Some Unsettled Issues in a Second Phase of the Cambridge-Cambridge Controversy (*)

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ABSTRACT

The aim of this paper is to clear up some issues in a second phase of the Cambridge-Cambridge capital theory controversies, when the neoclassical argument was chiefly conducted in terms of the Walrasian specification of capital in Intertemporal and Temporary General Equilibrium models. It is held that the response by the neoclassical side in that phase has not been as satisfactory to rebut the implications of reswitching and capital reversing as some neoclassical scholars have argued. The reason for this can be traced in the overlooking of the implications of the redefinition of equilibrium implied in those models.

1. Introduction

It is broadly acknowledged in the history of contemporary economics that the theoretical conflict known as the Cambridge-Cambridge capital theory controversies shook the tenets of neoclassical theory. One of the most significant outcomes of the controversy for that theory has been that the basic principle of factor substitution cannot be generalized to study market economies with heterogeneous capital goods. Indeed the results known as reswitching and capital reversing showed that factors demands functions will not necessarily be inversely related to their respective rates of remuneration. The reason behind those results ultimately lies in the fact that the capital endowment is specified in value terms among the data of equilibrium in neoclassical theory. But this implied, as was ultimately admitted (e.g., Samuelson, 1966), that the theory cannot robustly determine long-period equilibrium prices and distributive variables, and therefore provide a robust supply and demand framework to study real market economies. After more than fifty years since the first exchanges in those debates, one might ask: What happened in the discipline after that historical

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period in which even some of the most authoritative neoclassical scholars admitted that their theory was at stake?

One of the first and well-known reactions on the part of the neoclassical side was to play down the relevance of the capital debate, by asserting that reswitching is extremely unlikely to arise empirically (see, e.g., Ferguson, 1969). But secondly, later on the debates, the orthodox response was to abandon those versions of the theory that specify the capital factor in value terms, and to adopt the Intertemporal and Temporary general equilibrium models (IGE and TGE, respectively), which use a Walrasian notion of capital (i.e., the physically heterogeneous capital goods vector). This formal change in the structure of the theory however entails a radical departure from traditional economic analysis. Indeed, these models suffer from the same kind of problems impinging on Walras's theory of capitalization. One salient problem in that theory is the impossibility to determine a uniform rate of return on the supply prices of the capital goods (Garegnani, 1960; 1990). This response to the controversy actually meant a redefinition of the notion of equilibrium in neoclassical theory, as the latter did away with the traditional requirement of a uniform rate of return (Garegnani, 1976).

The aim of this paper is to provide an assessment of this second response to the controversy. Our purpose is not to establish any new analytical result; rather it is to analyze and contextualize this neoclassical way out of the controversy that may probably be unfamiliar to a wide range of non-specialists. With a view to presenting our reconstruction as clearly as possible, we shall hold that this orthodox response has not been as adequate to rebut the implications of capital reversing as some neoclassical scholars have argued (e.g., Bliss, 1970; 1975). One of the reasons for this to be so is that the consequences of the redefinition of equilibrium in marginalist theory were either misunderstood or not entirely grasped by the orthodox side. In particular we shall discuss that, once the traditional versions are abandoned due to the inconsistency of the value specification of capital, then equilibrium variables in IGE and TGE models cannot be taken as theoretical guides of real-economies behavior, as is the case for traditional theory.

The paper has six Sections. Section 2 examines the Walrasian specification of capital and the problems that arise from that notion. Section 3 discusses some implications of those problems for modern economic theory. Then, Section 4 analyzes how the redefinition of the traditional notion of equilibrium entailed by IGE and TGE models originates in the problem of using the Walrasian notion of capital. Accordingly, Section 5 will examine in more detail the issue of uniformity of rates of return in IGE models, and will discuss the different characteristics of such a uniformity with the uniform rate of return of traditional neoclassical theory. Section 6 offers some concluding remarks.
2. The change in the terms of the discussion in a second phase of the controversy: the Walrasian notion of capital

As sketched above, one of the most destructive results of the controversy in a first phase of its development was to show the flaws to derive well-behaved factors demand functions. This result implied that the basic principle of factor substitution cannot be generalized to analyze market economies with heterogeneous capital goods. Thus the marginalist theory lost one of its long standing foundations to derive a plausible explanation of equilibrium in terms of supply and demand. As argued in the relevant literature, one of the salient consequences of the controversy was the definitive abandonment of the marginalist versions that use a value specification of capital (Garegnani, 1976). One natural consequence for orthodox theory was, in fact, to adopt the other specification of the capital factor that we can find in the history of neoclassical theory, i.e., the Walrasian specification of capital.

This change in the terms of the discussion opened what we call here the second phase (1971-1976) in the development of this theoretical conflict.¹ This shift in the discourse to invalidate reswitching was carried out by having recourse to IGE and TGE models, which, as already pointed out, treat each physically heterogeneous capital good as a different factor of production from one another (Bliss, 1975; Hahn, 1975, 1982). So an important problem here arises: if these models use the Walrasian notion of capital, do not they also suffer from the same kinds of problems we find in Walras ([1954)? One may even wonder whether the problems entailed in Walras ‘s original framework were the responsible for the little influence Walras had among the most renowned neoclassical authors of his time (cf. Wicksell 1901 [1934], p. 171). So it is useful to begin with by restating the problem of Walras’s theory of capitalization.

In Walras’s original framework the analysis of market economies is set in three different levels. In the first one, a simple exchange economy without production is considered, where relative prices are explained only by individuals’ endowments of consumption goods and preferences (Walras, 1954,

¹ The results of the first phase of the debate were publicly acknowledged by neoclassical scholars (e.g., Samuelson, 1966). For an examination of the results of the first phase of the controversy, see Harcourt (1972, pp. 118-176) and Lazzarini (2011, pp. 55-90). Our breakdown of the controversy in two phases can also be found in Mirowski (1989, pp. 341-343). Although this scholar does not divide the controversy according to which notion of capital was at issue, he points out that the first phase was characterized by a “disoriented and disorganized” reply from the neoclassical side, while in the second phase it was “subtle and sophisticated” (Mirowski, 1989, p. 343). He identifies the second phase after the 1966 Symposium, in particular around 1975 with the work of Bliss (1975). In our paper, the reason to date the beginning of a second phase in the debate is due to Robinson (1971), where the Cambridge economist speaks of the “end of the controversy”, while 1976 is partly due to Garegnani (1976), which brings the implications of the controversy to the method in economic analysis, and partly to Harcourt (1976), the updated survey which took into account the neoclassical versions relying on Walrasian capital.
In the second level, Walras introduces into his analysis production of final goods only. Thus, besides the abovementioned data, technology is introduced to explain prices and distribution (Walras, 1954, pp. 211-63). Finally, because of his deeper goal to explain how real economies work, Walras considers production of capital goods in his general equilibrium system (Walras, 1954, pp. 278-312).

In the original Walrasian model it is possible to categorize factors and goods as follows: i) non reproducible factors; ii) reproducible final goods; iii) reproducible factors (i.e., capital goods). Factors are supplied by agents who are endowed with both kinds of factors; given the set of available techniques assumed by Walras to be fixed coefficients, preferences determine the consumption goods that will be effectively traded. Regarding the set of equilibrium relative prices, the remuneration of the non reproducible factors will be determined by their respective demands and their respective (fixed) supplies. Prices of final goods will be defined by their cost of production; given their (full employment) demands, supply will adjust up to the point in which both schedules match. As to capital goods, it is important to note their most important feature: they are a kind of hybrid of the other two. As productive factors they are fixed in quantity so its remuneration should be such that their (given) supply and demand match each other. But, as reproducible commodities, their long-period price must exactly cover their cost of production (Walras, 1954, p. 211). Therefore, the double nature of capital goods in the Walrasian scheme imposes two sets of independent equations to determine their long-period prices (Eatwell and Milgate, 1999). Formally, for the case of a one-capital good economy, we have:

\[ P^d_k = \frac{\nu_k}{(1 + \pi)} \]  

\[ P^s_k = c_k \]  

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As equation (2.1) shows, this demand price can be interpreted as the present net value of income that savers expect to gain through lending the capital good $K$ in the future.\footnote{Equation (2.1) abstracts from the insurance premium. Also, depreciation does not arise because we are assuming a circulating capital good.} As Walras assumes that the new capital good could be used in the following period, the rental price $\upsilon_k$, whose equilibrium value is unequivocally defined by the (fixed) supply and demand of $K$, should be discounted by the general rate of return ($\pi$). We can, at this juncture, note that the demand price of the capital good is a certain amount of value exclusively defined by arbitrage, without any reference to the conditions of production.

On the other hand, equation (2.2) defines that its supply price must cover the costs of production, $c_k$. So this price is the minimum amount that producers must receive in order to continually reproduce that capital good in the future. If this equation effectively defines the selling price of the capital good, we can figure out the uniform rate of return on the supply price of capital good $K$, that is:

$$\frac{\upsilon_k}{c_k} = 1 + \pi_k$$

(2.3)

The long-period equilibrium price implies that $P_k^s = P_k^d$, or replacing (2.2) and (2.3) with (2.1) we obtain:

$$\pi = \pi_k$$

(2.4)

Thus, the uniform rate of return can be determined by dividing the rental price of the capital good by its cost of production.

On the contrary, if we consider the more realistic assumption in which more capital goods are produced (for example $K$ capital goods), we shall have $K$ independent equations like (2.3) and so $K$ different rates of return on their supply prices. Some scholars showed that the system presents an over-determination of degree $K-1$.\footnote{This over-determination is evident in the case of a stationary economy where net savings are nil. In that case, the new quantities of capital goods disappear, and with them, $K$ unknowns of the system will vanish, but only one equation will disappear (i.e., the equilibrium equation between aggregate saving and total investment). See Garegnani (1990, pp. 15-6).} But, in fact, also Walras seemed to have perceived the impossibility to satisfy the condition of a uniform rate of return (‘rate of net income’) in a multi-capital good economy:

If we suppose that old fixed capital goods proper are already found in the economy in [given] quantities (...) and that their gross and net incomes are paid for at prices determined by the system of production equations and by the rates of depreciation and insurance, it is not at all certain that the amount of savings will be adequate for the manufacture of new capital goods proper in just such quantities as will satisfy the system. (...) In an economy like the above we have imagined which establishes its economic equilibrium ab ovo, it is probable that there would be no equality of the rates of net income. (Walras, 1954, p. 308).

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To assess this latter issue, let us assume that two kinds of physical capital goods, $K$ and $K'$, are produced in the economy, and that they are demanded in quantities $D_k$ and $D_{k'}$ respectively. Let us call $R$ the ratio between these two variables, that is:

$$R = \frac{D_{k'}}{D_k}$$

(2.5)

We can realistically assume that the respective rates of return on the capital goods' supply prices are such that $i_k < i_{k'}$. As $K'$ yields the highest rate, $R$ will increase because there will be an increase in the demand for $K'$ relatively to $K$ (and therefore an increase in the production of $K'$ relatively to $K$). The equalization of the rates of return must be verified over the supply prices because this condition ensures the permanent reproduction of the system under the same conditions as before. The effect of a rise in $R$ over the rate of return on each capital good can be better grasped if we rewrite condition (2.3) for each of them:

$$\begin{cases}
    \frac{u_{k'}}{c_{k'}} = 1 + i_{k'} \\
    \frac{u_k}{c_k} = 1 + i_k
\end{cases}$$

(2.6)

This change in relative demands for $K$ and $K'$ should trigger two different effects: 

a) Changes in $R$ will alter the cost of production of the capital goods.

b) Changes in $R$ will alter the rental price of the capital goods.

On the one hand, we should expect that the cost of production of $K'$ increases relative to the one of $K$, as the set of goods involved in the production of the former will be demanded in greater proportion than those goods involved in the production of the latter (effect a). If we keep the relative rental prices constant, this would imply that $i_k$ would decrease relative to $i_{k'}$. However, rental prices will be modified (effect b) and may more than compensate for the change in relative costs. Therefore, starting from any disequilibrium position, the final result over the yields is in general indeterminate. $^6$

$^6$ There are three possible cases. First, when the production of $K'$ is self-intensive, that is, when the production of $K'$ requires a higher amount of itself than of $K$, an increase in the demand for $K'$ will make its services price increase relatively to the corresponding price of $K$, hence the original discrepancies in the net yields will widen. It is then uncertain which effect, (a) or (b), will tend to prevail. If the production of $K'$ is hetero-intensive, both effects will reinforce each other, generating the desired effect over the net rates of return. However, it might well occur that, given the arbitrary character of the initial endowments of $K'$ and $K$, the initial differences among the net yields could be wide enough such that the elimination of those differences could be impossible to achieve. That is to say, the demand for $K$ could never fall below zero; and even in that case differences could still persist. Finally, if the intensity in the production of capital goods is the same among industries, changes in $R$ should not modify the net rates of return.
3. Implications of Walras's theory of capitalization

The indeterminateness into which the original Walrasian system fell placed Walras in front of a disquieting dilemma in his analysis of production of capital goods. In principle, there are two possible alternative solutions within Walras's model:

(i) Equality between the cost of production and the demand price of the capital goods can be maintained, thus allowing the existence of different effective rates of return in the economy (*i.e.*, different rates of return over the demand prices of capital goods).

(ii) Through arbitrage, a unique rate of return over the demand prices of the capital goods could be established (actually the highest), although the selling prices would be below the cost of production for some capital goods. As a result, some capital goods will not be reproduced in the future (*i.e.*, those which bear a rate of return lower than the highest one).

The first solution is unsatisfactory at first sight. It would imply that in a competitive market economy there are some agents (savers) who are not benefiting from simple arbitrage. Solution (i) merely assumes that there are individuals that could invest in more profitable activities and they do not take the advantage of this fact.

The second solution (ii) was the one finally pursued by Walras in the fourth edition (1900) of his Éléments. In fact, in that version of his work he assumed that the ‘rate of net income’ (rate of return) that would effectively prevail in the system would be the highest among all the possible ones. Accordingly Walras also envisaged that every capital good whose demand price—defined in terms of the former rate—fell below its supply value would not be reproduced in the following periods. Note that Walras's solution (*i.e.*, the highest rate of net income) is totally compatible with the broadly accepted assumption of competition. At first glance, then, this movement only presupposes a slightly formal adjustment, defined by equation (2.7) below:

\[ P_j^d \leq c_j \quad j = k, k' \quad (2.7) \]

The following is a note for clarification on the conditions for capital goods reproduction:

7 “All we could be sure of, under these circumstances, is: (1) that the utility of the new capital goods would be maximized if the first new capital goods to be manufactured were those yielding the highest rate of net income, and (2) that this is precisely the order in which new capital goods would be manufactured under a system of free competition.” (Walras, 1954, p. 308). It must be noted that a situation in which the demand price of a capital good exceeds its supply price must be taken to be a temporary disequilibrium which will be adjusted as savers invest their incomes in buying that specific capital good. Then, because of competition, the rate of return involved in the demand price will have to go down so as to make the latter increase enough up to the level compatible with its supply price.
The system of equations now admits the possibility of inequalities, and when a strict inequality holds, the quantity produced of the capital good in question will be nil. However, the qualitative content of this modification is much deeper than what it could resemble at first view.

In effect, if demand prices of some capital goods are lower than their costs of production, it is plausibly valid to expect that those capital goods will not be reproduced in the following periods. Walras believed that a state of general equilibrium could still be achieved “after the exclusion of those new capital goods which it was not worth while to produce” (Walras, 1954, p. 294). But if this were the case, the stocks of capital goods would undergo drastic changes as a result of the tendency to a uniform rate of return over the rest of capital goods, in the sense of solution (ii) above. For example, if a consumption good is produced with the help of capital goods whose costs are not covered by their selling prices, then neither the capital goods nor the consumption good will be produced. The issue that arises is precisely how to deal with the exogenously given capital good stock, which is likely to undergo changes in front of any disequilibrium situation. Moreover, in Walras’s theory a change in the whole system of prices and quantities would ensue, which is evident under the hypothesis of fixed coefficients.

Accepting equations (2.7) entails the abandonment of Walras’s original aim: to characterize the conditions under which the system is said to be in a proper equilibrium, where the condition of uniform rate of return on supply prices should prevail. The latter condition, however, needs of a value specification of capital. It is clear, therefore, the reason why the capital controversy in its first phase was significant in prompting a change in the structure of orthodox theory. Yet, along with this apparent formal shift, the neoclassical theory redefined the notion of equilibrium to which it had for so long adhered. It is thus necessary to examine the differences in these two notions of equilibrium in order to appraise the strategy pursued by the neoclassical side in the second phase of the controversies.

4. Two notions of equilibrium in the second phase of the controversy

It is precisely at this juncture in the development of the controversy that the issue of the consequences of dropping the condition of a uniform rate of return turns relevant, both historically and analytically. Historically, because discussion of Walrasian capital opens what I called the second phase of the capital debates; and analytically because Walrasian capital started being addressed by the neoclassical side exclusively in the way it is introduced in IGE and TGE models. By dropping the condition of uniformity of rates of return, these versions of the theory tried to overcome Walras’s inconsistency. This endeavor was essentially the neoclassical response
to *capital reversing*, thereby giving place to a redefinition of the notion of equilibrium. In order to assess if this way out of the controversy has been satisfactory, it will be necessary to address the implications of abandoning the traditional or long-period notion of equilibrium that is behind the uniform of return condition. Therefore, it is necessary to recall the main features of the traditional notion of equilibrium (Dvoskin and Lazzarini, 2013):

**i)** Equilibrium is conceived as a centre of gravitation of the empirical variables. It is therefore necessary for the theory to include within its system a robust explanation of the basic mechanisms that allow such equilibrium to come about.

**ii)** A centre of gravitation entails that the variables defining the equilibrium position are persistent enough so that they can determine the trend of the observable magnitudes without being substantially affected themselves by disequilibrium phenomena.

**iii)** As already discussed, equilibrium prices of the capital goods must yield a uniform rate of return on their supply prices. For this to be so, the physical composition of the capital stock must be endogenously determined.

**iv)** The endogenously determined composition of the capital stock needs a single-valued scalar endowment for the capital factor (capable of changing form without changing in terms of quantity). The outcome of the factor substitution processes turns out to be plausible only if capital is specified in such terms.

The Cambridge controversy undermines the fourth characteristic, while IGE and TGE models suffer from not attaining the third feature. In section 5 below, we address the issue of uniformity of returns since IGE and TGE actually do away with the third feature above. In what follows, although intrinsically connected with the third feature, we discuss whether a TGE provides a sound theory that allows connecting theoretical equilibrium variables with observable magnitudes (*i.e.*, if TGE theory meets the first and second features above).

To begin with, let us recall that Walras’s inconsistency arises once the competitive force pushes savers to buy those capital goods yielding the highest rate of return, thereby modifying the physical composition of the stock. As a result, production of certain capital goods will be brought to an end and the datum relative to capital will no longer be persistent. One must not fall into the mistake of thinking of a persistent variable as being unchanged. Rather, as will be discussed below, persistency of the data on which equilibrium depends is a vital attribute of the long period method (feature **ii**, above).\(^8\) If this is so

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\(^8\) The given variables may be *assumed* to be fixed for the purpose of explaining the long-period position, even though they may slowly change over time (Schlicht, 1985, pp. 2-3, 19-20).
for the traditional marginalist authors who openly agreed to that method and hence used a value specification of capital (e.g., Wicksell 1934), then the same connection seems to be true also for Walras who openly admitted:

Our problem is to reach equilibrium in capital formation *ab ovo* (...) we propose to start *by assuming the arbitrary data of our problem to be constant over a certain period of time*, and subsequently we shall suppose them to change in order to study the effects of such changes. (Walras, 1954, p. 282, emphasis added).

The question that may arise is why, as has been traditionally accepted in economic analysis, the data taken by the theory must be persistent. Such an importance, indeed, must be sought in the fact that economic analysis has, since Adam Smith up to comparatively recent decades, aimed at determining theoretical variables which could serve as guide to understand real economies behavior (Garegnani, 1990, p.47). These theoretical variables, in turn, cannot be the actually observed variables at any moment, which depend, by their own nature, on a myriad of accidental factors impossible to be ascertained *a priori*. Because of the countless number of factors influencing day-by-day market variables, the theory has to focus only on the general form of the phenomenon under analysis, leaving outside its purview of analysis the concrete determinants influencing the phenomenon at any particular time.

As has been well-documented by Garegnani (1990, pp.46-47), a distinctive method of analysis in order for the theory to arrive at the general form of economic phenomena, is that exemplified by A. Smith’s analysis of the tendency of market prices towards the natural price. In that explanation Smith (1976, Book I, pp.72-74) laid the grounds on which the method of economic inquiry had to rely: “the natural price is the central price to which [market] prices are continually gravitating”. This method presupposes that there are forces which are set off by the distance between the observable magnitudes (market prices) and the theoretical variables (natural prices). It is the force of free competition that, whenever discrepancies between market and natural prices exist, will activate transfers of capitals among sectors in their search of maximum profit thus giving rise to a permanent oscillation of market variables to their theoretical counterparts. In this regard, also marginalist theory advocated this method in economic analysis. For example, Walras, when referring to equilibrium of his system, says:

Equilibrium is the normal state in the sense that it is the state towards which things spontaneously tend under a regime of free competition (Walras, 1954, p. 224)

So this is feature 1) of the long period method, which is key to relate economic theory to observable phenomena. Gravitation establishes this connection precisely because it allows analysis to assume that over a sufficient interval of time, the deviations of market variables from their theoretical equivalent will
tend to balance each other. In other words, the theoretical variables may emerge as robust guides to some average of observable variables over a sufficient interval of time during which deviations should compensate for each other. However, this mechanism implies that the forces (data) considered by the theory to determine the theoretical variables should be persistent enough in comparison with the myriad of forces that make the actual magnitudes deviate from the theoretical counterpart. That is why the long period method also includes feature ii) in order to ascertain theoretical variables that can be related to observable phenomena.9

Returning to our main argument, we can now appraise that the Walrasian notion of capital jeopardizes the traditional method of economic analysis. Indeed, changes in the datum of the capital factor entail a drastic departure from the traditional way of assuming factors endowments: i.e., constant over time, while equilibrium adjustment mechanisms are triggered.10 Let us turn our attention to the TGE in order to assess if this theory can provide a sound foundation to connect theory with observable magnitudes.

Following the theoretical path of TGE models developed by Hicks in Value and Capital (Hicks, [1939] 1946), these models envisage the economic system as being divided into different periods. Because the economic system refers to different periods, the variables will be dated according to the period to which the latter correspond. At the first period in the time span of the economy, there are initial endowments of labor, land and capital goods. Besides the endowments, the data include the available technology, preferences and the definite agents’ price-expectations. Taking these data all into account they determine prices and distribution (Grandmont, 1989). Thus, for every period (e.g., a ‘week’) a TGE would consist of a price and quantity vector such that all spot markets clear. However, such an equilibrium position comes about only under the very restrictive condition of the action carried out by an auctioneer. The roles of the auctioneer are: (i) of modifying market prices when demand and supply do not match, and (ii) of forbidding transactions until agents’ plans are

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9 The issue of persistent forces was clearly understood also by Marshall (1920 [1890], p. 291): “Thus we may conclude that ... [t]he actual value at any time, the market value as it is often called, is often more influenced by passing events and by causes whose action is fitful and short lived, than by those which work persistently. But in long periods these fitful and irregular causes in large measure efface one another’s influence; so that in the long run persistent causes dominate value completely”.

10 Professor Michael Mandler, during the URPE-AEA session where the present paper was discussed, argued that my point on the consequences of the shutdown of production of capital goods relies on my not letting the price of a capital good and its cost adjust. However, it is precisely this issue of equalizing price and costs that turns out inconsistent in Walras’s capitalization theory. In Walras’s system the endowments of capital goods will rapidly change due to the adjustment in the composition entailed by the competitive force to obtain the highest rate of return. Moreover, that change in the proportions of capital goods can take any magnitude. The reason for this is that the physical composition of capital in Walras’s system is arbitrarily given. Thus, it is totally illegitimate to take as roughly constant the endowments in Walras’s system when the process of equilibration begins, because the very same data of capital goods will be changing. Of course, in traditional theory it is clearly by letting the price of capital goods and their costs adjust that equilibrium is reached. But this means taking the given quantity of capital in value as data, which is undermined by the results of the controversy.
fully balanced. Thus, TGE models have to rule out the rather plausible time-consuming adjustments, which in general will take place in market economies, during the time span of which economic data should be persistent enough. It is clear that a TGE position for any period would lose the second characteristic of the traditional notion of equilibrium. Were it not for the auctioneer, time-consuming adjustments would cause path dependence, with the resulting indeterminacy of the equilibrium. Precisely because the assumption of the auctioneer is highly questionable, contemporary general equilibrium theorists simply assume equilibrium and focus their analysis on the dynamic sequence of TGE positions. Therefore, the system is assumed to move from one equilibrium position to the next one, always being in equilibrium. Yet, the equilibrium notion behind TGE theory cannot explicitly address the mechanisms that explain the driving of the system towards equilibrium. Indeed, before permanent forces may correct deviations to guarantee a gravitational equilibrium variable, the data of TGE will change and hence will lack the property of being a centre of gravitation. Therefore, a TGE is unable to work as a centre of gravitation, which is the first characteristic of the traditional notion of equilibrium.

On the other hand, in long-period neoclassical theory the adjustment of the composition of the capital stock is endogenous. Indeed, a value specification of capital allows the system to bring about the uniform rate of return on supply prices as a long-period position is being reached. For this to be so there must be enough time to adjust the composition of the capital stock to the composition of demand so that all capital goods required by the prevailing technology are produced. If certain capital goods are still produced with obsolete techniques, this means that in due time these capital goods will no longer be produced, while in the long-period only those capital goods produced by dominant technology will be produced.

At this point one may legitimately doubt whether TGE theory determines actual magnitudes, instead of theoretical variables in the sense explained above. Some reflection, however, will be enough to discard this alternative reading. In fact, if a price vector determined by TGE theory were comparable

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11 Bliss (1975, p. 28) acknowledged that “even if the equilibrium were to be stable there might not be enough time within the space of a ‘week’ for prices to adjust to an equilibrium [...] In the face of the foregoing problems we could regard the object of our investigations not as ‘the economy’ but as an ‘economic equilibrium’.”

12 There is a further methodological problem arising from a physical specification of capital in IGE and TGE theories. This is the “substitutability problem” (Garegnani, 1990, pp. 57-59; Petri, 2004, p. 44). Since substitution among different processes of production can be generally distinguished by employing different capital goods, the property of substitution in marginalist theory will be immediately ruled out if the physical endowments of capital goods are part of the data. But by limiting substitutability to the system, it might well happen that, even if we assume that capital theoretic problems such as reswitching do not arise, the resulting factor demand curves for capital goods will be highly rigid, yielding zero rental prices. Labor demand elasticity, for example, can be very low “entailing an analogously implausible equilibrium real wage” (Petri, 2004, p. 44).
to the actually observable magnitudes, then the data taken as given by TGE theory should result from a myriad of influences, however short-lived those influences might be. But, as sketched above, the formulations of TGE we usually meet actually aim at determining a sequence of equilibria, avoiding in this way the changes in the data that endogenously may occur. As argued by Ciccone (1999, p. 70) the usual formalizations of the TGE is inhospitable to uneven influences on the data like those we can find in traditional theory.\textsuperscript{13} It seems, therefore, that the answer to the controversy by relying on TGE models is unconvincing. But what was actually the neoclassical response to these observations?

The neoclassical reply to these inconsistencies has essentially been that of reversing the sign of the critique by arguing that the critical side was exclusively concerned with constant equilibrium variables (Bliss, 1975; see also Mandler 2002; 2005). The fact, however, is that the property of persistency of the equilibrium variables in long period theory has been interpreted as steady state positions. For example, Mandler argues that

The convergence of relative prices in neoclassical models ... obtains only in the same time frame as capital accumulation comes to a stop or a steady state is reached. (Mandler, 2005, p.18)

It is important to note that this interpretation overlooks two issues. The first question is that convergence towards a steady state\textsuperscript{14} already implies accepting the validity of neoclassical price theory. However, if this were the case, the neoclassical theory should be capable of establishing the correspondence between the price path theoretically determined with observable magnitudes, as analyzed above. Indeed, notionally, if (theoretical) prices do converge to a price path, this is because theoretically determined equilibrium variables are both stable and persistent, and hence the theory allows for a correspondence between theory and observation. Since the latter situation cannot be proved in neoclassical theory, then neoclassical scholars argued that the sequence of prices converge towards a steady state, which is tantamount to assuming the correspondence between theory and observation.

The second basic problem of the above quoted reading of convergence towards a steady state is the further misunderstanding regarding the nature of the notion of a quantity of capital as is specified in the traditional versions. Indeed, the traditional marginalist theory has to rely on a value notion in order for the system to endogenously determine the physical composition of the stock while a

\textsuperscript{13} For an analysis of why TGE theory cannot be comparable with Marshall’s notion of temporary equilibrium, see Ciccone (1999, pp. 69-72).

\textsuperscript{14} This convergence towards a steady state is normally proved by resorting to turnpike theorems (Mandler, 2002).
uniform rate of interest on new investment is being ascertained. So this notion entails that capital as a valued magnitude is a *given* which does not change in quantity but adjusts its physical form. This, however, is not precisely the notion of capital if we refer to a steady state equilibrium position, in which a constant capital is instead an *unknown* variable of the system and not a *given* as in the traditional versions. We thus conclude that the TGE theory cannot connect theory with observable magnitudes, and therefore TGE theory cannot be taken to be a satisfactory answer to *capital reversing*.

5. **Own rates of return, uniform rate of return and intertemporal prices**

As examined above, the neoclassical side tried to overcome the results of the controversy by adopting the Walrasian specification of capital in TGE and IGE models. Also, since Walras’s system suffer from the inconsistency on that notion of capital which does not allow attaining a uniform rate of return on capital supply prices, then IGE and TGE models drop this traditional condition of equilibrium. However, during the second phase of the capital debates the issue of a uniform rate of return on capital supply prices got confused with the issue of uniformity of the own-commodity rates of return (Bliss, 1970, 1975; Hahn, 1975, 1982). This section tries to clear up some misunderstanding connected with the uniformity of the own-commodity rates of return and its difference with the (traditional) rate of return on the capitals’ supply prices.¹⁵

5.1. **Own rates of return**

We consider a two-period intertemporal economy (for brevity we shall refer to these periods as \( t=0 \) and \( t=1 \), respectively). There are two goods (\( a \) and \( b \)) in this economy, each being both a consumption and a circulating capital good.¹⁶ In addition, we assume that production is viable (i.e., there exists a surplus from the pure replacement of that capital), and wages (\( w \)) are paid ex-post (after the production cycle) in terms of one commodity.¹⁷

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¹⁵ As pointed out by Garegnani (2000; 2005), confusion over the nature of the difference between uniform own-commodity rates of interest and the traditional rate of interest was one of the elements in the second phase that marred the results of the controversy.

¹⁶ This example draws on Garegnani’s (2000) model.

¹⁷ Actually there are four commodities in this economy. In an intertemporal economy there are \( nT \) commodities; \( n \) refers to the number of heterogeneous goods and \( T \) indicates the number of periods the whole economy lasts. Debreu (1959) distinguishes the commodity as to location and time. However, we limit the analysis only to time and do not consider location.
In an intertemporal framework, all contracts happen in markets at the initial moment \((t=0)\) of the first period. For instance, if we consider \(P_{a0}=1\) then \(a\) in \(t=0\) is taken as the *numéraire*. Imagine now that someone wishes to buy one unit of that good in \(t=0\) (when the contract takes place) but to be delivered in \(t=1\). Accordingly, this agent will disburse \(P_{b1}\), *i.e.*, the amount of commodity \(b\) disbursed in \(t=0\) against the promise that this agent will receive one unit of that good in the future. This is equivalent to having this agent giving up one unit of \(b\) in \(t=0\) in order to consume \(1/P_{b1}\) in \(t=1\). Or, what is the same, the operation is tantamount to having someone lending the amount \(P_{b1}\) of \(b\) in \(t=0\) to some other agent, who will pay that debt back at the following period by providing one unit of \(b\). It is apparent that this operation can be regarded as a loan. This operation entails a rate of interest which we can call the own-commodity rate of interest (in this case, the rate of interest in terms of good \(b\)).

Assume, for instance, that \(P_{b1}=10/11\). This is the amount of commodity \(b\) to be disbursed in \(t=0\) in order to receive one unit of that good in \(t=1\). A loan in terms of \(b\) has been made in \(t=0\), and the rate of interest there involved is ten per cent:

\[
P_{b1}(1 + r_b) = P_{b0} \text{ hence } r_b = 10\%
\]

So, in general, own-commodity rates will be defined by:

\[
r_a = \frac{P_{a0}}{P_{a1}} - 1
\]

\[
r_b = \frac{P_{b0}}{P_{b1}} - 1
\]

### 5.2. Uniform rates of return in intertemporal equilibria

Having defined the two rates of interest in terms of commodity \(a\) and \(b\), a very important question arises: What is the form that the condition of a uniform rate of return on the capital goods’ supply prices takes if equilibrium relative prices change over time?

Now assume that one unit of commodity \(a\) in \(t=0\) has a value of 2 (*i.e.*, one unit of \(a\) is contemporarily exchangeable by 2 units of \(b\)). Assume further that paying for \(a\) at the moment of its delivery \((t=1)\) it would cost 1.8 (notice that we are assuming this price of good \(a\) both to be decreasing and to be undiscounted). The question is how to determine \(P_{a1}\), *i.e.*, the discounted price over time.
Before dealing with that problem, we have to face the issue regarding the (traditional) uniformity of rates of interest. In effect, it will first be necessary for us to consider that basic condition: the rate of interest, in equilibrium, cannot be different for the different loans made in terms of the different commodities. Indeed we can obtain, by lending $\frac{10}{11}$ units of $b$ in $t=0$, one unit of itself in $t=1$, the rate of interest there involved being ten per cent.\textsuperscript{18} Therefore, such a rate of interest will have to be the same as in the case we could obtain $\frac{10}{11}$ units of $b$ by renouncing to the acquisition of commodity $a$ in $t=0$, and with that quantity of $b$ ‘saved’ (since the $\frac{10}{11}$ units were not disbursed) buying commodity $a_1$ in that period. In that period ($t=1$) we assumed that one unit of $a$ costs 1.8 units of $b$, therefore, in order to know how much we must pay in $t=0$ for obtaining one unit of $a$ (and to be delivered in $t=1$ namely – $P_{a_1}$), we must discount 1.8 at the rate of 10%. Therefore

$$P_{a_1}=1.636$$

This will be the value of commodity $a$ in terms of $b$. In other words, this price spots the amount of $b$ obtained in $t=0$ by selling one unit of $a$ which will be delivered in $t=1$. That amount of $b$ could be lent to someone, then obtaining an interest rate of 10% when the borrower pays back in $t=1$ one unit of commodity $b$. On the other hand, since we know that $P_{a_0}=2$, then $r_b=22.2\%$.

It is clear from the above that the different own-commodity rates of interest will vary if relative prices change, as is the case in neo-Walrasian models. Moreover, the own-rate corresponding to the good $a$ is higher than $b$’s, for the relative value of the former commodity to the latter is assumed to be decreasing over time. On the other hand, if the loan were made in terms of $b$, while its relative price increases over time, it will follow that to compensate for those relative advantages (of the lender) and disadvantages (of the borrower) $r_b$ would have to be lower (and $r_a$ higher) as shown in the example. This is no more than a simple triangular arbitrage operation.\textsuperscript{19}

The above example also shows us that both own-commodity rates of interest would have the same value if and only if $P_{a_0}/P_{a_1}=P_{b_0}/P_{b_1}$, that is, when both intertemporal prices are equal, from which it can be deduced $P_{a_0}/P_{b_0}=P_{a_1}/P_{b_1}$; i.e., when both relative contemporary prices are equal. In general, however, relative prices will tend to be different and so will the own-commodity rates of interest. So, when equilibrium prices change over time the condition of a uniform rate of interest on the

\textsuperscript{18} Since commodity $b$ in $t=0$ is the numéraire by assumption ($P_{b_0}=1$), then both its own rate of return and the (traditional) uniform rate of return will coincide. It is clear, therefore, that that equality will entirely vary according as which good is chosen as numéraire.

\textsuperscript{19} In any intertemporal economy there are as many own-rates of interest as are goods for each pair of ‘periods’. In our case, we have two own-rates since we have two goods ($a$ and $b$) for only one pair of periods. In general, we can connect two generic own-rates by the following condition: $(1+r_{i(t,t+1)})/(1+r_{j(t,t+1)}) = (P_{i(t)}/P_{i(t+1)})/(P_{j(t)}/P_{j(t+1)})$ where $i$ and $j$ are two generic commodities. Cf. Bliss (1975, pp. 53-56).
capitals’ supply price will in general entail different own-commodity rates of interest. Accordingly
it is convenient to call the former uniformity of rate of interest as the effective rate (Garegnani, 2000).

As we saw above, the Walrasian specification of capital is not compatible with the traditional or
long-period notion of equilibrium. Yet in Walras’s equations we can explicitly see the equilibrium
condition of the uniformity of the effective rates of profit on the capitals’ supply prices. This is
present as soon as we recognize in those equilibrium price equations the equality sign between
demand and supply prices of those capitals. In fact, equalizing capital goods’ demand prices to
their corresponding supply prices, as Walras did up to his third edition of his Éléments, is
tantamount to assuming i) production of all capital goods and ii) the introduction of the traditional
condition of uniformity of the rate of return. But, because of assuming capital in physical terms,
Walras was compelled, in the fourth edition, to admit solutions with price equations bearing
inequality signs, instead of equality signs. This leads to some capital goods not to be produced. The
dropping of the uniformity of the effective rate of profit is apparent when we recognize the
inequality sign in the price equations of those versions of the theory, which is usual in formal
presentations of neo-Walrasian models. (Hahn, 1982).

5.3. Hahn’s (1975) ‘special case’

Having clarified the issues behind the use of Walrasian capital in general equilibrium models, let us
move on to some historical aspects in the second phase of the controversy. Indeed, the issue of
Walrasian capital and the criticism of that theory of capitalization was first raised by Garegnani
(1960) and later spread by Joan Robinson (1970). Robinson’s views were then echoed by Harcourt
(1975) whose arguments became the target of Hahn’s (1975) attack. The issue, not to be forgotten,
is the uniformity of the effective rate of return. The following message by Harcourt – though
literally echoing Robinson’s (1970) words – reflects Garegnani’s critique of Walrasian theory:20

20 It is perhaps useful to recall that Walras’s treatment of capital was thoroughly examined by Garegnani in his
Cambridge PhD dissertation in 1958 (see Garegnani 1960), and some years later taken up again to become in a
manuscript (1962), which was published only recently (Garegnani, 2008). The argument was known by Joan
Robinson, who was the internal examiner of Garegnani’s thesis. Also it must be noted that one of Robinson’s
closest collaborators, John Eatwell, made the reading of Garegnani’s dissertation almost compulsory for his
students. (I owe this information to the late Professor Pierangelo Garegnani and also to Prof. Geoff Harcourt).
Therefore, Robinson’s 1970 claim, that Walras does not have any theory of the profit rate can be taken as an
echo of the inconsistency between the impossibility to achieve a uniform rate of return on the supply prices
and Walrasian capital. It might be noticed though that the form chosen by Robinson to refer to the traditional
authors who conceived of capital in value terms – ‘Walrasian leets’ – might have misled participants in the
controversies, when it is well known that Walras does not explicitly use capital as a single magnitude.
The Walrasian model of general equilibrium has the implications that it is not possible to have a theory of the rate of profits. Harcourt (1975, p. 350).

No doubt, the impossibility in the “Walrasian model” referred to in that passage clearly means the non uniformity of the effective rate of return on capitals’ supply prices. This is what Hahn (1975, p. 360) took up in his reply to Harcourt (1975) to argue that this issue would only affect a “Special case” of the theory under attack.

Hahn’s contention can indeed be derived from what this author says in the following passage:

General equilibrium theory is general and so can discuss the equilibrium of an economy whatever its ‘initial conditions’, e.g. outfit of goods inherited from the past. For most such specifications it will not be the case that the equilibrium price of a good for future delivery in terms of the same good for current delivery will be the same for all goods. Hahn (1975, p. 360).

It is apparent that Hahn’s “special case” cannot but be the result of a confusion involving the two kinds of uniformities of the rate of interest that we have previously discussed. Indeed, according to Hahn, the impossibility of determining the effective rate of return owing to a Walrasian specification of capital is confined to a ‘special case’ in which relative intertemporal prices are all equal over time. If we recall the example we have previously used, Walras’s inconsistency would only affect the ‘case’ such that \( P_b1/P_b0 = P_a1/P_a0 \) (“the equilibrium price of a good for future delivery in terms of the same good for current delivery will be the same for all goods”). In other words, the original inconsistency would only occur when the own-commodity rates of return happen to be equal owing to the constancy of equilibrium prices over time. However, we have seen that the effective uniformity takes the form of non-uniform own-commodity rates of return when changes in the equilibrium prices are admitted.

Yet, analysis of Hahn’s passages reveals a further issue worth discussing. According to Hahn, ‘general equilibrium theory’ is the neo-Walrasian theory. This claim would however rule out that Walras’s inconsistency affects that ‘general theory’. But since neo-Walrasian theory shares with the original Walras’s model the datum of the physically heterogeneous capital stock, Walras’s inconsistency does affect that ‘general theory’, independently of whether we consider changes in equilibrium prices over time. Hahn thus misunderstands (Hahn, 1975, p. 360) the effective uniformity of the rate of return of “Wicksell (and others)” as the uniformity of own-commodities rates of return due to constancy of prices. The condition of uniformity will be present in the new formulations of the theory as soon as we reckon in the price equations of the capital goods the equality signs.
To rebut criticisms in this second phase of the controversies, Hahn also attempted to reverse the sign of the critique by arguing that the effective rate of return in a ‘Walrasian model’ is actually not an issue:

The crudest empirical observations will convince one that there is no unique rate of profit to be observed in the economy. Do we conclude from that that competition is functioning badly? Answer: No. Consult any general equilibrium text. Why is ‘the rate of profit’ an interesting unknown? Ask the neo-Ricardians. If we knew all relative prices from now to doomsday, could we ask more? Hahn (1975, p. 361).

Note that Hahn has apparently tried to show the existence of divergences in the own-commodity rates in reality, because if we know “all relative prices from now to doomsday” we shall also know future price changes. However, Hahn’s observations would be actually pointing to different rates of return in the different business sectors compared in terms of money or other numéraire. But, then, this divergence in rates of returns will have nothing to do with divergences of own-commodity rates. In fact the latter will be entailing the effective rate of returns expressed in different numéraires if the price of the numéraire is changing. Therefore, Hahn’s disputations shown in the passage misinterpret the divergence between own-commodity rates of returns: own-commodity rates may diverge from each other when prices change over time, while the effective rate is being ascertained when the physical composition of capital is adjusted. On the other hand, if the effective rate of return cannot be ascertained, due to Walras’s inconsistency entailed by the arbitrariness of the capital endowments, we may still have uniformity in the own-commodity rates whenever prices are assumed to be constant over time. Thus, Hahn downplays the essential role of the uniformity of the effective rate of interest because he believes that one could not “ask more” if “all relative prices” are known up to the end of the world.

5.4. Uniform rate of return in IGE models is not a uniform rate of return on the capital supply prices

As seen above, a uniform rate of return on all investment in an IGE can be obtained by means of choosing an appropriate numéraire. The possible uses of savings in an intertemporal system can be viewed as being equally profitable, and this finds expression in the equality of rates of interest in value terms once a numéraire has been chosen. On the other hand, in traditional neoclassical theory the uniform rate of return has to be on supply prices. In other words, to the rate of return in relation to the employment of savings which holds in an IGE the demand price of capital goods (i.e., the capitalization of future rentals) must be equal to the costs of production (see section 2). But, since the endowments in IGE and TGE models remain exogenous, the abovementioned condition of
demand price of capital good being equal to costs of production cannot be ascertained, as we have seen for the case of Walras. Indeed, if we focus on the IGE, it is in the very first period (or the first periods) in which such equilibrium condition should be verified, but it usually cannot. For example, if, because of the given endowment a certain capital good is so abundant initially, its demand price will fall below its costs of production. This arises due to its marginal product being very low (due to its relative abundance) and therefore the capital good will not be reproduced later as its rental will be zero.

Besides the methodological problems discussed above, it is clear that neo-Walrasian theory has to resort to a number of incredible assumptions in order to do away with problems that the presence of capital in IGE models may eventually create. Indeed, the question for neo-Walrasian authors has been how to accommodate capital goods in formal models where the economic system is formally treated as being an acapitalistic general equilibrium model of production and exchange. In those models production of capital goods is in fact done away with because the capital goods actually are formally treated as non-produced factors, like land or labour (Petri, 2009). Further, a more thorough assessment of IGE models should also cover further assumptions that are usually made in their most known formulations (Debreu, 1959). Just to name the most relevant: i) complete future markets (which do not exist in actual economies); ii) very rapid adjustment to equilibrium, hence equilibrium devoid of persistence and hence of doubtful use for interpreting real-economies behavior; iii) the indefensible assumption that the time horizon in IGE models is finite. This latter assumption is done in order to formally get rid of the savings investment markets: because the economy ends at some date T, there will be no need to produce capital goods in T-1, hence no need to analyze savings and investment behavior. As argued by Garegnani (2000; 2003), new versions of the neoclassical theory try to do away with any possible source for problems of stability or multiple equilibria to arise in the presence of savings investment markets.

6. Conclusion

We have revisited some issues underlying the Cambridge controversy in what I call its second phase. In this paper it has been shown that during this phase the defense of neoclassical theory was exclusively conducted in terms of Walrasian capital as introduced in the IGE and TGE models. The physical specification of the capital endowment entails the impossibility to determine theoretical equilibrium variables that satisfy the uniform rate of return condition. Thus, because the theory cannot rule out a permanent drive (i.e., the search for highest returns) for equilibrium to change constantly, equilibrium cannot be a persistent position and therefore a center of gravitation. As we
have argued, this strategy pursued by neoclassical theorists to sidestep the implications of capital reversing proved inadequate as their redefinition of the notion of equilibrium cannot provide a sound theory through which a correspondence between theoretical and observable variables can be established. It thus seems that economic analysis paid a high price for saving modern supply and demand theory in the wake of the unambiguous results of the controversy. Falling back on Walrasian capital in the IGE and TGE models, which only formally do away with the most apparent form of the issue of capital in value, actually entailed renouncing the determination of long-period theoretical variables that can be employed for analysis of empirical economies. Of course, returning to the long period method by restoring the neoclassical versions relying on capital in value would be to fall back on the inconsistencies pointed out fifty years ago. It is perhaps time for economic analysis to sweep away the different formulations of neoclassical theorizing and to finally revive a heterodox, Classical price and distribution theory which can determine long period variables on robust theoretical and empirical grounds.

References


