_t^{PW}} \right|_{Y_t} = T^N_t \left( \kappa^H_t \psi^{TW}_{(t-1)(t-1)} c^H_t + \left( h_2 + h_3 \right) \psi^{TW}_{(t-1)(t-1)} \right) \]  

(A4.264)

\[\left| \frac{\partial c^H_t}{\partial t_t^{PW}} \right|_{Y_t} = \left| \frac{\partial c^H_t}{\partial t_t^{PW}} \right|_{Y_t} \frac{1}{Y_t} \]  

(A4.265)

\[e_{TFW(t-1)}^{NF} = \left| \frac{\partial E_t^{NW}}{\partial t_t^{PW}} \right|_{Y_t} = -\frac{\beta_t^{NW} Y_t^{NW}}{(T_t^N)^2} \left| \frac{\partial T^N_t}{\partial t_t^{PW}} \right|_{Y_t} \]  

(A4.266)

\[e_{TFW(t-1)}^{NM} = \left| \frac{\partial E_t^{NW}}{\partial t_t^{PW}} \right|_{Y_t} = -\frac{1 - \beta_t^{NW} Y_t^{NW}}{(T_t^N)^2} \left| \frac{\partial T^N_t}{\partial t_t^{PW}} \right|_{Y_t} \]  

(A4.267)

\[e_{TFW(t-1)}^{HF} = 0 \]  

(A4.268)
\[
\frac{\partial \pi_t}{\partial t_t^{PW}} \bigg|_{Y_t} = \left( \frac{a_t^N \beta_t^N + \beta_t^{H, NF}}{(T_t^N)^2} \right) \frac{\partial T_t^N}{\partial t_t^{PW}} \bigg|_{Y_t} 
\]  (A4.269)

\[
\frac{\partial c_t^N}{\partial t_t^{PW}} \bigg|_{Y_t} = C_t^N \left( c_F \frac{e_{TW(t-1)}^{NF} w_t^{NF}}{W_{B_t}^F} + c_M \frac{e_{TW(t-1)}^{NM} w_t^{NF} \alpha_t^N}{W_{B_t}^M} \right.

\left. - c_R \frac{(e_{TW(t-1)}^{NM} \alpha_t^N + e_{TW(t-1)}^{NF} w_t^{NF})}{R_t} + c_{pw1} \frac{1}{pw1_t} \frac{\partial PW1_t}{\partial t_t^{PW}} \bigg|_{Y_t} \right)

+ c_{pw99} \frac{1}{pw99_t} \frac{\partial PW99_t}{\partial t_t^{PW}} \bigg|_{Y_t} \right) 
\]  (A4.270)

\[
\frac{\partial PW1_t}{\partial t_t^{PW}} \bigg|_{Y_t} = \lambda_t \frac{\partial PW_t}{\partial t_t^{PW}} \bigg|_{Y_t} + PW_t \left( \frac{\lambda_t}{\pi_t} \frac{\partial \pi_t}{\partial t_t^{PW}} \bigg|_{Y_t} \right) + \frac{s_1}{\pi_t} \frac{\partial \pi_t}{\partial t_t^{PW}} \bigg|_{Y_t} \right) \right) 
\]  (A4.271)

\[
\frac{\partial PW99_t}{\partial t_t^{PW}} \bigg|_{Y_t} = (1 - \lambda_t) \frac{\partial PW_t}{\partial t_t^{PW}} \bigg|_{Y_t} - PW_t \left( \frac{\lambda_t}{\pi_t} \frac{\partial \pi_t}{\partial t_t^{PW}} \bigg|_{Y_t} \right) + s_2 s_1 \frac{1}{t_t^{PW}} \right) \right) \right) 
\]  (A4.272)

where

\[
\frac{\partial PW_t}{\partial t_t^{PW}} \bigg|_{Y_t} = PW_t \left( a_F \frac{e_{TW(t-1)}^{NF} w_t^{NF}}{W_{B_t}^F} + a_M \frac{e_{TW(t-1)}^{NM} w_t^{NF}}{W_{B_t}^M} \right.

\left. - a_R \frac{(e_{TW(t-1)}^{NM} \alpha_t^N + e_{TW(t-1)}^{NF} w_t^{NF})}{R_t} \right) \right) 
\]  (A4.273)

\[
\frac{\partial PW_{t-1}}{\partial t_t^{PW}} \bigg|_{Y_t} = PW_{t-1} \left( a_F \frac{e_{Y(t-1)}^{NF} w_{t-1}^{NF} + e_{Y(t-1)}^{HF} w_{t-1}^{HF}}{W_{B_{t-1}}^F} \right. \right.

\left. + a_M \frac{(e_{Y(t-1)}^{NM} w_{t-1}^{NF} \alpha_{t-1}^N + e_{Y(t-1)}^{HM} w_{t-1}^{HF} \alpha_{t-1}^H)}{W_{B_{t-1}}^M} \right. \right.

\left. + a_R \frac{(1 - \kappa_{t-1}^H) \psi_{TW}^{(t-1)(t-1)}}{R_{t-1}} \right) \right) 
\]  (A4.274)

\[
\frac{\partial PW_{t-1}}{\partial t_t^{PW}} \bigg|_{Y_t} = PW_{t-1} \left( a_F \frac{e_{Y(t-1)}^{NF} w_{t-1}^{NF} + e_{Y(t-1)}^{NF} w_{t-1}^{HF} \psi_{TW}^{(t-1)(t-1)}}{W_{B_{t-1}}^F} \right. \right.

\left. + a_M \frac{(e_{Y(t-1)}^{NM} w_{t-1}^{NF} \alpha_{t-1}^N + e_{Y(t-1)}^{NM} w_{t-1}^{HF} \alpha_{t-1}^H)}{W_{B_{t-1}}^M} \right. \right.

\left. + a_R \frac{(1 - \kappa_{t-1}^H) \psi_{TW}^{(t-1)(t-1)}}{R_{t-1}} \right) \right) 
\]  (A4.275)
\[
\frac{\partial C^H_t}{\partial t_{t-1}^{PW} Y_t} = C_t^H \left( z_F \frac{e_{NM}^{NF}(t-1)W_{t}^{NF}}{WB_t^F} + z_M \frac{e_{NM}^{NF}(t-1)W_{t}^{NF} \alpha_t^N}{WB_t^M} \right) \\
- z_R \left( e_{TW(t-1)}^{NM} \alpha_t^N + e_{TW(t-1)}^{NF} \right) \frac{W_{t}^{NF}}{R_t} \\
+ z_{PW1}^{99} \frac{1}{P_{t}^{PW1}} \left| \frac{\partial PW1_{t}^{99}}{\partial t_{t-1}^{R} Y_t} \right|
\]
(A4.276)

\[
\frac{\partial I_t}{\partial t_{t-1}^{PW} Y_t} = I_t \left( \frac{\partial \pi_t}{\partial t_{t-1}^{PW} Y_t} \pi_t + \frac{d_{t}^{TW(t-1)}}{D(Y)_t} + i_4 \frac{1}{P_{t}^{PW1}} \left| \frac{\partial PW1_{t}^{99}}{\partial t_{t-1}^{R} Y_t} \right| \\
+ i_5 \frac{1}{P_{t}^{PW1}} \left| \frac{\partial PW99_{t}}{\partial t_{t-1}^{R} Y_t} \right| \right)
\]
(A4.277)

\[
d_{t}^{TW(t-1)} = \left| \frac{\partial (D/Y)_t}{\partial t_{t-1}^{PW} Y_t} \right| = \left| \frac{\partial D_{t}}{\partial t_{t-1}^{PW} Y_t} \right| \frac{1}{Y_t}
\]

\[
= \left( \frac{\partial D_{t-1}}{\partial t_{t-1}^{PW}} (1 + r_{t-1}^{W}) - t_{t}^{W} (e_{TW(t-1)}^{NM} \alpha_t^N + e_{TW(t-1)}^{NF} W_{t}^{NF}) \right) \\
+ t_{t}^{R} (e_{TW(t-1)}^{NM} \alpha_t^N + e_{TW(t-1)}^{NF} W_{t}^{NF}) - t_{t}^{PW} \left| \frac{\partial PW_{t}}{\partial t_{t-1}^{PW} Y_t} \right|
\]
(A4.278)

\[
\frac{\partial D_{t-1}}{\partial t_{t-1}^{PW}} = Y_{t-1} \frac{d(D/Y)_{t-1}}{dt_{t-1}^{PW}} + \Psi_{t-1}^{TW(t-1)(t-1)} \frac{D_{t-1}}{Y_{t-1}}
\]
(A4.279)

\[
\frac{\partial X_t}{\partial t_{t-1}^{PW} Y_t} = X_t \left( \frac{\partial \pi_t}{\partial t_{t-1}^{PW} Y_t} \right)
\]
(A4.280)

\[
\frac{\partial M_t}{\partial t_{t-1}^{PW} Y_t} = M_t \left( \frac{\partial \pi_t}{\partial t_{t-1}^{PW} Y_t} \right)
\]
(A4.281)

\[
\frac{\partial G_t^H}{\partial t_{t-1}^{PW} Y_t} = 0
\]
(A4.282)

\[
\frac{\partial G_t^C}{\partial t_{t-1}^{PW} Y_t} = 0
\]
(A4.283)
A4.6.3 The effect of a change in the wealth tax on employment

\[
\frac{dE_t^F}{dt_{t-1}^W} = \left( \beta_t^N \left( 1 - \kappa_t^H \right) T_t^N + \beta_t^H \frac{\kappa_t^H}{w_t^{HF}} \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right) \right) \psi_{tt}^{TW}
\]
\[
\frac{dE_t^M}{dt_{t-1}^W} = \left( 1 - \beta_t^N \right) \left( 1 - \kappa_t^H \right) T_t^N + \left( 1 - \beta_t^H \right) \frac{\kappa_t^H}{w_t^{HF}} \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right) \psi_{tt}^{TW}
\]
\[
\frac{dE_t}{dt_{t-1}^W} = \left( 1 - \kappa_t^H \right) T_t^N + \frac{\kappa_t^H}{w_t^{HF}} \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right) \psi_{tt}^{TW}
\]
\[
\frac{dE_t^F}{dt_{t-1}^W} = e_{tW(t-1)}^{NF} + \left( \beta_t^N \left( 1 - \kappa_t^H \right) T_t^N + \beta_t^H \frac{\kappa_t^H}{w_t^{HF}} \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right) \right) \psi_{t(t-1)}^{TW}
\]
\[
\frac{dE_t^M}{dt_{t-1}^W} = e_{tW(t-1)}^{NM} + \left( 1 - \beta_t^N \right) \left( 1 - \kappa_t^H \right) T_t^N + \left( 1 - \beta_t^H \right) \frac{\kappa_t^H}{w_t^{HF}} \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right) \psi_{t(t-1)}^{TW}
\]
\[
\frac{dE_t}{dt_{t-1}^W} = e_{tW(t-1)}^{NF} + e_{tW(t-1)}^{NM} + \left( 1 - \kappa_t^H \right) T_t^N + \frac{\kappa_t^H}{w_t^{HF}} \left( \beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H \right) \psi_{t(t-1)}^{TW}
\]

A4.6.4 The effect of a change in the wealth tax on public debt

\[
\frac{d\partial Y_t}{dt_{t}^W} = d_{tW}^{TW} + d_{tW}^{Y} \psi_{tt}^{TW}
\]
\[
\frac{d\partial Y_t}{dt_{t}^W} = d_{tW}^{TW} + d_{tW}^{Y} \psi_{tt}^{TW}
\]

A4.7.1 The short-run effect of a change in taxes on wages on aggregate output

\[
\psi_{tt}^{TW} = \frac{\frac{\partial C_t^N}{\partial t_t^W} + \frac{\partial C_t^H}{\partial t_t^W} + \frac{\partial l_t^H}{\partial t_t^W} + \frac{\partial x_t}{\partial t_t^W} - \frac{\partial M_t}{\partial t_t^W}}{1 - \varphi_{NF}}
\]

and the multiplier term is \( \left( \frac{1}{1 - \varphi_{NF}} \right) \).

\[
\left| \frac{\partial E_t^H}{\partial t_t^W} \right|_{Y_t} = \left| \frac{\partial E_t^M}{\partial t_t^W} \right|_{Y_t} = \left| \frac{\partial E_t^{NF}}{\partial t_t^W} \right|_{Y_t} = \left| \frac{\partial E_t^{NM}}{\partial t_t^W} \right|_{Y_t} = 0
\]

\[
\left| \frac{\partial l_t^H}{\partial t_t^W} \right|_{Y_t} = 0
\]

\[
\left| \frac{\partial C_t^N}{\partial t_t^W} \right|_{Y_t} = \left( \frac{c_f + c_m}{1 - t_t^W} + c_{PW1} \frac{1}{PW1_t} \right) \left| \frac{\partial PW1_t}{\partial t_t^W} \right|_{Y_t}
\]

\[
+ c_{PW99} \frac{1}{PW99_t} \left| \frac{\partial PW99_t}{\partial t_t^W} \right|_{Y_t}
\]

\[
\left( A4.284 \right)
\]

\[
\left( A4.285 \right)
\]

\[
\left( A4.286 \right)
\]

\[
\left( A4.287 \right)
\]

\[
\left( A4.288 \right)
\]

\[
\left( A4.289 \right)
\]

\[
\left( A4.290 \right)
\]

\[
\left( A4.291 \right)
\]

\[
\left( A4.292 \right)
\]

\[
\left( A4.293 \right)
\]

\[
\left( A4.294 \right)
\]

\[
\left( A4.295 \right)
\]

\[
\left( A4.296 \right)
\]

115
\[ \left| \frac{\partial PW_t}{\partial t_t} \right|_{Y_t} = PW_t \left( \frac{1}{PW_t} \left| \frac{\partial PW_t}{\partial t_t} \right|_{Y_t} \right) = \lambda_t \left| \frac{\partial PW_t}{\partial t_t} \right|_{Y_t} \]  
(A4.297)

\[ \left| \frac{\partial PW_{99_t}}{\partial t_t} \right|_{Y_t} = PW_{99_t} \left( \frac{1}{PW_t} \left| \frac{\partial PW_t}{\partial t_t} \right|_{Y_t} \right) = (1 - \lambda_t) \left| \frac{\partial PW_t}{\partial t_t} \right|_{Y_t} \]  
(A4.298)

\[ \left| \frac{\partial PW_t}{\partial t_t} \right|_{Y_t} = -PW_t \left( \frac{(a_F + a_M)}{(1 - t_t)} \right) \]  
(A4.299)

\[ \left| \frac{\partial C^H_t}{\partial t_t} \right|_{Y_t} = C^H_t \left( \frac{(z_F + z_M)}{(1 - t_t)} + z_{PW1} \frac{1}{PW_t} \left| \frac{\partial PW_t}{\partial t_t} \right|_{Y_t} \right) \]  
(A4.300)

\[ \left| \frac{\partial I_t}{\partial t_t} \right|_{Y_t} = I_t \left( i_3 \frac{d_{tt}^W}{D_t} + i_4 \frac{1}{PW_t} \left| \frac{\partial PW_t}{\partial t_t} \right|_{Y_t} + i_5 \frac{1}{PW_{99_t}} \left| \frac{\partial PW_{99_t}}{\partial t_t} \right|_{Y_t} \right) \]  
(A4.301)

where

\[ d_{tt}^W = \left| \frac{\partial (D/Y)_t}{\partial t_t} \right|_{Y_t} = \left| \frac{\partial D_t}{\partial t_t} \right|_{Y_t} \frac{1}{Y_t} \]  
(A4.302)

\[ \left| \frac{\partial D_t}{\partial t_t} \right|_{Y_t} = -(W_B^F + W_B^M) - t_t^{PW} \left| \frac{\partial PW_t}{\partial t_t} \right|_{Y_t} - t_t^C \left( \left| \frac{\partial C_t^N}{\partial t_t} \right|_{Y_t} + \left| \frac{\partial C_t^H}{\partial t_t} \right|_{Y_t} \right) \]  
(A4.303)

\[ \left| \frac{\partial X_t}{\partial t_t} \right|_{Y_t} = 0 \]  
(A4.304)

\[ \left| \frac{\partial M_t}{\partial t_t} \right|_{Y_t} = 0 \]  
(A4.305)

A4.7.2 The effect of a change in the taxes on wages on aggregate output in the next period

\[ \Psi_t^{NW} = \left| \frac{\partial Y_t}{\partial t_t} \right|_{Y_t} = \left| \frac{\partial C_t^N}{\partial t_t} \right|_{Y_t} + \left| \frac{\partial C_t^H}{\partial t_t} \right|_{Y_t} + \left| \frac{\partial I_t}{\partial t_t} \right|_{Y_t} + \left| \frac{\partial X_t}{\partial t_t} \right|_{Y_t} - \left| \frac{\partial M_t}{\partial t_t} \right|_{Y_t} \]  
(A4.306)

\[ \left| \frac{\partial G_t^H}{\partial t_t} \right|_{Y_t} + \left| \frac{\partial G_t^C}{\partial t_t} \right|_{Y_t} + \left| \frac{\partial I_t^G}{\partial t_t} \right|_{Y_t} \right) \]  

\[ \left| \frac{\partial T_t^{NW}}{\partial t_t} \right|_{Y_t} = T_t^N \left( h_1 \frac{\partial C_t^H}{\partial t_t} \left| \frac{\partial t_t}{\partial Y_t} \right|_{Y_t} + (h_2 + h_3) \frac{\Psi_t^{NW}}{Y_t} \right) \]  
(A4.307)

\[ \left| \frac{\partial C_{t-1}^H}{\partial t_t} \right|_{Y_t} = \left| \frac{\partial C_{t-1}^H}{\partial t_t} \right|_{Y_t} + \left| \frac{\partial C_{t-1}^H}{\partial Y_t} \right|_{Y_t} \Psi_t^{NW} \]  
(A4.308)
\[ e_{TW(t-1)}^{NF} = \frac{\partial E_{t}^{NF}}{\partial t_{t-1}^{W} Y_{t}} = -\frac{\beta_{t}^{N} Y_{t}^{N}}{(T_{t}^{N})^{2}} \frac{\partial T_{t}^{N}}{\partial t_{t-1}^{W} Y_{t}} \] (A4.309)

\[ e_{TW(t-1)}^{NM} = \frac{\partial E_{t}^{NM}}{\partial t_{t-1}^{W} Y_{t}} = -\frac{(1 - \beta_{t}^{N} Y_{t}^{N})}{(T_{t}^{N})^{2}} \frac{\partial T_{t}^{N}}{\partial t_{t-1}^{W} Y_{t}} \] (A4.310)

\[ e_{TW(t-1)}^{HF} = 0 \] (A4.311)

\[ e_{TW(t-1)}^{HM} = 0 \] (A4.312)

\[ \frac{\partial \pi_{t}}{\partial t_{t-1}^{W} Y_{t}} = \left( \frac{\alpha_{t}^{N} - \alpha_{t}^{N} \beta_{t}^{N} + \beta_{t}^{N} w_{t}^{NF}}{(T_{t}^{N})^{2}} \right) \frac{\partial T_{t}^{N}}{\partial t_{t-1}^{W} Y_{t}} \] (A4.313)

\[ \frac{\partial c_{t}^{N}}{\partial t_{t-1}^{W} Y_{t}} = C_{t}^{N} \left( c_{F} - \frac{e_{TW(t-1)}^{NF} w_{t}^{NF}}{W_{B_{t}^{F}}^{F}} + c_{M} \frac{e_{TW(t-1)}^{NM} w_{t}^{NF} \alpha_{t}^{N}}{W_{B_{t}^{M}}^{M}} \right) \\
- c_{R} \left( \frac{e_{TW(t-1)}^{NM} \alpha_{t}^{N} + e_{TW(t-1)}^{NF} w_{t}^{NF}}{R_{t}} \right) + c_{PW1} \frac{1}{PW_{t}} \frac{\partial PW_{1,t}}{\partial t_{t-1}^{W} Y_{t}} \] (A4.314)

\[ + c_{PW99} \frac{1}{PW_{99,t}} \frac{\partial PW_{99,t}}{\partial t_{t-1}^{W} Y_{t}} \]

\[ \frac{\partial PW_{t}}{\partial t_{t-1}^{W} Y_{t}} = \lambda_{t} \frac{\partial PW_{t}}{\partial t_{t-1}^{W} Y_{t}} + PW_{t} \left( s_{1} \frac{\lambda_{t}}{\pi_{t}} \frac{\partial \pi_{t}}{\partial t_{t-1}^{W} Y_{t}} \right) \] (A4.315)

\[ \frac{\partial PW_{99,t}}{\partial t_{t-1}^{W} Y_{t}} = (1 - \lambda_{t}) \left( \frac{\partial PW_{t}}{\partial t_{t-1}^{W} Y_{t}} - PW_{t} \left( s_{1} \frac{\lambda_{t}}{\pi_{t}} \frac{\partial \pi_{t}}{\partial t_{t-1}^{W} Y_{t}} \right) \right) \] (A4.316)

where

\[ \frac{\partial PW_{t}}{\partial t_{t-1}^{W} Y_{t}} = PW_{t} \left( a_{F} \frac{e_{TW(t-1)}^{NF} w_{t}^{NF}}{W_{B_{t}^{F}}^{F}} + a_{M} \frac{e_{TW(t-1)}^{NM} w_{t}^{NF} \alpha_{t}^{N}}{W_{B_{t}^{M}}^{M}} \right) \\
- a_{R} \frac{e_{TW(t-1)}^{NM} \alpha_{t}^{N} + e_{TW(t-1)}^{NF} w_{t}^{NF}}{R_{t}} + a_{c} \frac{\partial PW_{t}}{\partial t_{t-1}^{W} Y_{t}} \frac{1}{PW_{t-1}} \] (A4.317)

and

\[ \frac{\partial PW_{t}}{\partial t_{t-1}^{W} Y_{t}} = PW_{t-1} \left( a_{F} \frac{(e_{Y(t-1)}^{NF} w_{t-1}^{NF} + e_{Y(t-1)}^{HF} w_{t-1}^{HF}) \psi_{(t-1)(t-1)}}{W_{B_{t-1}^{F}}} \right) \\
+ a_{M} \frac{(e_{Y(t-1)}^{NM} w_{t-1}^{NF} \alpha_{t}^{N} + e_{Y(t-1)}^{HM} w_{t-1}^{NF} \alpha_{t}^{H}) \psi_{(t-1)(t-1)}}{W_{B_{t-1}^{M}}} \\
- \frac{(a_{F} + a_{M})}{1 - t_{t-1}^{W}} + a_{R} \frac{(1 - \kappa_{t-1}^{H}) \psi_{(t-1)(t-1)}}{R_{t-1}} \] (A4.318)
\[
\left| \frac{\partial c_t^H}{\partial t_{t-1}^W} \right| = C_t \left( e_{TW(t-1)}^{NF} w_{t}^{NF} + e_{TW(t-1)}^{NM} w_{t}^{NF} \alpha_t^N + z_{R} \left( e_{TW(t-1)}^{NM} \alpha_t^N + e_{TW(t-1)}^{NF} w_{t}^{NF} \right) + z_{PW1} \frac{1}{pW1_t} \left| \frac{\partial PW1_t}{\partial t_{t-1}^W} \right|_{Y_t} \right)
\]

\[
+ z_{PW99}^R \left| \frac{\partial PW99_t}{\partial t_{t-1}^W} \right|_{Y_t}
\]

\[
\frac{\partial i_t}{\partial t_{t-1}^W} = i_t \left( \frac{|\partial \pi_t|}{\partial t_{t-1}^W} \right) \left( \frac{Y_t}{Y_t} \right) + i_3 \frac{1}{pW1_t} \left| \frac{\partial PW1_t}{\partial t_{t-1}^W} \right|_{Y_t}
\]

\[
+ i_5 \left| \frac{\partial PW99_t}{\partial t_{t-1}^W} \right|_{Y_t}
\]

\[
d_{t(t-1)}^{TW} = \left| \frac{\partial (D/Y)_t}{\partial t_{t-1}^W} \right|_{Y_t} = \left| \frac{\partial D_t}{\partial t_{t-1}^W} \right|_{Y_t} \frac{1}{Y_t}
\]

\[
= \left( \frac{\partial D_t}{\partial t_{t-1}^W} \right)_{Y_t} \left( 1 + \tau_{t-1} \right) - t_t^W \left( e_{TW(t-1)}^{NM} \alpha_t^N + e_{TW(t-1)}^{NF} w_{t}^{NF} \right)
\]

\[
+ t_t^R \left( e_{TW(t-1)}^{NM} \alpha_t^N + e_{TW(t-1)}^{NF} w_{t}^{NF} \right) w_{t}^{NF} - t_t^{PW} \left| \frac{\partial PW_t}{\partial t_{t-1}^W} \right|_{Y_t}
\]

\[
\frac{\partial D_{t-1}}{\partial t_{t-1}^W} = Y_{t-1} \left( \frac{\partial (D/Y)_t}{\partial t_{t-1}^W} \right) + \psi_{TW(t-1)(t-1)}^{TW(t-1)} D_{t-1}^{Y_{t-1}} \left( \frac{D_{t-1}}{Y_{t-1}} \right)
\]

\[
\left| \frac{\partial X_t}{\partial t_{t-1}^W} \right|_{Y_t} = X_t \left( \frac{\partial \pi_t}{\partial t_{t-1}^W} \right) \left( \frac{Y_t}{Y_t} \right)
\]

\[
\left| \frac{\partial M_t}{\partial t_{t-1}^W} \right|_{Y_t} = M_t \left( \frac{\partial \pi_t}{\partial t_{t-1}^W} \right) \left( \frac{Y_t}{Y_t} \right)
\]

\[
\frac{\partial C_t^H}{\partial t_{t-1}^W} = 0
\]

\[
\frac{\partial C_t^C}{\partial t_{t-1}^W} = 0
\]
The effect of a change in the taxes on wages on employment

\[ \frac{dE_t^F}{dt_{t-1}^W} = \left( \beta_t^N \frac{1 - \kappa_t^H}{\tau_t^N} + \beta_t^H \frac{\kappa_t^H}{w_{\tau t}^H (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi_{tt}^{TW} \]  
(A4.328)

\[ \frac{dE_t^M}{dt_{t-1}^W} = \left( 1 - \beta_t^N \right) \frac{1 - \kappa_t^H}{\tau_t^N} + \left( 1 - \beta_t^H \right) \frac{\kappa_t^H}{w^H_{\tau t} (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi_{tt}^{TW} \]  
(A4.329)

\[ \frac{dE_t}{dt_{t-1}^W} = \left( 1 - \kappa_t^H \right) \frac{\kappa_t^H}{\tau_t^N} + \left( 1 - \beta_t^H \kappa_t^H \right) \psi_{tt}^{TW} \]  
(A4.330)

\[ \frac{dE_t^F}{dt_{t-1}^W} = e_{TW(t-1)}^{NF} + \left( \beta_t^N \frac{1 - \kappa_t^H}{\tau_t^N} + \frac{\beta_t^H \kappa_t^H}{w^H_{\tau t} (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi_{tt}^{TW(t-1)} \]  
(A4.331)

\[ \frac{dE_t^M}{dt_{t-1}^W} = e_{TW(t-1)}^{NM} + \left( 1 - \beta_t^N \right) \frac{1 - \kappa_t^H}{\tau_t^N} + \frac{\kappa_t^H}{w^H_{\tau t} (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi_{tt}^{TW(t-1)} \]  
(A4.332)

\[ \frac{dE_t}{dt_{t-1}^W} = e_{TW(t-1)}^{NF} + e_{TW(t-1)}^{NM} + \left( \beta_t^N \frac{1 - \kappa_t^H}{\tau_t^N} + \frac{\kappa_t^H}{w^H_{\tau t} (\beta_t^H + \alpha_t^H - \beta_t^H \alpha_t^H)} \right) \psi_{tt}^{TW(t-1)} \]  
(A4.333)

A4.7.4 The effect of a change in the taxes on wages on public debt

\[ \frac{d(D_t)}{dt_{t}^W} = a_{tt}^{TW} + a_{tt}^{Yr} \psi_{tt}^{TW} \]  
(A4.334)

\[ \frac{d(D_t)}{dt_{t-1}^W} = a_{tt}^{TW(t-1)} + a_{tt}^{Yr(t-1)} \psi_{tt}^{TW(t-1)} \]  
(A4.335)
## Appendix 5. Stylised facts of the data

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>1161.194</td>
<td>359.168</td>
<td>47</td>
</tr>
<tr>
<td>WP</td>
<td>273.802</td>
<td>121.957</td>
<td>46</td>
</tr>
<tr>
<td>WPx</td>
<td>465.495</td>
<td>115.115</td>
<td>46</td>
</tr>
<tr>
<td>x</td>
<td>7.171</td>
<td>1.764</td>
<td>46</td>
</tr>
<tr>
<td>p</td>
<td>44.793</td>
<td>1.408</td>
<td>46</td>
</tr>
<tr>
<td>p2</td>
<td>5.426</td>
<td>1.385</td>
<td>46</td>
</tr>
<tr>
<td>p3</td>
<td>1.745</td>
<td>0.399</td>
<td>46</td>
</tr>
<tr>
<td>A</td>
<td>17.697</td>
<td>0.783</td>
<td>46</td>
</tr>
<tr>
<td>a</td>
<td>27.096</td>
<td>1.088</td>
<td>46</td>
</tr>
<tr>
<td>b</td>
<td>13.143</td>
<td>4.217</td>
<td>46</td>
</tr>
<tr>
<td>b2</td>
<td>20.321</td>
<td>3.626</td>
<td>46</td>
</tr>
<tr>
<td>b3</td>
<td>11.017</td>
<td>4.144</td>
<td>46</td>
</tr>
<tr>
<td>c</td>
<td>15.874</td>
<td>3.825</td>
<td>46</td>
</tr>
<tr>
<td>c2</td>
<td>1.621</td>
<td>0.256</td>
<td>46</td>
</tr>
<tr>
<td>c3</td>
<td>1.519</td>
<td>0.222</td>
<td>46</td>
</tr>
<tr>
<td>D</td>
<td>29.913</td>
<td>9.006</td>
<td>47</td>
</tr>
<tr>
<td>D2</td>
<td>684.148</td>
<td>254.500</td>
<td>47</td>
</tr>
<tr>
<td>D3</td>
<td>174.792</td>
<td>47.024</td>
<td>47</td>
</tr>
<tr>
<td>E</td>
<td>110.120</td>
<td>11.600</td>
<td>47</td>
</tr>
<tr>
<td>E2</td>
<td>32.162</td>
<td>9.919</td>
<td>47</td>
</tr>
<tr>
<td>E3</td>
<td>142.017</td>
<td>48.692</td>
<td>47</td>
</tr>
<tr>
<td>M</td>
<td>276.036</td>
<td>167.121</td>
<td>47</td>
</tr>
<tr>
<td>X</td>
<td>265.690</td>
<td>143.665</td>
<td>47</td>
</tr>
<tr>
<td>X2</td>
<td>1019.177</td>
<td>311.513</td>
<td>47</td>
</tr>
<tr>
<td>x</td>
<td>0.122</td>
<td>0.008</td>
<td>47</td>
</tr>
<tr>
<td>x2</td>
<td>0.102</td>
<td>0.025</td>
<td>47</td>
</tr>
<tr>
<td>x3</td>
<td>0.030</td>
<td>0.013</td>
<td>47</td>
</tr>
<tr>
<td>f</td>
<td>22.473</td>
<td>6.613</td>
<td>46</td>
</tr>
<tr>
<td>f2</td>
<td>0.755</td>
<td>0.020</td>
<td>46</td>
</tr>
<tr>
<td>f3</td>
<td>0.395</td>
<td>0.014</td>
<td>46</td>
</tr>
<tr>
<td>g</td>
<td>412.358</td>
<td>121.270</td>
<td>47</td>
</tr>
<tr>
<td>g2</td>
<td>0.406</td>
<td>0.026</td>
<td>47</td>
</tr>
<tr>
<td>g3</td>
<td>25.102</td>
<td>1.433</td>
<td>47</td>
</tr>
<tr>
<td>t</td>
<td>29.881</td>
<td>5.084</td>
<td>47</td>
</tr>
<tr>
<td>tW</td>
<td>1.286</td>
<td>0.426</td>
<td>47</td>
</tr>
<tr>
<td>tC</td>
<td>18.494</td>
<td>1.405</td>
<td>47</td>
</tr>
<tr>
<td>IVY</td>
<td>0.516</td>
<td>0.171</td>
<td>47</td>
</tr>
<tr>
<td>PW</td>
<td>4359.685</td>
<td>2413.936</td>
<td>47</td>
</tr>
<tr>
<td>PW1</td>
<td>852.214</td>
<td>513.752</td>
<td>47</td>
</tr>
<tr>
<td>PW99</td>
<td>3507.471</td>
<td>1914.337</td>
<td>47</td>
</tr>
<tr>
<td>l</td>
<td>0.196</td>
<td>0.033</td>
<td>47</td>
</tr>
<tr>
<td>l2</td>
<td>417000000000000</td>
<td>167000000000000</td>
<td>47</td>
</tr>
</tbody>
</table>
Appendix 6. Estimation results with instrumental variables

**Table A6.1 Regression results for Consumption in N and H**

<table>
<thead>
<tr>
<th>Variable</th>
<th>ΔlogC&lt;sub&gt;N&lt;/sub&gt;</th>
<th>p-value</th>
<th>ΔlogC&lt;sub&gt;H&lt;/sub&gt;</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.007</td>
<td>0.007</td>
<td>0.011</td>
<td>0.091</td>
</tr>
<tr>
<td>Δlog(R&lt;sub&gt;t&lt;/sub&gt;(1-t&lt;sub&gt;R&lt;/sub&gt;))</td>
<td>0.058</td>
<td>0.052</td>
<td>0.001</td>
<td>0.993</td>
</tr>
<tr>
<td>Δlog(WB&lt;sup&gt;F&lt;/sup&gt; t(1-t&lt;sub&gt;W&lt;/sub&gt;))</td>
<td>0.139</td>
<td>0.092</td>
<td>0.292</td>
<td>0.168</td>
</tr>
<tr>
<td>Δlog(WB&lt;sup&gt;M&lt;/sup&gt; t(1-t&lt;sub&gt;W&lt;/sub&gt;))</td>
<td>0.373</td>
<td>0.062</td>
<td>0.224</td>
<td>0.452</td>
</tr>
<tr>
<td>Δlog(PW99&lt;sub&gt;t&lt;/sub&gt;(1-t&lt;sub&gt;PW&lt;/sub&gt;))</td>
<td>0.172</td>
<td>0.009</td>
<td>-0.089</td>
<td>0.586</td>
</tr>
<tr>
<td>Δlog(PW1&lt;sub&gt;t&lt;/sub&gt;(1-t&lt;sub&gt;PW&lt;/sub&gt;))</td>
<td>-0.005</td>
<td>0.861</td>
<td>-0.016</td>
<td>0.834</td>
</tr>
<tr>
<td>Adj. R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.681</td>
<td>0.067</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DW statistic</td>
<td>1.504618</td>
<td></td>
<td>1.406538</td>
<td></td>
</tr>
<tr>
<td>Sample</td>
<td>1975 2015</td>
<td></td>
<td>1975 2015</td>
<td></td>
</tr>
</tbody>
</table>

Note: Instruments are w<sub>F</sub>, α, β in H and N, t<sub>R</sub>, t<sub>W</sub>, t<sub>PW</sub> all in t, t-1, t-2

Estimation Method: Three-Stage Least Squares

**Table A6.2 Regression results for private investment**

<table>
<thead>
<tr>
<th>Variable</th>
<th>ΔlogI</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-1.800</td>
<td>0.001</td>
</tr>
<tr>
<td>Δlog(π&lt;sub&gt;t&lt;/sub&gt;(1-t&lt;sub&gt;R&lt;/sub&gt;))</td>
<td>0.081</td>
<td>0.543</td>
</tr>
<tr>
<td>ΔlogY&lt;sub&gt;t&lt;/sub&gt;</td>
<td>1.730</td>
<td>0.033</td>
</tr>
<tr>
<td>Δlog(PW1&lt;sub&gt;t&lt;/sub&gt;(1-t&lt;sub&gt;PW&lt;/sub&gt;))</td>
<td>-0.213</td>
<td>0.079</td>
</tr>
<tr>
<td>Δlog(PW99&lt;sub&gt;t&lt;/sub&gt;(1-t&lt;sub&gt;PW&lt;/sub&gt;))</td>
<td>0.415</td>
<td>0.122</td>
</tr>
<tr>
<td>Δlog(D/Y)&lt;sub&gt;t&lt;/sub&gt;</td>
<td>-0.167</td>
<td>0.249</td>
</tr>
<tr>
<td>logI&lt;sub&gt;t&lt;/sub&gt;</td>
<td>-0.322</td>
<td>0.000</td>
</tr>
<tr>
<td>logY&lt;sub&gt;t&lt;/sub&gt;</td>
<td>0.6395</td>
<td>0.0002</td>
</tr>
<tr>
<td>log(PW1&lt;sub&gt;t&lt;/sub&gt;(1-t&lt;sub&gt;PW&lt;/sub&gt;))</td>
<td>-0.161969</td>
<td>0.0078</td>
</tr>
<tr>
<td>Adj. R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.714379</td>
<td></td>
</tr>
<tr>
<td>DW statistic</td>
<td>1.735481</td>
<td></td>
</tr>
<tr>
<td>Sample</td>
<td>1973 2015</td>
<td></td>
</tr>
</tbody>
</table>

Note: Instruments are w<sub>F</sub>, α, β in H and N, t<sub>R</sub>, t<sub>W</sub>, t<sub>PW</sub> all in t, t-1, t-2

Estimation Method: Two-Stage Least Squares with ECM
### Table A6.3 Regression results for exports

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.020</td>
<td>0.074</td>
</tr>
<tr>
<td>$\Delta \log(\pi_t)$</td>
<td>0.100</td>
<td>0.422</td>
</tr>
<tr>
<td>$\Delta \log Y_{\text{World}}$</td>
<td>1.992</td>
<td>0.000</td>
</tr>
<tr>
<td>Adj. R$^2$</td>
<td>0.494</td>
<td></td>
</tr>
<tr>
<td>DW statistic</td>
<td>1.643</td>
<td></td>
</tr>
<tr>
<td>Sample</td>
<td>1973 2015</td>
<td></td>
</tr>
</tbody>
</table>

Note: Instruments are wF, α, β in H and N, tR, tW, tPW, and Y^world all in t, t-1, t-2

Estimation Method: Two-Stage Least Squares

### Table A6.4 Regression results for imports

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coeff.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-1.915</td>
<td>0.048</td>
</tr>
<tr>
<td>$\Delta \log(\pi_t)$</td>
<td>-0.191</td>
<td>0.197</td>
</tr>
<tr>
<td>$\Delta \log Y_t$</td>
<td>1.502</td>
<td>0.000</td>
</tr>
<tr>
<td>$\log M_{t-1}$</td>
<td>-0.241</td>
<td>0.038</td>
</tr>
<tr>
<td>$\log Y_{t-1}$</td>
<td>0.470</td>
<td>0.043</td>
</tr>
<tr>
<td>Adj. R$^2$</td>
<td>0.638</td>
<td></td>
</tr>
<tr>
<td>DW statistic</td>
<td>2.409</td>
<td></td>
</tr>
<tr>
<td>Sample</td>
<td>1973 2015</td>
<td></td>
</tr>
</tbody>
</table>

Note: Instruments are wF, α, β in H and N, tR, tW, tPW, all in t, t-1, t-2

Estimation Method: Two-Stage Least Squares with ECM