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'+1': Scholem and the Paradoxes of the Infinite

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Abstract This article draws on several crucial and unpublished manuscripts from the Scholem Archive in exploration of Gershom Scholem's youthful statements on mathematics and its relation to extra-mathematical facts and, more broadly, to a concept of history that would prove to be consequential for Walter Benjamin's own thinking on "messianism" and a "futuristic politics." In context of critiquing the German Youth Movement's subsumption of active life to the nationalistic conditions of the "earth" during the First World War, Scholem turns to mathematics for a genuine and self-consistent theory of action. In the concept of actual infinity (in Cantor and Bolzano) he finds an explanation of how mathematics relates to "the physical" without reducing the former to an "image" of the latter, and without relying on the concept of geometric intuition. This explanation, insofar as it relies on the notion of actual infinity, provides Scholem with a conception of mathematics (and the history of mathematics) that reconciles freedom and necessity—remarks on which he outlines in his diaries and communicates to Benjamin in early March 1916.

Keywords: history, Zionism, infinity, set theory, intuition, tautology

1. Getting Physical

1.1 In March 1916, an eighteen-year-old freshman by the name of Gerhard Scholem found himself walking around campus between classes, holding imaginary lectures on the foundations of mathematics in his head.[1] However precocious the young Gerhard may have been, the premises are preposterous to say the least. At the time, Scholem was in his second semester as a mathematics student at the University of Berlin, where among the courses he had taken for credit in the preceding term—which included analytic geometry, introduction to philosophy, and the "hygiene of male sexual life"[2]—he seems to have been the most impressed by a course on "cosmic cognition and ethics" taught by the director of the Berlin planetarium at the time, Wilhelm Förster (Scholem 1995: 137; 178). In the winter semester of 1915 to

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[1] Scholem 1995 I: 258. —All translations of this and other texts from the German are my own.

[2] According to his university transcripts, which are preserved in the Scholem Archive in Jerusalem, Scholem attended courses in the summer semester of 1915 that included the following: "Analytische Geometrie" with Frobenius; "Einleitung in die Philosophie" with Frischeisen-Köhler; and "Hygiene des männlichen Geschlechtslebens" with Posner. Cf. Scholem 1915b.

1916, Scholem attended courses on differential calculus, the "elementary geometry of conic sections," and the philosophy of religion, but seems to have been captivated primarily by a lecture he heard on experimental physics and optics given by the director of the physics institute, Heinrich Rubens.[3]

1.2 Indeed, in a couple of pages from his youthful diaries that were posthumously omitted from publication by his editors, Scholem reproduces formulae and diagrams from Rubens' lecture, indicating that they were to serve as the "*Wegweiser*," or signposts that would point towards the understanding of what would follow in the notebook of March 1916 (Scholem 1995: 257; n.1).[4] What ensues in the pages to follow is the record of several fast and furious days during which Scholem thinks out and writes down the foundational lectures he imagines himself delivering to his professors, and does in fact deliver to his friends, including in a letter to Walter Benjamin that is now lost (Scholem 1995: 258). The "signposts" that point the way towards these reflections on the foundations of mathematics, however, apparently attain their significance not from mathematics or from the philosophy thereof, but from another application altogether of Rubens' technical elaborations on optical phenomena resulting from prisms and mirrors of various curvatures. To cite from another passage redacted out of the published diary pages immediately preceding these diagrams, Scholem was concerned that a certain "Stefanie," with whom he had lamentably been "forced" to take up "*Verkehr*" or "relations" again, was a name that "appears this time as the symbol of the inessential"—for, as "Rubens says, symbols are worth nothing!!"[5]

1.3 Scholem's intimation that the foundations of mathematics may correlate at all to the problem of how to "get physical" certainly contains an element of the overheated fantasy of youth—as Scholem himself admits, he had in recent times been in the thrall of a "very excited imagination" when he delivered the "truly shattered and shattering lecture" to himself and his circle (Scholem 1995: 258). The young

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[3] According to the editors of the published version of the *Tagebücher*, Rubens taught "Experimentalphysik, II. Teil (Elektrizität und Optik)," "Mathematische Ergänzung zur Experimentalphysik" and "Physikalisches Colloquium" in the winter semester of 1915/1916, though judging from the content of the notes that follow in the manuscript of the diaries, it is the first course that Scholem must have attended. Cf. Scholem 1995: 257 n.1.

[4] The diary entry of March 1, 1916, which is the first in the notebook in which it is recorded, commences with the following statement: "There is no reason why I should not write in this notebook, even though the next two pages are covered with the results of the two hours I spent with Rubens, because they function as signposts for what will be the main concern of this notebook I have just begun." (Scholem 1995: 257)
The omitted pages pertaining to Rubens' lecture are referenced by the editors of the diaries in Scholem 1995: 257 n.1 and marked with the ellipses inserted in the middle of the page. Cf. the manuscript notebook entitled "Nach meinem Aufbruch zu mir: Auf dem Wege [*ba-derech*]" and containing the diary entries from March 1-11, 1916. (Scholem 1916: Ms 2-3) —I would like to thank the Scholem Archive for granting me access to the unredacted manuscripts of the diaries.

[5] These remarks are summarized from a passage that appears on the first page of the same notebook, and which is also part of the omitted text that is marked with the ellipses on Scholem 1995: 257. Cf. Scholem 1916: Ms 1.

"anarcho-messianist," as Scholem is widely held to be by intellectual historians, was deeply involved in the Jewish youth movement of the day. It was under these auspices that he had made the acquaintance of Benjamin a year earlier, in July 1915, some days after Scholem participated in a discussion hosted by the *Freie Studentenschaft* on a lecture held by Kurt Hiller. In his lecture, Hiller, "so to speak in Nietzsche's footsteps," had denounced history as a power that was inimical to spirit and life[6]: youth, by which Hiller meant the Jewish youth of the movement that was being encouraged by the education reformer Gustav Wyneken to enlist themselves in the German war effort of the First World War, was understood to be defined by its natural drive towards spiritual and intellectual independence from the burden of responsibility to the past. For Hiller, youth was equivalent to the will to cast off history and create the world anew, and "we" admonished to embrace our "youth" *viz.* capacity to exercise this will in our "own fullness of life."[7] Scholem's reaction to Hiller's concept of history, however, was critical; in his response a week later, which Benjamin was not only privy to but also took as cause to introduce himself to Scholem a few days later, Scholem criticized the youth leader's reduction of the complexity of political "movement" to a fiat of the will to do away with the continuity with the past by mere contradiction. Scholem saw in the life of youth not the solution but the problem of conceiving of agency in view of an existence that he regarded as ineluctably historical, and whose historical burden could not be so easily cast off. Far from being by definition the avant-garde, as it were, "youth" represented each moment that was mired between the desires of the day and the longing of tradition so as to be unable to "move." During Scholem's very first conversations with Benjamin they discussed, *contra* Hiller, the concept of youthful life as an ineluctable process *of* history (Scholem 1995: 123), which then led them to speak of the solutions that the Zionist, Socialist, and anarchist movements defining their milieu each presented for the antinomy between the event and history.

1.4 For his part, Benjamin was sympathetic to Scholem's intervention because of the recent suicide of his friend Fritz Heinle in the meeting room of the *Freie Studentenschaft* in 1914, four days after Germany's invasion of Belgium; Heinle's protest against the youth movement's complicity with the war effort later served as the model for Benjamin's discussion of the ends of life in his interpretation of Dostoevsky's novel *The Idiot*. (Benjamin 1977) In the essay, which was written in 1917, Benjamin identifies in the novel a lament of "the failure of the movement of youth," youth whose life is rendered dissolute and ultimately dispensable in regard to the equivalence that the movement sets up between the ends of humanity and the ends of nationalism (Benjamin 1997: 240); youth's overheating, as it were, dissipates in the absence of an adequate form on earth in which life might be coincident with its ends, rather than with its end. Shortly after reading Benjamin's essay on *The Idiot*, Scholem presented Benjamin with the gift of a novel by the turn-of-the-century

[6] Scholem 1975: 12.

[7] Scholem's recapitulation of Hiller's lecture, as well as a version of Scholem's critical response to Hiller, can be reconstructed from Scholem 1995: 123.

science fiction writer and satirist Paul Scheerbart: the novel, entitled *Lesabéndio: An Asteroid Novel*, was a fictional account of life under physical conditions other than those found on earth. (Scheerbart 1964) By all indications, Scholem regarded the fictional account of life under conditions not of this earth as the only adequate counterpart to Dostoevsky's *Idiot*, whose centrifugal force around the idea of the "national" had rendered individual, youthful life, in war as in words, entirely incompetent and incontinent—Dostoevsky's *Idiot* had illustrated that where the ends of the nation coincide with the ends of youth, a situation so mundane as to be populated even by idiots, only fantasy or failure present themselves as domains proper to the pursuit of life's ends. In early 1916, Scholem had already long discovered Scheerbart's work; indeed, Scheerbart's accounts of fictional extraterrestrial life were his unspoken lifelong pursuit[8], and in which he likely started to take an interest in 1915. According to a list of books that the young bibliophile maintained right out of secondary school, one of his earliest purchases for his personal library was Paul Scheerbart's *Das Perpetuum Mobile* (Scheerbart 1910), a satirical, pseudo-scientific account of the author's attempt to produce a perpetual motion machine, which Scholem acquired in January 1915[9], around the same time that he attended Wilhelm Förster's aforementioned course on astronomy and ethics.

1.5 All signs therefore point towards the exigency of understanding the products of Scholem's "very excited imagination" circa 1916 in the same context of his (and eventually Benjamin's) speculations on the physical conditions that might be adequate to a viable alternative to the politics of the day—that is, an alternative to any idea of the national whose ends are fulfilled by "life" achieved on its disability and self-destruction. For Benjamin, the thinkability of a life not subsumable under the identification of humanity with nationalism hinges on the projection of an optimum of thought that he would eventually call a "futuristic politics."[10] As for the one who played no small role in originating that thought, the projection of such an optimum would only have been conceivable for a student of mathematics conversant with contemporaneous theories of number and limit but unencumbered by the history of philosophy or the philosophy of mathematics save for a cursory introduction to Kantian cosmology in Förster's course in 1915.[11] Förster's lectures had advocated a view of the historicity of the intellect that was roughly equivalent to keeping the place of humanity relative to the spectrum of rational and embodied beings within the framework of natural scientific law; natural science, according to the docent lecturing months after the outbreak of the First World War, was regarded as

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[8] In his personal library, Scholem had kept a collection of clippings by and about Paul Scheerbart, some of which date to 1981. Cf. Scholem 1981.

[9] Scholem, "Verzeichnis anzuschaffender Bücher," 1.1.1915-7.6.1917 (Scholem 1917).

[10] As recorded by Scholem in his diary entry of June 13, 1919. Cf. Scholem 2000: 452.

[11] Scholem gives indication of the content of Förster's course in Scholem 1995: 137 and 178.

equivalent to a universal natural history with an end in a genuine peace.[12] In turn, Scholem saw in Förster's insight a demonstration that the state and its political-economic dogmatisms were actually "forms of inorganic violence" whose force derived from claiming "exclusive ownership of cosmic cognition" (Scholem 1995: 137); therefore, and in spite of prevailing views on the possibility and efficacy of scientific cognitions at the time, it was thinkable that the history of the intellect is not equivalent to the history of the cosmos, and that an optimum of thinking might be located at the point where thinking ceases to simply mirror the natural world. At the very least, Scholem's postulate that the foundations of mathematics lie in the relation of mathematics to "the physical" suggests, therefore, that mathematical foundations, the premises for why what is taught is taught, lie in *overstepping* the boundaries of mathematics, or at least of a certain understanding of mathematics that he finds being taught in the schools and university classrooms, in which he is one of just a handful still waiting to be called up for military service.[13] Not only must the evocation of Kant as a guarantor of cosmic harmony have seemed glaringly untimely, but as a student, Scholem was living as it were in the disjunction between the teachings of life and its ends.

1.6 Scholem's eagerness to expound on the foundations of mathematics as a relation to a certain something beyond the mathematical is in any case not easily dismissed as youthful exuberance, though the redaction of the notebooks in which his reflections on the topic appear is inclined to focus attention exclusively on their "anarcho-messianic" character. Scholem, whose main contribution to the life of letters and to Benjamin scholarship has been conceived by many as a function of his study of the Kabbalah, appears as such precisely due to interventions such as the redaction of technical mentions of physics and the physical from his diaries in particular and from the understanding of his thinking on "messianism" at large. Under this light the type of "messianism" he also then claims widely to have influenced Benjamin is painted as derivative of a mathematics so rarified that Scholem could ultimately "give it up" around 1917 or 1918 for its lack of essential difference from the metaphysical pursuits to which he would then "turn." The diary pages on which Scholem reflects on the "imaginary lecture" indicate, on the contrary, that Scholem's understanding of mathematics is inflected by an anticipatory structure that he develops from within mathematics itself (Scholem 1995: 265)—founding mathematics, as it were, on a different conception of mathematics that might "get physical" without coming into self-contradiction. For Scholem, the mathematics student, projecting a mathematics at whose foundation there would be no perpetuation of a false equivalence between the set of all possible (human) cognitions and the physical real under the guise of scientific objectivity was tantamount to sustaining and rigorously reconceptualizing history and the conditions for a genuine agency, such as may begin to abrogate the "forms of inorganic violence" (Scholem 1995: 137). Calling it a mathematics

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[12] The content of Förster's lectures is reconstructed from Scholem's remarks around Scholem 1995: 137 as well as Förster 1887.

[13] Unpublished passage from the diary entry of March 1, 1916. Cf. Scholem 1916: Ms 2-3.

"without semblance" (Scholem 1995: 261)—a mathematics in which the "symbol" does not signify in the manner of resembling or reproducing the "essential," but which instead hinges on the freedom of construction—Scholem spends the first six days of March investigating how a genuinely, rigorously mathematical understanding of mathematics might arrive at a relation between cognition and the real, and a concept of history, without falling back into a self-cancellation of life and world.

2. An Imaginary Lecture on the Foundations of Mathematics

2.1 At the center of the problem of founding mathematics, according to Scholem, is this: how to get from n to n + 1, or in other words, how to define the mathematical such that it may be systematically brought into relation with the physical real. For Scholem, the problem of "concluding from n to n + 1" describes the entire problem of how thinking can model itself on the sensible world without reducing the former to an "image" of the latter. If mathematics is to serve as the "equation" for all the other sciences, it must be grounded in itself, rather than imparted by way of "images" or "resemblances": the world cannot very well turn on a "line," and the "line" can only be expressed by the word "line" and not by means of a symbol like the "axis" (Scholem 1995: 265). "Mathematics," Scholem writes, "is *the* attempt to approach a thing naked" (Scholem 1995: 265).

2.2 For some, however, this is the equivalent to suggesting that mathematics amounts to nothing more than a "tautology"—that all possible cognitions would already be contained in its axioms, not unlike the idea that the world of living organisms might emerge from a single seed without the influence of any external agents.[14] To whatever extent he was aware of it, Scholem had in fact inserted himself into one of the most contentious debates in the history of mathematics in the nineteenth century, which did concern the foundations and definition of mathematics. This debate, as Scholem had earlier intuited, revolved around the question of whether or not there are any synthetic judgments in mathematical symbolization. Scholem's answer, which he gives in a lengthy disquisition on another page that was omitted from publication and dated August 1, 1915[15], is a "no." All synthetic judgments can only lead to a tautology, according to Scholem, because in mathematics the "concept" of a thing is its "definition"; for this reason, a "concept" of a straight line must first define, by way of a tautology, the thing that we wish to name a "straight line" with the characteristics thereof. Scholem then runs through the three classes of statements upon which mathematical propositions are formed: definitions, postulates, and axioms. First, definitions are not synthetic because it is not a judgment at all, but rather an assertion, just as we "name" a certain type of milk "buttermilk." Definitions posit what a thing is, whether or not it exists, and do not add anything to the concept. Second, postulates, which are not to be mixed up with axioms (as Scholem thinks

[14] One of the sources that Scholem draws upon during these six days in March 1916 is the work of the mathematician Aurel Voss. Cf. Voss 1913: 107.

[15] This missing page is marked with the ellipses inserted by the editors at the end of the fragmentary entry on August 1, 1915, Scholem 1995: 140. Cf. Scholem 1915a: Ms 28-31.

Kant does), are analytic judgments because they are given in a form that one can only take them for a priori. Nothing is added to the "concept" A if one postulates that A is also C because A is B and B is C. Third, axioms appear to be synthetic, since they cannot be derived analytically from any "concepts." But in fact, either axioms can be derived from the principle of contradiction to the extent that they are false, or they can be derived easily because there are no contradictions, in which case the axiom is not a synthetic judgment any longer because (according to Scholem) the opposite claim would be just as free of contradiction as the axiom that is claimed to begin with.

2.3 For this reason, Scholem argues, the parallel axiom or any other axiom in general is not a judgment (and thus not a synthetic judgment) since it would be contradictory. In any case, the axiom is an assertion that is presupposed for calculation and convenience, and it is because the parallel axiom is convenient to "us" that we regard it as "true." In mathematics, there is therefore no role played by intuition; axioms are employed out of convenience for the calculation at hand. Scholem concludes the note by remarking that, as it so happens, he had recently come across a statement made by the mathematician Carl Friedrich Gauss to his friend Heinrich Christian F. Schumacher that seems to support his view: "But even in Kant matters are often not much better. His distinction between analytic and synthetic proposition seems to me to be either a triviality or false![16]

2.4 Gauss was, long before Karl Weierstrass, Richard Dedekind and Georg Cantor, insistent that mathematics was a free creation of the mind: he extended the concept of number to include complex numbers—whose legitimacy was highly contentious at the time—and accorded them the same rights as the real numbers in order to ensure the scientific independence of analysis.[17] His insistence that the complex numbers can be geometrically represented in order to legitimate their existence, but that they were essentially the creation of the mathematician, is positioned radically against Kant's synthetic view that mathematics proceeds by the construction of concepts for which a priori intuition has to be presented which corresponds with the concept. Gauss' understanding of the foundations of the new numbers as essentially free of intuition later defined the position taken by Bernard Bolzano, Dedekind and Cantor, who successively extended the number concept to include those with which Cantor would eventually prove that the set of real numbers could not be put into a one-to-one correspondence with the set of natural numbers. Essentially, Cantor's proof that the set of real numbers, that is, the continuum, is "uncountably infinite," not only overturned the Euclidean axiom that the whole must be greater than its part by making the whole equal to a subset of itself. Cantor's proof also generated the concept that there are differently sized infinities, based on the existence of irrational numbers (specifically, the set of non-algebraic real numbers, an article about which

[16] Scholem 1915a: Ms 31. Cf. Letter from Gauss to Schumacher, Göttingen, November 1, 1844. In Gauss 1929: 63.

[17] Gauss, letter to Bessel, December 18, 1811. Cited in Boniface 2007: 325.

Cantor wrote as a way of surreptitiously announcing his findings). (Cantor 1874) Cantor's coup upset an entire tradition of thinking about the infinite that dated back to Aristotle's pronouncement that the infinite was to be understood mathematically, with the caveat that we must resist the idea that the infinite can be "given all at once"—a view that limited mathematics to the infinite repetition of the operations of addition and division and that would find its way in inverse form in the nineteenth century as Hegel's contrived opposition between "bad" or mathematical infinity, and "good" infinity or the Absolute. The birth of transfinite set theory, in which infinities are added and collected, swept aside this opposition and its attendant prejudices about the construction of number by proving that the actual infinite could be grasped mathematically through a defense of the existence of irrational numbers that is rooted in the fundamental belief that "das Wesen der Mathematik liegt in ihrer Freiheit." (Cantor 1883) (Incidentally, this phrase was adopted by Benjamin's uncle, the mathematician Arthur Schoenflies, as the epigraph to a text that served to popularize Cantorian set theory, *Die Entwicklung der Mengenlehre und ihrer Anwendungen*.[18])

2.5 Cantor encountered a lifetime of fierce opposition for his views from Leopold Kronecker, who for his part believed that all of arithmetic could be based on whole numbers—and that therefore irrational numbers simply do not exist: "Die ganzen Zahlen hat der liebe Gott gemacht, alles andere ist Menschenwerk."[19] In his devotion to what he regarded as the truth of mathematics, and no small amount of anxiety about how what appeared to him as false mathematical logic would affect the integrity of the integer, Kronecker accused Cantor of corrupting the youth (Dauben 1990: 1), and worked actively behind the scenes to try to prevent the publication of Cantor's results.[20] For his part, Kronecker's opposition of actual infinity to irrational, transcendental and transfinite numbers also forced him to reconstruct mathematics on a radically new basis, in the course of which he foreshadowed twentieth-century intuitionists such as the French logician, mathematician and philosopher of science Henri Poincaré, who picked up on Kronecker's work after the paradoxes related to the continuum hypothesis were discovered. Thus Poincaré would utter phrases echoing Kronecker's, such as "it is by logic that we prove, but it is by intuition that

[18] Schoenflies' *Die Entwicklung der Lehre von den Punktmannigfaltigkeiten* (1900/1908 [1913]) was the first popularization of Cantorian set theory, and was originally published in two parts: a) *Jahresbericht der Deutschen Mathematiker-Vereinigung* 8 (1900), 1-250; and b) in *Jahresbericht der Deutschen Mathematiker-Vereinigung*, Ergänzungsband II (1908). It was later republished as Schoenflies 1913.

[19] Attributed to Kronecker by Weber in Weber 1893.

[20] Kronecker famously machinated behind the scenes and used his position as an editor in *Crelle's Journal* [*Journal für die reine und angewandte Mathematik*] (or so Cantor suspected) to delay the publication of Cantor's paper on dimension, "Ein Beitrag zur Mannigfaltigkeitslehre," in 1877. Kronecker might also have suspected an underlying mental instability in Cantor and tried to provoke him into destroying his ties to his own supporters by suggesting that he would publish on the insignificance of set theory with Gösta Mittag-Leffler's *Acta Mathematica*, one of Cantor's only major publishing outlets.

we discover" (Poincaré 1908: 129), and that "logic sometimes makes monsters."[21]

2.6 In the pages detailing his imaginary lecture on the foundation of mathematics from March 1-6, 1916, Scholem makes reference to Poincaré to illustrate the counterposition represented by those who regarded a mathematics grounded in itself rather than imparted by way of "resemblances" to be merely tautological. According to Poincaré, so Scholem, a solution to the problem that mathematics in that case would already contain all possible cognitions in its axioms is provided by the fact that mathematics itself contains a "creative," "synthetic" force in the inductive process, whereby some statement that is true for a number n is shown to also hold when n + 1 is substituted for n, and so forth, *ad infinitum*, until one concludes that the statement that is true for n is true for all numbers. Poincaré argued that inductive reasoning was a synthetic judgment a priori (in the Kantian sense) because it allows one to skip over the tedium of infinite steps of reasoning by way of assuming a hypothesis—that the statement is true of n—that cannot be logically derived.

2.7 For Scholem, however, Poincaré's argument proves precisely that all of mathematics is analytic, because the "n + 1" is only a "genial aid" to proofs: "there is," so Scholem, "no statement that cannot be proven without the inductive hypothesis (Scholem 1995: 268). Moreover, if for Poincaré inductive reasoning is synthetic by the sheer "violence" of its consummating act, which for him is testimony to the power of the mind to represent to itself the infinite, for Scholem this does not seem correct since "between the nth member and the first for which the statement is valid, there are not infinitely many interim members, but only a finite number," for which reason a "potentially infinite power of imagination" is decisively *not* required to effectuate the conclusion from n to n + 1, since there is no need for imagining the infinite repetition of the same (Scholem 1995: 268). At the same time, Poincaré does not show that the application of the statement could ever lead to "actual infinity," since for an "arbitrary number" of cases the mere principle of non-contradiction would suffice to prove the generality of the statement (Scholem 1995: 277; agreeing with Voss 1913: 108). Having failed to prove the synthetic a priori nature of inductive reasoning, Poincaré would seem to be left with the tautology he set out to resolve, namely a mathematics reduced to analytic judgments with no extra-logical content, or an infinite number of indirect ways of saying A = A.

2.8 At this juncture Scholem proposes that Poincaré has simply misunderstood the nature of the "great tautology A = A," that in fact it is "nothing frightening" in need of resolution, since "infinite mathematics, as paradoxical as it sounds, is already completed, and [the only question is] that the human mind grasps every one of its infinitely many propositions by way of logical links between what is already known" (Scholem 1995: 277). In mathematics, the "entire world" lies folded within the fundamental definitions and concepts, whose immense fullness is exhausted in their infinite combinations (ibid.). Mathematics does not say anything *new*; mathematics only concerns itself with relations between beings considered to be indifferent in

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[21] Poincaré 1899. Cited in Kline 1972 3: 973.

themselves, and absolutizes the relative to the extent that it says *what* something is. History, at least in contradistinction to this definition of mathematics, says *how* something has come into being, and to this extent it relativizes the absolute into accidents and acts of will (Scholem 1995: 260). It is at the end of history that one will have arrived at a concept of science adequate to its articulations in time; mathematics, however, "abides by the momentary disposition of things" and as such is the "only way" for the mind to approach nature "without a leap" (Scholem 1995: 261).

3. Paradoxes of the Infinite

3.1 The approach to things thus cannot be accomplished in the manner of Hegel who, as Scholem recalls, once tried to reinvent Kepler by attempting to philosophically derive the positions of the planets[22], and then gave up in favor of a view of a theory of nature that unfolds with the development and completion of the individual sciences (Scholem 1995: 259)—as if the mathematical laws of natural science were only adequate for grasping infinity quantitatively, as the possibly infinite accumulation of scientific discoveries, while philosophical principles assumed to lie at their foundation are associated with qualitative infinity, or the qualified way in which something is assumed to exist in actuality, which is treated as an infinity much higher than the quantitative. For Scholem, the approach to things is realizable because "infinite mathematics … is already completed" (Scholem 1995: 277): that is, the infinite is actually existent and not limited to the simple possibility of repeatedly adding the finite. For this idea, Scholem borrows from the Catholic theologian and logician Bernard Bolzano, who attempted towards the end of his life to lay down a new foundation for all of mathematics. In his posthumously published *Paradoxes of the Infinite* (1851), which Scholem studied during this time, Bolzano gave the first complete mathematical presentation of the problems of the infinite, in which Scholem finds precisely the basis for collapsing the contrived *viz.* Hegelian opposition between a "bad" quantitative infinity and a "good" qualitative infinity.

3.2 The findings presented in *Paradoxes of the Infinite* were, in fact, influential for Cantor's own thinking on the equivalences between sets and their subsets, and Scholem was probably attracted to Bolzano's untimeliness as a representative of Cantor before Cantor himself lent his name to the sea-change in mathematical thought. Indeed, Bolzano was recognized only *ex post facto*, after Cantor became recognized for his innovations in conceiving of infinite numbers. Just as Scholem likely imagined himself as he delivered his imaginary lectures on the foundations of mathematics, Bolzano simply breaks away through the Hegelian contrived opposition between bad and good infinities, seemingly without grounding in established philosophical or mathematical argumentation.

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[22] As the editors of Scholem's diaries point out, there is no indication that Scholem had read the text in which Hegel achieves this—his *Dissertatio philosophica de Orbitis Planetarium* (Jena 1801). However, references to Hegel's dissertation are made in an essay on Kierkegaard that Scholem did read by Georg Brandes (1902: 395). —Cf. Scholem 1995: 259 n.10.

3.3 And in fact, Scholem's notes on Bolzano, which he made on the margins and on loose leaves inserted into his personal copy of the book[23], are the most extensive on the passages that describe the new premises upon which mathematics might make statements about actual infinity that do not lead to contradictions doing away with these statements. The central point for Scholem (as for many others interpreting Bolzano) is this: that some infinities are greater or smaller than others because one might include another as a part of it, or consist in a sum of infinitely many finite entities (Bolzano 1851: 28 [§19]). That is, "two sets that are both infinite can be in relation to one another such that it is possible that each element of one set be paired with an element from the other set, such that no one element is unpaired, and such that no element appears in two or more pairs," and still it is possible that one of these sets is merely a part of the other set (Bolzano 1851: 28 [§20]). From these observations regarding the mapping of elements in infinite sets it follows that mathematics is not reducible to the infinite adding and dividing of finite magnitudes, but can also manipulate the actual infinite. The opposition between a quantitative infinity represented by certain mathematical operations' generating the repetition of the same, on the one hand, and a qualitative infinity represented by the absolute, by truth, or by god, is thereby done away with.

3.4 As an outcome, mathematics is capable of grasping actuality, but this, to reiterate, is based on the insight that the mathematical *per se* is neither "creative," nor "constructive," but stands in some other relation to the real. As Scholem remarks to Bolzano's discussion of an infinite series consisting in an endless alternation of + 1, - 1, + 1, - 1, and so forth *ad infinitum*, it is not only clear that the series does not express a real quantity, since it oscillates around some intermediate value. It is also clear, to Scholem, that were one to insist that it does express a real quantity, one would have "proven *creatio ex nihilo*"—and this is constructed by Bolzano by adding brackets around pairs of terms such that one acquires a series that amounts to 0 + 0 + 0 + 0 + 0 … *etcetera*, which can only equal 0. The question remaining is that of how the links are indeed forged between the alternating entities and reality if not by bracketing off their infinite progression, at which point Scholem hesitates. For Bolzano, the Catholic theologian, "We call god infinite because we have to ascribe to him powers of more than one kind that are of infinite magnitude" (Bolzano 1851: 8). For Scholem, this remark elicits a question mark on the margin of the page, since at the very least the adequation of the mathematical to the "momentary disposition of things" (Scholem 1995: 261), entirely "indifferent" to the mind's advances, would likely look nothing other than like the infinite alternation between + 1 and - 1, as a principle, perhaps, of reality's having momentarily already arrived.

3.5 "And yet," Scholem writes, "history is the unfree thought of science while mathematics is the free thought of science: history fulfills with horror the confusion that humans call freedom, while mathematics fulfills in necessary construction with the deepest joy" (Scholem 1995: 260). Echoing Cantor's definition of mathematics

[23] Bolzano 1851. —Scholem's personal copy is located at the Scholem Collection Nº 8059.

as freedom, of which Bolzano was made the representative *ex post facto*, Scholem comes to the conclusion that freedom in mathematics lies somewhere other than in the "confusion" wreaked from the history *in toto* of the intellect's pursuits. For "history" fulfills "freedom" only as "confusion" if, as Kronecker in the guise of Poincaré believes, the integer is divinely given; if God created all the integers, mathematical science would be the history of man. From this standpoint mathematics can only appear as the domain of synthetic judgments—and tautological. If all synthetic judgments in mathematics rely on definitions contained within mathematics itself, however, then all of mathematics is in fact non-intuitional, containing all of its definitions and containing only definitions *qua* constructions. In such a case, as Scholem writes in paraphrase of the letter from Gauss to Schumacher that he had cited on August 1, 1915, "the differentiation between analytic and synthetic judgments is an insignificant game" (Scholem 1995: 278). For in this case, what for Poincaré appears tautological only appears as such given the presupposition that there are necessarily synthetic judgments in mathematics—and that therefore integers are divinely given, while all calculation therewith is "Menschenwerk," self-limiting in scope to acts of attempted adequation with the truth of a statement.

3.6 In fact, Scholem suggests, all of mathematics conceived as non-intuitional is "an infinite tautology," which "is not [equivalent to] a tautology as seen from the human standpoint" (Scholem 1995: 278), but rather to an infinity of definitions contained within mathematics such that its symbols, regardless of how they are construed, express A = A. Genuine freedom therefore lies not with history but with "necessary construction" in mathematics: in a mathematics where analysis is given its foundational freedom from intuitive *viz.* geometric proof, the terrestrial condition is no longer limited to everyday impossibilities by the presumed necessity of synthetic judgment, but rather expresses all of the science-fictional possibilities expressible by a mathematical symbolism thus construed.

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3.7 At least in 1916, Scholem's fantasy of founding the material of reality anew through mathematics seems to point beyond the language of conditions, in the schema of which freedom might only appear in the series of infinite quantitative repetition—in Kantian terms, in the representation of our terrestrial condition in the form of an infinite given magnitude. For "an infinite tautology is not a tautology as seen from the human standpoint" (Scholem 1995: 278). How another standpoint might look, from which "the differentiation between analytic and synthetic judgments is an insignificant game" (Scholem 1995: 278) and such that the abrogation of "forms of inorganic violence" deriving from claims to "exclusive ownership of cosmic cognition" (Scholem 1995: 137) might occur, is not merely science-fictional. Just a year later, in a letter to Werner Kraft, he writes that "Zion and mathematics" are at the center of his life; the question that concerns him as a result is "whether the intellectual / spiritual essence of the world is expressible (in any manner whatsoever, so that what is said by this is not yet that it is also cognizable, because pure action too, for instance, has expression and also language) [*ob das geistige Wesen der Welt ausdrückbar sei (auf irgend eine Weise, so dass noch nicht damit gesagt wäre: auch erkennbar, denn auch reine Handlung z.B. hat Ausdruck und auch Sprache*]."[24] This great question, to be unfolded over the course of several further occasions upon which Scholem would attempt to articulate the relation between "mathematics and language, i.e. mathematics and thinking, mathematics and Zion,"[25] and which even predicated his later, well-documented "turn" towards the study of the Kabbalah[26], finds germination in the moment when, conceiving of "freedom" as the "freedom of necessary construction" in a mathematical sense, Scholem discovers a viable figure for thinking the possibility of pursuing the ends of life, and of Jewish life in particular: as an "infinite tautology" that, free from the primacy of a European humanism amplified to its warring and nationalistic extremes, might be capable of expression a state of "Zionistic" possibility.

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[24] Scholem, Letter to Werner Kraft, 3.8.1917, in Scholem 1994 I: 86.

[25] Benjamin 1966 II: 128.

[26] In an open letter to the publisher Schocken from 1937, in which Scholem declares his reasons for turning to the investigation of Jewish mysticism, Scholem refers to the exigency of dealing with "myth" and "pantheism" not by erecting "antitheses" against them (which would be a natural-scientific attitude) but by "canceling them out" in a "higher order"; by this Scholem means to "penetrate through the symbolic plane, the wall of history," where the "mystical totality of the system" otherwise "disappears" when projected onto it. Cf. Scholem 1994 I: 471.

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