EFFECTIVE DEMAND AND PRICES OF PRODUCTION: AN EVOLUTIONARY APPROACH

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ABSTRACT

The paper develops a critique of and a (partial) solution to *Say's Law Marxism,* defined as the employment of Say's Law within the Marxist framework. The critique is based on the notion that expectations and effective demand are constitutive of labor values and prices of production. Socially necessary labor cannot be determined solely from the supply side. The (partial) solution to Say's Law Marxism rests on the development of an evolutionary model that integrates the principle of effective demand in the short run and the formation of prices of production in the long run. The equalization of profit rates in the long run and the formation of prices of production as attractors of market prices depend upon the realization of profits in the short run and, therefore, are subject to the principle of effective demand. The model employs replicator dynamics from evolutionary game theory to formalize competition at the firm level. The replicator equations also formalize the adoption of new production techniques and the nonlinear feedback effects between individual decisions and unintended macroeconomic outcomes.

Key words: Effective Demand, Prices of Production, Marx, Keynes, Kalecki

JEL codes: B51, C73, D20

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1. Introduction

Despite Marx's criticisms of Say's Law in the early drafts and many editions of Volume I of *Capital*, Say's Law remained one of the assumptions in the model of accumulation, technological progress, and competition of Volume III. Say's Law has also been a regular feature within the Marxist branch of Political Economy, often through the assumptions that the values created in the productive sphere are fully realized, that savings or surplus value precede investment, or that the realized rate of exploitation is exogenous. In this sense, I label *Say's Law Marxism* the employment of Say's Law within the Marxist framework.

The paper offers a critique of and a (partial) solution to Say's Law Marxism. The critique against it rests on the idea that Say's Law obscures the concept of value in Marxist theory, for it removes the demand side from the determination of socially necessary labor and of prices of production. Say's Law Marxism gives the false impression that commodity values can be derived from the production sphere regardless of the ex-post realization of values, therefore abstracting from demand and the role of expectations. The paper will demonstrate that, on the contrary, expectations and effective demand are constitutive of labor values and prices of production. Socially necessary labor and prices of production cannot be, and should not be determined solely from the supply side.

The partial solution to Say's Law Marxism rests on the partial integration of the Keynesian-Kaleckian principle of effective demand into the Marxist theory of value. The Marxist branch of the literature tends to focus mostly on technology and supply-side factors in the determination of values and prices of production. The Keynesian-Kaleckian branch, on the contrary, tends to focus on the demand side but does not have a theory of value as Marx does. The paper (partially) bridges this gap by partially integrating Marx's value theory and the Keynesian-Kaleckian theory of effective demand and expectations.

The adjective *partial* is necessary since my approach applies to a competitive economy with free capital mobility between two sectors, with no fixed capital or joint production, and neither unemployment nor business cycles. It therefore does not cover the much more relevant and realistic cases of imperfect competition in a multi-sector economy with fixed assets, joint productions, and the possibility of unemployment and business cycles.

More specifically, the paper develops an evolutionary model that combines the principle of effective demand in the short run and the formation of prices of production in the long run. The incorporation of the Keynesian-Kaleckian principle of effective demand within the Marxist framework offers a better theory of how expected demand determines the realization of values, the rates of exploitation, and the rates of profit in a competitive economy.

In Keynesian and Kaleckian theories, the principle of effective demand states that, given the technology and cost structure of firms in the production period, the *expected* level of revenues determines the current level of production and employment, regardless of whether or not these expectations are correct. The current level of production, profits, and employment can influence the expectations of future profitability as well as the confidence in such expectations. But, ultimately, it is the level of expectations and the confidence in these expectations that determine the current level of production, profit, and employment. Expectations are therefore partially autonomous from the current level of profitability, even though the current level of profitability is fully determined by the level of expectations.

In Marxist theory, a competitive economy with free capital mobility tends to equalize the rates of profit in the long run, adjusted for the corresponding levels of risk. In the absence of monopoly power, market prices gravitate around their prices of production (or natural prices in Adam Smith's terms), which are long-run prices under equalized profit rates. The equalization of profit rates and the formation of prices of production in the long run, however, are both dependent upon the realization of profits in the short run and, hence, are subject to the principle of effective demand. Current and past levels of profitability certainly exert an influence on the expectations of future profitability. But given the technology and cost structure of firms in the production period, the *expected* level of profits determines the level of expenditures that realizes the current profit rate. In this way, the partially autonomous *expected* profit rate ultimately determines the current realized profit rate. Expected demand thus matters in the determination of both values and prices of production.

The evolutionary model in the paper formalizes a competitive economy with technical change and free capital mobility. Within each production period, given the technology and cost structure, the expected profit rates determine the realized profit rates. In the long run, the free flow of capital across sectors equalizes the rates of profit. Expected demand determines the expenditures at the beginning of each production period, which then realize the values previously produced. By realizing the values of the commodities produced, the expected demand and the beginning-of-period expenditures associated with it also realize the surplus values and rates of exploitation. Profit rates then equalize across sectors if profitability responds negatively to the capital committed to production in each sector. Oversupply in one sector will erode profits in that sector and firms will search for more profitable opportunities elsewhere.

Once effective demand is brought into the Marxist framework, the distinction between the *source* of value and the determination of the *quantity* of value becomes crucial. In this paper I take the stance that *value* is the social form of labor in capitalism, whose quantity is determined by the abstract labor time *socially necessary* to reproduce commodities. As Rubin ([1928]1972) noted, the term "socially necessary" has a double meaning. It means "socially necessary" in the supply-side sense that the existing technology and cost structures determine what and how firms can produce. But "socially necessarily" also has a demand-side dimension under which value needs social validation to become value in the first place. Hence, the "socially necessary" character of value determination must be understood at the intersection of supply and demand aspects. There is no such a thing as non-realized values, for value must be realized to be conceptualized as value. Value that is not realized is not value: "It is only by being exchanged that the products of labour acquire a socially uniform objectivity as values" (Marx [1887]1990, p.166), as the "different kinds of individual labour represented in these particular use values, in fact, become labour in general, and in this way social labour, only by actually being exchanged" (Marx [1859]1989, p. 286).

The model presented in this paper employs replicator dynamics from evolutionary game theory to describe the competitive selection that occurs simultaneously at the micro intra-sector and at the macro inter-sector levels. Firms compete by both moving production across sectors and by adopting cost-reducing technologies within each sector, amid nonlinear feedback effects between individual decisions and unintended macroeconomic outcomes. The replicator dynamics describes an updating process with random interactions in which behaviors with higher payoffs proliferate. It is a useful device to mimic the competitive struggle for survival in natural and social environments, for it models the process of equilibration by tracking the results of individual interactions (Bowles 2006; Prado 2006, 2002). Even though rationality is not bounded in a replicator system, agents have limited and localized knowledge and, hence, the distribution of information does remain bounded (Gintis 2009, p.273).

The model combines the principle of effective demand from Keynes and Kalecki, the theory of value from Marx, and the evolutionary approach to modelling from game theory. In this way, the model can (partially) circumvent the limitations of Say's Law Marxism and thus offer a better understanding of how expected demand determines the behavior of profit rates and prices of production in a competitive economy.

2. Comparison to Previous Studies

The paper makes the following claims: (i) the Keynesian-Kaleckian principle of effective demand offers a better approach to the determination of labor values and the short-run equilibrium in an extended Marxist framework; (ii) the Keynesian-Kaleckian principle of effective demand is compatible with equalized profit rates and prices of production operating as long-run attractors of market prices in a competitive economy with free capital mobility; (iii) the nonlinear adjustments in the replicator dynamics make the long-run stationary state stable over a wider range of parameter values compared to models with exogenous adjustment coefficients. The present section explains these claims in more detail and compares my approach to previous studies.

The principle of effective demand remains a debated concept even within the post-Keynesian tradition. I follow Chick (1983), Hayes (2019; 2007), Hartwig (2007), Allain (2009), and Casarosa (1981) in their understanding of effective demand as the firms' effective commitment to production. Given the technology and cost structure, effective demand refers to how *expected* profitability determines supply and employment decisions at the *beginning* of the production period, whether or not these expectations are correct. Hence, the *ex-ante* commitment to produce is not identical to the *ex post* aggregate expenditures with consumption and investment. Effective demand is the firms' profit-maximizing expected proceeds, an *ex ante* concept relating to expectations but revised in line with *ex post* realized incomes. Even though aggregate demand and aggregate expenditure are not identical, they can be equal when *ex ante* expectations are fulfilled *ex post*, but not otherwise. Therefore, "effective demand is an unfortunate term, for it really refers to the output that will be supplied; in general there is no assurance that it will also be demanded" (Chick 1983, p.65).

Marx, in Volume III of *Capital*, took account of the role of aggregate expenditures with consumption and investment in the realization of values in the following way:

> The conditions for *immediate exploitation* and for the *realization of that exploitation* are *not identical*. Not only are they separate in time and space, they are also separate in theory. The former is restricted only by the society's productive forces, the latter by the proportionality between the different branches of production and by the society's power of consumption. And this is determined … by the power of consumption within a given framework of antagonistic conditions of distribution […]. It is further restricted by the drive for accumulation, the drive to expand capital and produce surplus-value on a larger scale (Marx [1894]1994, p.352-353 – emphasis added).

In an extended Marxist approach that includes the roles of expectations and expected demand, the principle of effective demand means that the *expected* profit rate determines the firms' constant and variable capitals advanced at the *beginning* of the production period, as well as the firms' *supply* of commodities in the current production period. Market prices can change during the production period, but output changes only in the transition from one production period to the next. The beginning-of-period expenditures, which comprise the firms' effective commitment to production, will then realize the values created at the *end of the previous* production period. Because the economy is structured as a chain of production periods (or circuits of capital as Marx put it), the *ex-ante* aggregate demand at the beginning of a production period is also at the same time the expenditure that realizes *ex post* the values created in the preceding production period.

The principle of effective demand, in this regard, determines the causality between the core elements of Marxist theory in the opposite direction from that of Say's Law (Trigg 2006). The causal direction between aggregate demand and the profit rate offers the best example. A substantial branch of the Marxist tradition would assign the profit rate as the cause and demand as the effect, which amounts to deploying Say's Law and making the economy supply led. Discussions of the tendency of the profit rate to fall feature this type of reasoning (as in Moseley 2016; Kliman and McGlone 1999; Freeman and Carchedi 1995; Nikaido 1985,1983; Okishio 1961, 2001; Duménil and Lévy 1995; Steedman 1977). In models of business cycles or underconsumption, on the contrary, the profit rate is the effect and aggregate demand is the cause, in which case the profit rate becomes itself endogenous to demand (as in Dutt 2011; Shaikh 1989; Foley 1983, 1985).

Kalecki then built on Marx's insights to claim that the volume of real aggregate gross profit is determined at the macro level by the capitalists' aggregate expenditures. Assuming workers do not save, capitalists cannot realize more surplus value in the aggregate than their own expenditures (Sardoni 2011; 2009; 1989). No matter how large the rate of exploitation in the production sphere, the capitalist class can only realize a rate of exploitation that makes total profits match their own expenditures. Expenditures are partially autonomous from surplus value, but surplus value is not autonomous from expenditures. Given the technology and the cost structure of firms in the production period, the firms' expenditures fully determine the level of surplus value realized.

There is an important distinction, however, in how Marx, Keynes, and Kalecki derive their macro results from different microfoundations (Sardoni 2011). In Marx, companies operate in a competitive market with free capital mobility. In the short run, marginal costs are either constant or decreasing in the relevant range, and the firms' demand curves are flat at the market price. Profit maximization therefore implies output maximization. Firms always produce to maximum capacity and invest as much as possible as long as the market price is above the average variable cost. Otherwise, if the market price is below the average variable cost, firms will not produce or invest at all. Firms operate at one of two extremes: they either produce and invest as much as possible or they do not produce and invest at all. Keynes, on the contrary, stayed as close as possible to the Marshallian model of short-run perfect competition under increasing marginal costs and increasing supply curves. Because marginal costs are rising in the relevant production range, and firms' demand curves are flat at the market price, profit maximization does not imply output maximization.

Kalecki, unlike Keynes and Marx, combined imperfect competition and marginal cost curves with an inverted-L shape: firms set a markup on prime costs that are flat up to capacity but increasing rapidly close to full utilization. Production is restricted by market shares (downward-sloping individual demand curves) and credit rationing by banks, while firms usually keep some degree of capacity underutilization even in the long run. Because of imperfect competition, entry barriers, and the fixed markups, profit rates cannot equalize, and prices of production cannot operate as gravitational centers for market prices unless the degree of capacity utilization of firms varies endogenously. Equalized profit rates are compatible with differential markups if capacity utilization is endogenous in the long run (Duménil and Lévy 1999, p.696; Vianello 1989).

The Keynesian-Kaleckian literature is further divided in terms of the short-run adjustment between supply and demand (Kurz and Salvadori 2010), which can take place via: (i) changes in output, with exogenous markups per unit of output (Kaleckian version); (ii) changes in prices and profit rates for a given level of output (Kaldor-Pasinetti-Robinson version); or (iii) changes in both output and prices.

The model in this paper employs both price and output adjustments in an extended Marxist approach with full capacity utilization in which the beginning-of-period expenditures on wages and means of production are advanced capital, set at their nominal levels at the start of each production period. The model combines price adjustments *within* the production periods (ensuring market clearing for given outputs) and output adjustments *between* production periods (ensuring that output expands to meet the new levels of demand across sectors, for given prices). For Keynes and Kalecki, different expected levels of demand imply different levels of production at given capacity within the period of production. In my Marxist model, even though firms always produce to capacity within the production period, the level of maximum capacity changes from one production period to the next based on the level of effective demand and on the fact that firms are moving (and most likely expanding) their capacity across sectors *between* production periods. The supply equations in my model assume that firms produce at maximum capacity *within* each production period, but *between* production periods the level of maximum capacity expands or contracts in response to the level of effective demand and the mobility of firms across sectors.

The gravitation of market prices around prices of production has also been the subject of rigorous studies in the literature. Boggio (1985; 1990) notes that the equalization of profit rates and the convergence to production prices depends on the nature of time (continuous versus discrete) and on the nature of pricing (excess demand pricing versus fixed markups on costs). Continuous time models tend to focus solely on contemporaneous effects, while discrete time models can include complex lagged effects across variables. Fixed markup models are easier to handle mathematically and better suited for sectors in which cost changes are more important to prices than changes in demand, even though fixed markup models do not usually have a theory of long-run pricing. Boggio argues that continuous time models (as in Flaschel and Semmler 1985, 1987; Franke 1988) should be dropped as they portray production periods as lasting an infinitesimal amount of time. Discrete time models, however, tend to have unstable long-run equilibria while continuous-time models tend to be stable. Production prices can operate as attractors of market prices if consumption out of profit income is allowed for, reaction coefficients are not too large, and price substitutions effects are large enough. In a similar model to Boggio's, Duménil and Lévy (1989) add inventories and capacity utilization, but stability holds only for very small reaction coefficients of prices and output in relation to excess demand.

Boggio (1990), in particular, argues in favor of an alternative definition of prices of production. Instead of prices that equalize profits at a uniform rate across sectors, which is a feature not observed in reality, prices of production should be interpreted as long-run equilibrium prices associated with profit rates that *do not induce net movements of capital* across sectors. Under this alternative definition, the concept of prices of production would cover not only perfect competition with free capital mobility but also markets with imperfect competition in which barriers to entry exist and net capital flows take place only when profit differentials rise above certain thresholds.

Nikaido (1983; 1985) shows that in a Marxist framework in continuous time with consumption out of profit income, the long-run equilibrium is unstable if the organic composition of capital is higher in the capital good producing sector relative to the consumption good producing sector, but stable otherwise. Nikaido's mathematical proofs, however, depend on two assumptions. First, labor values are defined in a *dual system* in which labor values are independent of market prices, an approach that Steedman (1977) showed to be logically inconsistent and inadequate as a theory of prices and profits. Second, Say's Law is assumed to hold: "accumulation of capital through investment of part or all of surplus value" (Nikaido 1985, p.197), or "investment, the driving engine of the scheme, that is, allocation among sectors of the savings from surplus value" (Nikaido 1985, p.198), or "savings is invested to increase the money capital" (Nikaido 1985, p.200). At the aggregate level, however, how can surplus value be realized prior to reinvestment expenditures? Nonetheless, Nikaido's (1983; 1985) stability results are confirmed in my simulations: a high technical composition of capital in the capital good sector tends to make the long-run equilibrium unstable.

Kubin (1990) develops a model of prices of production in discrete time but assumes that the long-run profit rates and the growth rate are exogenous. More importantly, Kubin notes that the introduction of lagged effects in the reaction functions in a discrete time system increases the range of parameter values within which the long-run equilibrium is stable. Steedman (1984) notes that industries with more fixed capital will have less mobility and slower adjustment towards equalized profit rates, and that prices at which goods are purchased at the beginning of a production period may differ from those at which goods are sold for at the end of the production period. More importantly, Steedman makes two key arguments. First, with three or more commodities, a sector in which the market price is higher than its price of production could have a rate of profit below average if the market prices of its inputs where proportionally much higher than their respective prices of production. The solution in this case is to link the output levels in each industry to the profit rate differentials rather than linking output levels to market prices. Second, Steedman notes that under economies of scale the long-run prices of production become endogenous to the level of output. This is an important observation given that the literature on prices of production and profit rate equalization always assumes a technology with fixed coefficients and constant returns to scale.

Duménil and Lévy (1995) model the convergence of market prices toward prices of production amid stochastic technological change. However, supply grows around an exogenous growth rate and "a form of Say's Law is, therefore, assumed for simplicity" (p.400), hence "aggregate realized profit (as in sales) is equal to aggregate appropriated profit (as in production)" (p.403). Duménil and Lévy (1999) develop a traverse model in which autonomous demand matters in the short run but reverts to Say's Law in the long run. Because the traverse model is constructed in logical time, the short and long runs are logical states, and it becomes difficult to understand how the economy would make the transition from short- to long-run equilibria in historical time. Duménil and Lévy (1993, chapter 6) is one of the very few studies acknowledging that the theory of effective demand offers a superior result compared to Say's Law. They show that models that use the realized rates of profit instead of the "appropriated rates of profit" tend to have stable long-run equilibria. The use of realized profit rates implies that sectoral revenues are computed based on quantities purchased, not based on quantities produced, which counterbalances the destabilizing forces that would push market prices away from prices of production.

Prado (2002) shows that the replicator equation can be used to model the gradual adoption of new techniques of production, and Prado (2006) shows that the replicator equation can also be used to model the movement of firms across sectors and the formation of long-run natural prices à la Adam Smith. More recently, Cockshott (2017) demonstrated using computer simulations that market prices that ensure balanced reproduction across sectors are not necessarily the set of prices that can also ensure profit rate equalization. My model incorporates the game-theoretic insights from Prado and, as Cockshott, it ensures that the within-period price adjustments and between-period quantity adjustments imply both balanced reproduction as well as equalized profit rates. Foley (2018) and Cogliano (2011) further argue that if free capital mobility leads to the equalization of profit rates, then free labor mobility and competition among workers should lead to the equalization of the rates of exploitation. In section 8 I explain how my model would behave with equalized rates of exploitation.

Bellino and Serrano (2018) show that most of the models developed in the 1980s and 1990s shared a common caveat, namely that changes in relative output levels cause changes not in the levels of market prices but in the rates of change of market prices. Building on previous work by Lippi (1990) and Garegnani (1997), Bellino and Serrano demonstrate that the undue focus on the rates of change of prices is responsible for the instability of the long-run equilibrium and the associated price overshooting that prevents the convergence toward prices of production. For profit rates to equalize, excess demand needs to induce an adjustment in the levels of prices rather than in their rates of change. Without this correction the short-run level of output will overshoot its long-run equilibrium level, since the adjustment of relative prices is disconnected from the long-run level of natural output, implying that there is no mechanism that brings back market prices to prices of production when output reaches its natural level. But even with the proposed correction, Bellino and Serrano find that the convergence to natural prices depends on the size of the reaction coefficients being sufficiently small.

Like Bellino and Serrano (2018), my model makes the levels of prices, not their rates of change, depend upon the level of output, thus removing this source of instability. Furthermore, I also follow

Duménil and Lévy (1993, chapter 6) by replacing the supply-led "appropriated rate of profit" with the demand-led realized profit rate, thus removing Say's Law as another source of instability. Unlike Nell (1998), my model does not suppose that price or quantity adjustments occur because of deviations of prices from natural prices, or of output from natural output, or of profit rates from the natural profit rate, for the simple reason that in decentralized competition no one knows what the natural prices, natural outputs, and natural profit rates are a priori.

In the literature it is often assumed that adjustment coefficients are exogenous parameters, which need to be sufficiently small to ensure that the fixed point is a stable attractor. In contrast to the literature, my model based on replicator equations has no exogenous adjustment coefficients. The replicator equations include the term $p_t(1-p_t)$, where p_t is the share of a certain trait in the population at time t, and this term has a nonlinear sigmoid shape like a logistic equation. The term $p_t(1-p_t)$ therefore operates as an endogenous nonlinear adjustment coefficient that reaches maximum speed at $p_t = 50\%$. But its nonlinear sigmoid shape also reduces the adjustment speed at the extreme values of p_t (close to 0% and 100%) and thus contributes to make the stationary state stable under a wider range of parameter values.

Okishio (1961; 2001) had originally demonstrated that in a Marxist framework with an exogenous real wage, technical change leads the average profit rate not to fall but in fact to rise. Okishio's theorem is valid, however, only if labor productivity rises faster than the real wage, such that the fall in the wage share is translated to a rise in the profit share and in the profit rate (Basu 2019). In my approach, on the contrary, the real wage is endogenous to aggregate demand and, hence, the Okishio theorem no longer holds. Technical change can reduce profitability over the long run even if it temporarily raises profit rates for the initial adopters of the new cost-reducing technology. If the gains from technology are channeled to higher real wages, and real wages rise faster than the productivity of labor, then average profitability will decrease over time despite an initial spike for early adopters.

In the literature on the relations between prices and labor values, Steedman (1977; 1992) proved that a *dual system* in which labor values are independent of market prices produces inconsistent and meaningless results such as, for example, profit rates and rates of exploitation computed in price terms that differ substantially from profit rates and rates of exploitation computed in terms of labor values. Labor values, Steedman concluded, are not a priori quantities from which market prices can be derived. This important result led to the development of *single systems* in which labor values and market prices are interdependent and linked directly to each other through the monetary expression of labor time (MELT), defined as the ratio of the nominal net product to the total amount of labor hours in productive activities (Foley 2000, 2018; Mohun 1993; Wolff, Callari, and Roberts 1984; Kliman and McGlone 1999; Duménil and Lévy 2000). In a single system, the rates of profit and the rates of exploitation are the same whether computed in labor values or in market prices, and surplus value maps into gross profits while the total value maps into the gross output. Kliman and McGlone (1999, p.51), however, unnecessarily assume that Say's Law holds in their approach since "the profit rate is determined before and independently of output prices, the movements in which lead to different distributions of profit". The model developed in this paper builds on the single system tradition and further integrates the role of effective demand.

On the empirical side of the literature (Scharfenaker and Foley 2017; Scharfenaker and Semieniuk 2017; Fröhlich 2013; Farjoun and Machover 1983) there has been a growing consensus that profitability converges not to a single uniform profit rate but to a statistical equilibrium distribution of profit rates with a tent shape around a single peak. Shaikh (2016) shows, however, that profit rates on *new* investment projects (what Keynes labeled the marginal efficiency of capital) tend to equalize over time when adjusted for company size. Scharfenaker and Foley (2017), in particular, developed a model based on thermodynamics and quantal responses to explain the tent-shaped distribution of profit rates across firms. My model, however, assumes perfect competition and free capital mobility and, hence, cannot produce a non-uniform statistical distribution of profit rates that is caused by imperfect competition and entry barriers.

In the next sections I build on the existing literature to develop the evolutionary model of accumulation and competition as an extension of the Marxist framework. First, I formalize the macro inter-sector competition through which the aggregate and growing monetary capital of an economy is continuously redistributed between two sectors: sector I produces the means of production and sector II produces the final consumption good. The continuous redirection of monetary capital between sectors takes place according to profit rate differentials. Second, I formalize the micro intra-sector competition in which firms within each sector compete against each other via cost-reducing technical change. Innovations are gradually adopted based on profit rate differentials within sectors. Last, I close the model using the principle of effective demand. This closure makes exploitation, profits, growth, labor values, and production prices all dependent upon the level of expected demand. Profit rates equalize and market prices converge toward production prices. I present computer simulations of the model, an analysis of the evolutionary stability of the long-run equilibria, and an explanation of the conditions under which market prices converge to prices of production.

3. Macro Inter-Sector Competition

The economy-wide circuit of capital, which starts and ends with capital in the form of money, and which represents the production period during time t can be summarized through the aggregation in (1). Variables with primes ($'$) are *ex post* (after production has taken place) while variables without primes are *ex ante* (before production takes place):

$$M_t - C_t \begin{cases} LP \\ MP \end{cases} \ldots P \ldots\ C_t' -\ M_t' \qquad (1)$$

An initial amount of monetary capital M_t purchases two types of commodities as inputs C_t: labor power (LP) and means of production (MP). During the subsequent production phase (... P ...) labor power creates more value than its own. The difference between the value that labor power creates and the value of labor power itself is the surplus value. The total value of the gross output C_t' contains the new value added created by productive workers plus the cost of the means of production. The gross output exchanges for a sum of money represented by the aggregate gross expenditures M_t'. The extra value that workers create and for which they receive no compensation (unpaid labor) is the basis for the gross profits $\Delta M_t = M_t' - M_t$ in the system.

The economy comprises two sectors, each producing a single type of output using both labor power and means of production. Sector I supplies a homogenous type of means of production. Sector II supplies a homogenous type of final consumption good. Economic events take place temporally, therefore the overlap of any two consecutive circuits of capital can be represented as follows:

$$
\begin{array}{ll}
M_t - C_t \begin{cases} LP \\ MP \end{cases} \ldots P \ldots\ C_t' - \ M_t' & \\
& M_{t+1} - C_{t+1} \begin{cases} LP \\ MP \end{cases} \ldots P \ldots\ C_{t+1}' - \ M_{t+1}'
\end{array} \tag{2}
$$

The new circuit formally repeats the preceding one. The crucial causal relation is between the total monetary capital M_{t+1} advanced at the beginning of the new circuit and the total value realized M_t' at the end of the first circuit. Because of its supply-led principle, Say's Law would mean that causality runs from M_t to M_t' and then to M_{t+1}. The principle of effective demand, on the contrary, implies that the direction of causality runs from the *ex-ante* demand M_{t+1} at the *beginning of the new production period* to the realization of the total value M_t' at the *end of the previous production period*. If the expected demand M_{t+1} is too low, then $M_t' < M_t$ and firms experience a negative profit rate. As we shall see in the next sections, for some firms the profit rate will indeed turn out to be negative, forcing them to either move to a different sector and/or adopt a new technique of production with lower unit costs.

The adjustment from M_{t+1} to M_t' occurs as explained in section 2. *Within* each production period, output is set at maximum capacity at the start of the period, and prices adjust to ensure market clearing. *Between* production periods, prices remain constant, firms move across sectors, and capacity adjusts such that the levels of maximum output ensure balanced reproduction across sectors. Even though firms always produce to capacity, the level of maximum capacity changes from one period to the next based on the level of effective demand and on the mobility of firms across sectors. The model therefore has three time frames:

the production period, the short run (comprising a small number of production periods), and the long run (comprising a long sequence of short runs).

There are no fixed capital or joint products in this economy. Firms operate with flat marginal cost curves within each production period. As in Marx, the average cost and the supply curves are both constant, so profit maximization means output maximization, implying that firms either produce to capacity or do not produce at all. The means of production that enter as inputs in sectors I and II are the previous output of sector I in the preceding production period. Capital, therefore, is itself the product of labor rather than an independent source of output or a contribution of its legal owner. Technology is represented by a linear production structure with fixed coefficients and constant returns to scale. Using a_{ji} to indicate the quantity of input from sector j per unit of output in sector i, the matrix of input-output coefficients is:

$$A = [a_{ji}] = \begin{bmatrix} a_{11} & a_{12} \\ 0 & 0 \end{bmatrix} \quad \text{with} \quad 0 \leq a_{ji} < 1 \tag{3}$$

Using l_i to indicate the quantity of labor hours per unit of output in sector i, $r_{i,t}$ to indicate the within-sector profit rate per unit of output, p_i to indicate the market price per unit of output, and w to indicate the exogenous money wage per work hour, then per unit of output we have $[p_{j,t-1}a_{ji} + wl_i](1 + r_{i,t}) = p_{i,t}$. For each sector the price system is:

$$\begin{aligned} [p_{1,t-1}a_{11} + wl_1](1 + r_{1,t}) &= p_{1,t} \\ [p_{1,t-1}a_{12} + wl_2](1 + r_{2,t}) &= p_{2,t} \end{aligned} \tag{4}$$

The first term inside the brackets on the left-hand side represents constant capital, and the second term represents variable capital or the value of labor power, both in money terms. Their summation $[p_{j,t-1}a_{ji} + wl_i]$ is the unit (or average) cost, which is constant within the production period. Competition in each sector then simultaneously determines profit rates and prices.

Although the nominal wage per work hour w is exogenously given by the bargaining power between workers and firms, the real wage $\frac{w}{p_{2,t}}$ in terms of quantities of the consumption good produced in sector II is determined endogenously. As in the *single system* approach, workers get their money wages and spend it as they like, not bound to any real wage specified in terms of a bundle of goods. Labor supply and credit are assumed not to be binding constraints on growth.

In each sector there is a collection of several firms and each of them can switch between sectors depending on the average profitability $\bar{r}_{i,t}$. Firms commit their capitals to where they expect to profit the most. But once firms flow into a sector aiming at the prevailing $\bar{r}_{i,t}$ they will immediately and unintentionally alter this average profitability. Supposing a very large collection of firms in the economy, we can normalize the total number of firms to unity and then consider only the evolution of population shares, with $f_{1,t}$ representing the fraction committed to sector I and $f_{2,t}$ the fraction committed to sector II:

$$M_t = f_{1,t}M_t + f_{2,t}M_t = M_{1,t} + M_{2,t} \quad \text{with} \quad f_{1,t} + f_{2,t} = 1 \tag{5}$$

Outputs $x_{i,t}$ supplied by each sector are the sectoral monetary capitals advanced divided by the respective unit costs. This implies that firms produce at maximum capacity in each production period. Sectoral supply expands when more monetary capital is advanced in the sector at the beginning of the production period, and it contracts when capitalists withdraw their initial expenditures:

$$x_{i,t} = \frac{f_{i,t}M_t}{\left[p_{1,t-1}a_{1i} + wl_i\right]} = \frac{M_{i,t}}{\left[p_{1,t-1}a_{1i} + wl_i\right]} \tag{6}$$

Within each sector, $M'_{i,t}$ indicates the end-of-period gross expenditures or the valorized monetary capitals that comprise the original monetary capitals $M_{i,t}$ advanced plus the surplus value realized. Market prices $p_{i,t}$ are the end-of-period expenditures divided by quantities supplied:

$$p_{i,t} = \frac{M'_{i,t}}{x_{i,t}} = \frac{M_{i,t}\left(1+ \bar{r}_{i,t}\right)}{x_{i,t}} = \frac{f_{i,t}\, M_t\, \left(1+ \bar{r}_{i,t}\right)}{x_{i,t}} \tag{7}$$

The monetary capital $M_{i,t}$ committed to sector i at the beginning of the production period t is valorized on average to $\left(1+\bar{r}_{i,t}\right)$ after the output is sold. The fraction $\left(1+\bar{r}_{i,t}\right)$ includes the replication of the money initially spent plus average profits. Hence, the valorized capital in each sector is $M'_{i,t} = M_{i,t}\left(1+\bar{r}_{i,t}\right) = f_{i,t}\, M_t \left(1+\bar{r}_{i,t}\right)$. The endogenous profit rates $\bar{r}_{i,t}$ can take on any value: positive, negative, or zero. There is no assurance, therefore, that realized profits are positive. Using $\widetilde{r_t}$ to indicate the economy-wide weighted average profit rate, such that $(1+ \widetilde{r_t}) = \sum_i f_{i,t}\left(1+\bar{r}_{i,t}\right)$, the aggregate valorized capital for the entire economy is:

$$M'_t = M_t\,(1+ \widetilde{r_t}) = \sum_i M_{i,t}\left(1+\bar{r}_{i,t}\right) = \sum_i f_{i,t}\, M_t \left(1+\bar{r}_{i,t}\right) \tag{8}$$

Equation (8) also implies that the nominal growth rate of the gross output is equal to the economy-wide average nominal profit rate $\widetilde{r_t}$. The shares of the total monetary capital advanced at the beginning of period $t+1$ then change according to the average profitability obtained in period t in each sector:

$$f_{i,t+1} = \frac{M'_{i,t}}{M'_t} = \frac{f_{i,t}\, M_t\left(1+\bar{r}_{i,t}\right)}{M_t\,(1+ \widetilde{r_t})} = f_{i,t}\,\frac{\left(1+\bar{r}_{i,t}\right)}{(1+ \widetilde{r_t})} \tag{9}$$

Rewriting it as $\frac{f_{i,t+1}}{f_{i,t}} = \frac{(1+\bar{r}_{i,t})}{(1+\widetilde{r_t})}$, subtracting 1 from both sides and using $\Delta f_{i,t+1} = f_{i,t+1} - f_{i,t}$, we then obtain the replicator equation that formalizes the macro competition between firms across sectors:

$$\Delta f_{i,t+1} = f_{i,t}\left(\frac{1}{1+\widetilde{r_t}}\right)\left[\bar{r}_{i,t} - \widetilde{r_t}\right] \tag{10}$$

The profitability gap in relation to the economy-wide average determines how firms allocate their monetary capitals across sectors. Firms make decisions based on profit rates prevailing in each sector, but they end up affecting aggregate profitability through their decentralized individual actions to move their capitals from one sector to another. The effects on the aggregate profit rate then feed back into individual decisions about where to commit the monetary capital in the following period.

The equations so far presented describe the growth of output at the macro level and the nonlinear adjustments that regulate the shares of the monetary capital flowing to each sector. In the next section I turn to the competition for profits through cost-reducing technical change that characterizes the micro-adjustments within each sector.

4. Micro Intra-Sector Competition

Large collections of firms compete for profits within each sector. Markets are intensely competitive, forcing firms to sell at prevailing market prices. The way to increase individual profit lies therefore with the adoption of cost-reducing technologies. Innovations are generated exogenously and then adopted conditional on enhancing individual profitability. When an individual firm decides upon the adoption of a new productive structure it does so taking the prevailing market price as given. But the individual adoption of the newer technique changes the sector cost structure, and it therefore unintentionally affects the market price. The new market price then operates as a signal for the remaining firms to also adopt the cost-reducing technique. Each sector will thus display a production structure that is a combination of firms producing with the new technique and firms still producing with the old technique.

The economy has three evolutionary processes taking place concurrently. The first is the adoption of new techniques in the sector producing means of production. The second is the adoption of new techniques in the sector producing final consumption goods. The third is the distribution of the growing monetary capital between sectors. An individual decision to adopt a new technique thus triggers a chain of nonlinear feedback effects that no individual firm can anticipate.

The prevailing technique of production is represented by a set of four technical parameters $(a_{11}^o, a_{12}^o, l_1^o, l_2^o)$. An innovation $(a_{11}^n, a_{12}^n, l_1^n, l_2^n)$ can imply the use of more of labor power and means of production, less of both inputs, or more of one input and less of the other (where superscript o stands for 'old' and n for 'new' technique). By rearranging the price equations in (4) we get the profit rate per unit of output using the old technology:

$$r_{i,t}^o = \frac{p_{i,t}}{\left[p_{1,t-1}a_{1i}^o + wl_i^o\right]} - 1 \tag{11}$$

Similarly, the profit rate associated with the new technology is:

$$r_{i,t}^n = \frac{p_{i,t}}{\left[p_{1,t-1}a_{1i}^n + wl_i^n\right]} - 1 \tag{12}$$

The diffusion of a new technique of production is formalized with the dynamics of replication. The variable $\upsilon_{i,t} \in [0,1]$ indicates the share of firms in sector i that adopt the new technique at time t, while $(1 - \upsilon_{i,t})$ indicates the share that remains with the older technique. Because each sector has a large collection of firms, and if they interact through random pairwise matching, we can use a simple replicator equation for the diffusion of innovations. Normalizing population sizes to unity allows us to work with population shares in each sector as follows:

$$\begin{gathered} \upsilon_{i,t+1} = \upsilon_{i,t} + \upsilon_{i,t}\left(1 - \upsilon_{i,t}\right)[r_{i,t}^n - r_{i,t}^o] \\ \Delta\upsilon_{i,t+1} = \upsilon_{i,t}\left(1 - \upsilon_{i,t}\right)[r_{i,t}^n - r_{i,t}^o] \\ \Delta\upsilon_{i,t+1} = \upsilon_{i,t}[r_{i,t}^n - \bar{r}_{i,t}] \end{gathered} \tag{13}$$

The term $\upsilon_{i,t}\left(1 - \upsilon_{i,t}\right)$ is the variance of the firms within each sector and the term $\left[r_{i,t}^n - r_{i,t}^o\right]$ is the differential replication selection, so that the updating process is payoff monotonic. The third line in equation (13) follows from the fact that the average profit rate in each sector is: $\bar{r}_{i,t} = \left(\upsilon_{i,t}\right)\left[r_{i,t}^n\right] + \left(1 - \upsilon_{i,t}\right)\left[r_{i,t}^o\right]$.

The gradual adoption of new technologies implies that older and newer cost structures coexist until the newer technique completely replaces the older one. Given the monetary capital $M_{i,t}$ committed to each sector, the new quantities supplied can be found by dividing the monetary capital advanced by the mixed cost structure:

$$x_{i,t} = \frac{M_{i,t}}{(v_{i,t})\left[p_{1,t-1}a_{1i}^{n} + wl_{i}^{n}\right] + (1 - v_{i,t})\left[p_{1,t-1}a_{1i}^{o} + wl_{i}^{o}\right]} \tag{14}$$

The supply equation in (14) thus replaces the supply equation in (6), which only applied to production under a single technology. Average rates of profit in each sector now depend on the prevailing market prices and on the linear combination between older and newer techniques:

$$\bar{r}_{i,t} = \frac{p_{i,t}}{(v_{i,t})\left[p_{1,t-1}a_{1i}^{n} + wl_{i}^{n}\right] + (1 - v_{i,t})\left[p_{1,t-1}a_{1i}^{o} + wl_{i}^{o}\right]} - 1 = \frac{\Delta M_{i,t}}{M_{i,t}} \tag{15}$$

As soon as profit rates in each sector change from their previous position they trigger intra-sector competition via the micro replicator equations (13) as well as inter-sector competition via the macro replicator equation (10). In the next section I analyze the demand side of the model.

5. Effective Demand and the Realization of Value

The realization of value and surplus value are endogenous and dependent upon the principle of effective demand. Once effective demand is brought into the Marxist framework, the *expected* level of demand will determine the realized rates of exploitation, the realized profit rates in each sector, and consequently the long-run prices of production. Profit rates equalize if the average profit rate of a sector increases less than the competing profit rate when the firms committing their capital to that sector increase their share in the population, or simply $\frac{\mathrm{d}\bar{r}_{1,t}}{\mathrm{d}f_{1,t}} < \frac{\mathrm{d}\bar{r}_{2,t}}{\mathrm{d}f_{1,t}}$. In section 8 I show under what parameter values this stability condition is met.

The total expenditures on labor power and means of production take place at the *beginning* of each production period. Constant capital and variable capital are both *advanced* before production takes place. The hours worked per unit of output, l_i, then generate the value added that corresponds to the summation of wages and profits. Workers in sector i produce $wl_i(1 + e_{i,t})$ of value added per unit of output, but they only get back the value of their labor power corresponding to wl_i, leaving the surplus $e_{i,t}wl_i$ to the firms hiring them.

The wage share of value added is $\frac{V}{V+S} = \frac{1}{1+e}$, in which V is the value of labor power (the total wage bill advanced in the economy), S is realized surplus value or profits, $e = \frac{S}{V}$ is the average realized rate of exploitation, and V+S is the flow of value added. Given that V is advanced capital, if the realized rate of exploitation were assumed to be exogenous, as has often been the case in the Marxist literature, it would imply that Say's Law holds since the realized surplus value and value added would be determined before exchange takes place. This assumption is avoided once the realized rates of exploitation $e_{i,t}$ become endogenous to the level of effective demand. Profits originate from unpaid labor, hence:

$$M'_{i,t} = \left[p_{1,t-1}a_{1i} + wl_i(1 + e_{i,t})\right]x_{i,t} \tag{16}$$

$$\Delta M_{i,t} = M'_{i,t} - M_{i,t} = e_{i,t}\, wl_i\, x_{i,t} \tag{17}$$

This relation between profitability and exploitation derives from the fact that the price system is such that $p_{i,t} = p_{1,t-1}a_{1i} + wl_i + e_{i,t}wl_i = \left[p_{1,t-1}a_{1i} + wl_i\right]\left(1 + r_{i,t}\right)$. Rearranging terms and solving for the profit rate gives us:

$$r_{i,t} = \frac{e_{i,t}}{1 + \left(\frac{p_{1,t-1}}{w}\right)\left(\frac{a_{1i}}{l_i}\right)} \tag{18}$$

Equation (18) is the usual Marxist relation in which the profit rate is the rate of exploitation divided by one plus the organic composition of capital. The organic composition is, in turn, the relative price $\frac{p_{1,t-1}}{w}$ times the technical composition $\frac{a_{1i}}{l_i}$ between constant and variable capital.

Once firms begin to adopt technological innovations, the mixed productive structure requires weighting the surplus value produced by the respective shares of firms employing the newer and older technologies. Equations (19) and (20) replace equations (16) and (17) as soon as a new technique is introduced:

$$M'_{i,t} = \{(v_{i,t})[p_{1,t-1}a^n_{1i} + wl^n_i(1 + e_{i,t})] + (1 - v_{i,t})[p_{1,t-1}a^o_{1i} + wl^o_i(1 + e_{i,t})]\}x_{i,t} \quad (19)$$

$$\Delta M_{i,t} = M'_{i,t} - M_{i,t} = \{(v_{i,t})[wl^n_i e_{i,t}] + (1 - v_{i,t})[wl^o_i e_{i,t}]\}x_{i,t} \quad (20)$$

Increments in the share of firms adopting the new technology ($v_{i,t}$) can reduce or increase the average profit rate prevailing in a sector, or even turn the profit rate negative. The final effect on profitability can only be known after the repricing of both the means of production and the final consumption good.

The key element in this extended Marxist model is the determination of aggregate demand. I use a neo-Keynesian autonomous investment function à la Joan Robinson (1962; Dutt 2011) which assumes that firms operate at full capacity utilization and that the amount of monetary capital committed to the investment good sector is a function of both animal spirits and past profitability. The autonomous component of investment in constant capital consists of the investment carried out in the previous period ($M'_{1,t-1}$) times a parameter β that measures the degree of animal spirits. $\beta = 1$ is the reference value and deviations from unity indicate the degree of firms' pessimism (if $\beta < 1$) and optimism (if $\beta > 1$). The parameters γ_i indicate the sensitivity of *ex ante* investment demand to the realized profit rates in each sector. Given that there are firms operating with the newer and older technologies simultaneously in each sector, we have that:

$$M_{1,t+1} = M'_{1,t} = \beta\, M'_{1,t-1} + \gamma_1\{(\upsilon_{1,t})[r^n_{1,t-1}] + (1-\upsilon_{1,t})[r^0_{1,t-1}]\}\, M_{1,t-1} + \gamma_2\{(\upsilon_{2,t})[r^n_{2,t-1}] + (1-\upsilon_{2,t})[r^0_{2,t-1}]\}\, M_{2,t-1} \quad (21)$$

The monetary capital $M_{1,t+1}$ effectively committed to production in sector I at the beginning of period $t+1$ thus reflects the firms' level of animal spirits and the expected profitability in that sector. Expected profitability is based on the realized profit rate in the previous period. The principle of effective demand implies that causality runs from $M_{1,t+1}$ at the beginning of production period in $t+1$ to $M'_{1,t}$ at the end of production period in t. The monetary capital $M_{1,t+1}$ advanced is the *ex-ante* demand at the beginning of period $t+1$, and as such it simultaneously comprises the expenditure $M'_{1,t}$ necessary to realize the value produced in sector I at the end of the previous production period t.

In sector II, likewise, the monetary capital $M_{2,t+1}$ effectively committed to production at the beginning of period $t+1$ reflects the firms' expected profitability for that sector. Supposing that workers do not save and that there is no consumption credit, the total expenditure $M'_{2,t}$ with the consumption goods produced in sector II is simply the total wage bill in the economy. At the beginning of period $t+1$, firms commit to sector II an amount of monetary capital proportional to the aggregate consumption of out wages realized in the previous production period t. Given that the wage bills in each sector must be weighted by the shares of firms using the old and the new technologies, we have that:

$$M_{2,t+1} = M'_{2,t} = \{(\upsilon_{1,t})[wl^n_1] + (1-\upsilon_{1,t})[wl^0_1]\}\, x_{1,t} + \{(\upsilon_{2,t})[wl^n_2] + (1-\upsilon_{2,t})[wl^0_2]\}\, x_{2,t} \quad (22)$$

Effective demand at the beginning of period $t+1$ is $M_{t+1} = M_{1,t+1} + M_{2,t+1} = M'_{1,t} + M'_{2,t}$, in which the second equality follows directly from equations (5) and (9). The endogenous rates of exploitation $e_{i,t}$ within each sector are the surplus values realized over the nominal wage bill advanced:

$$e_{i,t} = \frac{M_{i,t+1} - M_{i,t}}{\{(v_{i,t})[wl_i^n] + (1 - v_{i,t})[wl_i^0]\}\, x_{i,t}} = \frac{M'_{i,t} - M_{i,t}}{\{(v_{i,t})[wl_i^n] + (1 - v_{i,t})[wl_i^0]\}\, x_{i,t}} \tag{23}$$

The rates of exploitation realized in each sector depend directly on the level of aggregate demand from equations (21) and (22). In *qualitative* terms, profits originate from surplus value (unpaid labor). The principle of effective demand then implies that, in *quantitative* terms, the determination runs from the partially autonomous expectations in the demand functions to the realized surplus values and exploitation rates. Even though profits originate *qualitatively* from surplus value at the point of production, under the principle of effective demand the amount of profits is the *quantity* of surplus value realized at the point of exchange.

6. Long-Run Equilibria and Evolutionary Stability

This section analyzes the long-run behavior of the model by focusing on the trajectories of the three replicator equations $(f_{1,t}, v_{1,t}, v_{2,t})$ considered separately. Section 7 simulates the model and section 8 analyzes the stability of the stationary states considering the replicator equations both separately and simultaneously.

Stationary states are those states at which the replicator reaches a fixed point with no further changes in the replication process $(\Delta f_{1,t} = 0, \Delta v_{1,t} = 0, \Delta v_{2,t} = 0)$. In an evolutionary game with replicator dynamics, we know that the *evolutionarily stable strategies* (ESS) prevail over the long run. A strategy is an ESS if a population using that strategy cannot be invaded by a small group using an alternative strategy. An ESS is, therefore, a best response to itself and hence it is a symmetric pure-strategy Nash equilibrium that is also asymptotically stable in its respective replicator equation. *Evolutionary stability* implies both self-correction and asymptotic attractiveness (stable attractor), hence the system converges over time to a stationary point that is evolutionarily stable (Bowles 2006; Gintis 2009; Elaydi 2005; Scheinerman 2000).

Table 1 summarizes the stationary states and asymptotic properties of each replicator equation considered separately. The long-run stationary state of the full model with the three replicator equations can be any combination of the possibilities listed in Table 1. If adopting the new techniques is an ESS in sectors I and II, for example, and if moving into either sector is not an ESS, then the model will converge to a stationary state with full technological diffusion in both sectors and equalized profit rates across sectors. When there is no ESS in the macro inter-sector replicator, the system converges to an interior stable solution f_1^* such that average profit rates are equalized asymptotically. In this case, profit rates are not just *equal* across sectors but truly *equalized* in the sense that the equality in sector profitability is evolutionarily stable.

[Table 1 about here]

Because the technical coefficients in the input-output matrix are exogenous but not constant, a strategy that was an ESS *before* the technical change might not be an ESS *after* the innovation is introduced. As long as we have exogenous innovations brought into the system, the ESSs themselves will change over time. The stationary long-run equilibria $(f_1^*, r^*, p_1^*, p_2^*)$ at which there is full adoption of the new techniques in both sectors $(v_1^* = 1, v_2^* = 1)$ and profit rate equalization $(\bar{r}_1^* = \bar{r}_2^* = \tilde{r}^* = r^*)$ are determined implicitly by the equations (24-27), where $\beta, \gamma_i, a_{1i}^n, l_i^n, w$ are exogenous parameters as defined in sections 4 and 5:

$$f_1^* = \frac{\beta + \left[\gamma_1 + \gamma_2 \frac{1-f_1^*}{f_1^*}\right] \frac{r^*}{(1+r^*)}}{\beta + \left[\gamma_1 + \gamma_2 \frac{1-f_1^*}{f_1^*}\right] \frac{r^*}{(1+r^*)} + (1 - a_{11}^n - a_{11}^n r^*) + \frac{1-f_1^*}{f_1^*} \frac{l_2^n (1 - a_{11}^n - a_{11}^n r^*)}{a_{12}^n l_1^n (1+r^*) + l_2^n (1 - a_{11}^n - a_{11}^n r^*)}} \tag{24}$$

$$\frac{(1+r^*)^2}{r^*} = \beta + \gamma_1 + \gamma_2 \frac{1-f_1^*}{f_1^*} \tag{25}$$

$$p_1^* = \frac{w\, l_1^n \frac{r^*}{1+r^*}\left\{\beta + \gamma_1 + \gamma_2 \frac{1-f_1^*}{f_1^*}\right\}}{1 - a_{11}^n \frac{r^*}{1+r^*}\left\{\beta + \gamma_1 + \gamma_2 \frac{1-f_1^*}{f_1^*}\right\}} \tag{26}$$

$$p_2^* = [p_1^* a_{12}^n + w l_2^n](1+r^*) \tag{27}$$

Because of the highly nonlinear structure and lagged feedback effects, there is no explicit analytical solution to the model. We must therefore analyze it numerically via computer simulations. The next section simulates the model and discusses the results and implications, in particular how the stationary long-run states described by equations (24-27) imply long-run prices and values whose absolute levels and relative ratios are both functions of the demand parameters β and γ_i. Section 8 then provides a complete analysis of the stability properties of the long-run equilibria under different parameter values.

7. Model Simulation and Discussion

To simulate the model, it is necessary to fix parameters and initial conditions. Section 8 shows that the long-run stationary states are dependent on the parameter values but independent from the arbitrary initial conditions. In this example the initial technical coefficients are set to (a_{11}^o, a_{12}^o, l_1^o, l_2^o) = (0.2, 0.1, 0.7, 0.7) representing the old technology. The exogenous nominal wage w is set to 10 dollars per work hour. The initial aggregate monetary capital $M_{t=1}$ is set to 100 dollars, and the initial distribution is set at 60% to sector I ($f_{1,t=1} = 0.6$) and 40% to sector II ($f_{2,t=1} = 0.4$). The means of production are initially priced at 50 dollars per unit ($p_{1,t=0} = 50$). The investment function assumes $\beta = 1$, $\gamma_1 = \gamma_2 = 0.5$, and investment demand begins at 50 dollars ($M'_{1,t=1} = 50$).

The model is set to run for 200 production periods. For the first 49 rounds the trajectories evolve without technical change. At period $t = 50$ an innovation in sector II increases labor productivity by 100% while increasing the use of machines by 100% per unit of output, hence (a_{11}^o, a_{12}^n, l_1^o, l_2^n) = (0.2, 0.2, 0.7, 0.35). This machine-intensive, labor-saving innovation generates a strong increase in the technical composition of capital in the sector producing the consumption good. At time $t = 100$ an innovation in sector I increases labor productivity by 150% and the use of machines by 100% per unit of output such that (a_{11}^n, a_{12}^n, l_1^n, l_2^n) = (0.4, 0.2, 0.28, 0.35). This innovation implies a strong machine-intensive labor-saving technical change in the sector producing the means of production.

[Figure 1 about here]

Figure 1 plots the simulation results. The model and the simulation were coded and compiled in R version 4.0.4. Panel (a) shows the equalization of profit rates over time, such that the economy-wide average profit rate is a stable attractor to the sectoral average profit rates. Panel (b) shows the movement of firms across sectors in search of higher returns. Panels (c) and (d) show the shares of firms operating with the old and new technologies within each sector. Panel (e) and (f) show the profit rates of firms employing the old and new technologies within each sector. The uncoordinated implementation of the new technologies increases the profit rate only for those firms initially adopting the innovation, but the gradual diffusion of the new technologies results in lower levels of profitability for all firms over time. Panel (g) shows the real wage and the average rates of exploitation in both sectors. Because the nominal wage is constant and the new technology reduces the price of the consumption good, the real wage rises. Hence, the gains from technology reduce the rates of exploitation. Finally, panel (f) shows the wage and profit shares of value added. The reduction in the production price of the consumption good causes the real wage to rise faster than the productivity of labor, contributing to the fall in the average rate of profit. As the real wage rises, so does the wage share.

New techniques of production can reduce the average profitability if the repricing of means of production and consumption goods increases the real wage faster than the productivity of labor. As Basu (2019) demonstrated, since technical change impacts both the real wage and the productivity of labor, the trend of the profit rate derives from the relation between these two factors. Okishio's (1961; 2001) theorem holds true only if labor productivity rises faster than the real wage, which imparts a positive trend to the profit rate over time. If technical change causes the real wage to rise faster than labor productivity, the profit rate falls. For the rate of exploitation to increase over time the model would need to include equations for the labor market, allowing for the existence of unemployment and bargaining power. This extension of the model will be pursued in further work.

Labor values $\lambda_{i,t}$ are defined as weighted averages of the individual values of the commodities produced with the different techniques, all measured in labor hours. The value of each type of output thus depends on the technical coefficients and also on the evolution of the technology shares $v_{i,t}$, which are in turn functions of the prevailing profits rates $r_{i,t}^{n}$ and $r_{i,t}^{o}$ in each sector:

$$\lambda_{i,t} = \lambda_{j,t}\left[(v_{i,t})a_{ji}^{n} + (1 - v_{i,t})a_{ji}^{o}\right] + \left[(v_{i,t})l_{i}^{n} + (1 - v_{i,t})l_{i}^{o}\right] \qquad (28)$$

As Steedman (1977, p.147) demonstrated, "when there is a choice of technique, the determination of the profit rate is logically prior to the determination of value magnitudes". But because commodity values depend upon the profit rates in each sector, and the realized profit rates are endogenous to effective demand, value magnitudes also depend upon the level of expected demand. In this regard, Figure 2 plots labor values as functions of the investment parameters (animal spirits β and sensitivity to profits γ_i) at time period $t = 150$. In each figure the model was simulated 500 times over with very small gradual increases in the parameters values of either β or γ_i, and assuming the same pattern of technical change as in the previous simulation. The investment demand parameters affect the profitability $r_{i,t}^{n}$ of the new techniques and the profitability $r_{i,t}^{o}$ of the old techniques within each sector and, hence, impact the evolution of the technology shares $v_{i,t}$ which, in turn, determine labor values $\lambda_{i,t}$.

[Figure 2 about here]

Figure 3 plots the long-run states for different levels of animal spirits β. Figure 4 plots the long-run states for different parameter values of γ_i, supposing these are equal across sectors ($\gamma_i = \gamma$). In each figure the model was again simulated 500 times over under very small gradual increases in the parameters values for β and γ_i, and assuming the same pattern of technical change as before. The figures use color coding where blue regions indicate that the model converges to an asymptotically stable long-run state, while red regions indicate asymptotic instability. Asymptotic stability is defined as the convergence to a stationary state with equalized profit rates and full adoption of the new techniques in both sectors.

[Figure 3 about here]

[Figure 4 about here]

Figures 3 and 4 reveal that the long-run market prices (prices of production), the rates of exploitation, the ratios of prices of production, the ratios of the rates of exploitation, the economy-wide average profit rate, the distribution of capital across sectors, and the adoption of new techniques within each sector are all endogenous to the level of animal spirits and to the sensitivity of investment to profitability.

Demand and expectations are therefore constitutive of labor values and prices of production. Hence, socially necessary labor cannot be determined solely from the supply side. Figures 3 and 4 also demonstrate that both the absolute levels and the relative ratios of long-run prices and values are endogenous to the components of aggregate demand.

8. Stability Analysis of the Stationary States

This section analyzes the stability of the stationary states. The first part analyzes the model assuming no technological progress (or full adoption, which is equivalent) and focuses solely on the inter-sector replicator for $f_{1,t}$. The second part analyzes the stability of the full model in its three dimensions $(f_{1,t}, v_{1,t}, v_{2,t})$ simultaneously. The simulations, iterations, and stability plots were coded and compiled in R version 4.0.4.

8.1. Stability Analysis in One Dimension

In this section I present the stability analysis of the long-run stationary state supposing no technical change or, equivalently, full technical change in both sectors. Hence, either $v_i^* = 1 \; or \; 0$ and we focus on the one-dimensional behavior of $f_{1,t}$.

Asymptotic stability means that the stationary state is both *stable* and an *attractor*, so the system converges to it over time (Scheinerman, 2000; Elaydi, 2005). In the one-dimensional replicator equation, asymptotic stability requires the payoff of a strategy to increase less than the competing payoff when the agents adopting that strategy increase their share in the population (Bowles 2006; Gintis 2009). The expected payoffs are the average profit rates within each sector. Thus, the stability condition is:

$$\frac{\mathrm{d}\bar{r}_{1,t}}{\mathrm{d}f_{1,t}} < \frac{\mathrm{d}\bar{r}_{2,t}}{\mathrm{d}f_{1,t}} \tag{29}$$

For $\beta = 1$, the model has a stationary state with equalized profit rates across sectors at $p_1^* = \frac{wl_1(1+r^*)}{1-a_{11}(1+r^*)}$; $p_2^* = \left[\frac{wl_1 a_{12}(1+r^*)}{1-a_{11}(1+r^*)} + wl_2\right](1+r^*)$; $\left(\frac{e_1}{e_2}\right)^* = \frac{\frac{p_1^*}{w}.\frac{a_{11}}{l_1}+1}{\frac{p_1^*}{w}.\frac{a_{12}}{l_2}+1}$; $\left(\frac{x_1}{x_2}\right)^* = \left(\frac{f_1^*}{1-f_1^*}\right)\frac{p_1^* a_{12}+wl_2}{p_1^* a_{11}+wl_1}$. With some algebraic manipulation, the stability condition (29) is satisfied when:

$$\gamma_1\left(\frac{f_{1,t-1}}{1+\tilde{r}_{t-1}}\right) - \gamma_2\left(\frac{f_{1,t-1}+\tilde{r}_{t-1}}{1+\tilde{r}_{t-1}}\right) < \left(\frac{f_{1,t}}{1-f_{1,t}}\right)^2\left(\frac{1}{\frac{p_{1,t-1}}{w}\frac{a_{11}}{l_1}+1}\right) \tag{30}$$

This inequality means that the interior stationary state $0 < f_1^* < 1$ tends to become less stable when, ceteris paribus: (i) the organic composition of capital in sector I $\left(\frac{p_{1,t-1}}{w}\frac{a_{11}}{l_1}\right)$ is high; (ii) the ratio between the sector investment coefficients $\left(\frac{\gamma_1}{\gamma_2}\right)$ is high. But if γ_1 and γ_2 are not too far apart and the technical composition $\frac{a_{11}}{l_1}$ in sector I is not too high, the stationary state with equalized profit rates is asymptotically stable under the principle of effective demand.

The stability condition (29) refers to the one-dimensional replicator in which the economy has only two sectors, so the interior solution $0 < f_1^* < 1$ is either stable or unstable. In an economy with three or more sectors the stability condition in higher dimensions would cover cases with saddle path stability and limit cycles. This issue is beyond the scope of this paper but will be pursued in further work.

Foley (2018) and Cogliano (2011) argue that while competition between firms equalizes profit rates, competition between workers equalizes the rates of exploitation. For the free flow of labor to generate

$e_1^* = e_2^*$ in the long run, the nominal wage in either one of the two sectors would need to vary such that $w_1 = \frac{\frac{a_{11}}{l_1}}{\frac{a_{12}}{l_2}} w_2$. The adjustment of nominal wages across sectors according to this rule, however, would make little difference to the dynamics of the model.

8.2. Stability Analysis in Three Dimensions

The full model comprising the three replicator equations is a three-dimensional system of nonlinear difference equations with complex lagged feedback effects that has no closed-form solution. To study the properties of this system without solving it explicitly, I implement a numerical analysis using very small incremental changes in the parameters. I focus on the fixed points for which the inter-sector replicator reaches a stable interior solution with equalized profit rates ($0 < f_1^* < 1$, such that entering either sector is not an ESS) and all firms in both sectors adopt the new technologies ($v_1^* = v_2^* = 1$, such that adopting the new techniques is an ESS in sectors I and II).

The stability analysis is implemented in two steps. In the first step, parameters values and initial conditions vary one at a time, holding all else constant. In each case the model is simulated 10,000 times over, and in each iteration the respective parameter value is increased by 0.001 unit. Stability regions are reported in Table 2. The results reveal that if we exclude the trivial case in which the economy begins with zero monetary capital ($M_{t=1} = 0$), then asymptotic stability also implies *global* asymptotic stability.

[Table 2 about here]

[Figure 5 about here]

In the second step, I compute a total of 387,500 simulations in which the values of the parameters change simultaneously. The first row of Figure 5 plots the regions of asymptotic stability for different combinations of parameters a_{11}^o, a_{12}^o, and $\frac{\gamma_1}{\gamma_2}$, where a_{11}^o and a_{12}^o are the pre-technical change parameters. The second row of Figure 5 plots the regions of asymptotic stability for different combinations of parameters β, a_{11}^n, and γ_i assuming that $\gamma_i = \gamma$. Brighter regions indicate a stable fixed point with equalized profit rates ($0 < f_1^* < 1$) and full adoption of new production techniques ($v_1^* = v_2^* = 1$). Darker regions indicate instability. Simulations suppose the same pattern of technological change as in section 7. As in Table 2, as long as we exclude the trivial initial condition $M_{t=1} = 0$, then asymptotic stability also implies *global* asymptotic stability.

The results summarized in Table 2 and in Figure 5 confirm the previous results from the one-dimensional case and further demonstrate that: the model is insensitive to the initial conditions, and hence asymptotic stability implies global asymptotic stability; the long-run state tends to become more stable for lower values of the input-output coefficients; and that the long-run state tends to become more stable when the demand parameters for animal spirits β and sensitivity to past profitability γ_i increase simultaneously.

9. Final Remarks

The paper developed a critique of and a (partial) solution to Say's Law Marxism by demonstrating that effective demand and expectations are constitutive of labor values and prices of production. This result was achieved by showing that both labor values and long-run prices of production depend upon the principle of effective demand and the realization of surplus value in the short run. The principle of effective demand within a Marxist framework implies that labor values and prices of production cannot be, and should not be determined solely from the supply side. Supply and expected demand jointly determine the "socially necessary" character of abstract labor time. Even more, the principle of effective demand is compatible with equalized profit rates and with prices of production operating as long-run attractors of market prices.

The evolutionary model of competition and technical change presented in the paper applies to an economy operating in perfect competition with no fixed assets and no joint products, where companies operate with fixed-coefficient technologies and constant returns to scale, always producing at maximum capacity. The next step, evidently, is to investigate whether the same result applies to the more realistic and relevant case of a multi-sector economy with fixed assets, joint production, and increasing returns to scale in imperfectly competitive markets.

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Tables and Figures

Table 1: Stationary States of the Replicator Equations Considered Separately

(a) Macro Inter-Sector Replicator		
Moving into Sector I is ESS Moving into Sector II is ESS	$\bar{r}_1 \gtrless \bar{r}_2$	$f_i^* = 1$ is stable $f_i^* = 0$ is stable $0 < f_i^* < 1$ is unstable
Moving into Sector I is ESS Moving into Sector II is not ESS	$\bar{r}_1 > \bar{r}_2$	$f_i^* = 1$ is stable $f_i^* = 0$ is unstable
Moving into Sector I is not ESS Moving into Sector II is ESS	$\bar{r}_1 < \bar{r}_2$	$f_i^* = 1$ is unstable $f_i^* = 0$ is stable
No ESS	$\bar{r}_1 = \bar{r}_2$	$0 < f_i^* < 1$ is stable $f_i^* = 1$ is unstable $f_i^* = 0$ is unstable
(b) Micro Intra-Sector Replicator in Sector I		
Innovate is ESS Not innovate is ESS	$r_1^n \gtrless r_1^o$	$v_1^* = 1$ is stable $v_1^* = 0$ is stable $0 < v_1^* < 1$ is unstable
Innovate is ESS Not innovate is not ESS	$r_1^n > r_1^o$	$v_1^* = 1$ is stable $v_1^* = 0$ is unstable
Innovate is not ESS Not innovate is ESS	$r_1^n < r_1^o$	$v_1^* = 1$ is unstable $v_1^* = 0$ is stable
No ESS	$r_1^n = r_1^o$	$0 < v_1^* < 1$ is stable $v_1^* = 1$ is unstable $v_1^* = 0$ is unstable
(c) Micro Intra-Sector Replicator in Sector II		
Innovate is ESS Not innovate is ESS	$r_2^n \gtrless r_2^o$	$v_2^* = 1$ is stable $v_2^* = 0$ is stable $0 < v_2^* < 1$ is unstable
Innovate is ESS Not innovate is not ESS	$r_2^n > r_2^o$	$v_2^* = 1$ is stable $v_2^* = 0$ is unstable
Innovate is not ESS Not innovate is ESS	$r_2^n < r_2^o$	$v_2^* = 1$ is unstable $v_2^* = 0$ is stable
No ESS	$r_2^n = r_2^o$	$0 < v_2^* < 1$ is stable $v_2^* = 1$ is unstable $v_2^* = 0$ is unstable

Table 2: Stability of the Long-Run Equilibria

Asymptotic Stability Region	Main Assumptions
$0.98 < \boldsymbol{\beta} < 1.50$	$\gamma_1 = \gamma_2 = 0.50$
$0.00 < \boldsymbol{\gamma_1} = \boldsymbol{\gamma_2} < 1.26$	$\gamma_1 = \gamma_2; \beta = 1$
$0.01 < \frac{\boldsymbol{\gamma_1}}{\boldsymbol{\gamma_2}} < 1.377$	$\gamma_2 = 1;\ \beta = 1$
$0.01 < \frac{\boldsymbol{\gamma_2}}{\boldsymbol{\gamma_1}} < 1.832$	$\gamma_1 = 1; \beta = 1$
$0.01 < \boldsymbol{a_{11}^o} < 0.315$	$\gamma_1 = \gamma_2 = 0.50;\ \beta = 1$
$0.01 < \boldsymbol{a_{12}^o} < 0.44$	$\gamma_1 = \gamma_2 = 0.50;\ \beta = 1$
$0.01 < \boldsymbol{l_1^o} < 3.084$	$\gamma_1 = \gamma_2 = 0.50;\ \beta = 1$
$0.159 < \boldsymbol{l_2^o} < 10.009$	$\gamma_1 = \gamma_2 = 0.50;\ \beta = 1$
$0 < \boldsymbol{w} < \infty$	$\gamma_1 = \gamma_2 = 0.50;\ \beta = 1$
$0 < \boldsymbol{M_{t=1}} < \infty$	$\gamma_1 = \gamma_2 = 0.50;\ \beta = 1$

Note: In each row the model is iterated 10,000 times over 500 production periods, with gradual increases of 0.001 unit in the respective parameter value. Simulations suppose the same pattern of technological change as in section 7. The simulated parameter values for (a_{11}^o, a_{12}^o, l_1^o, l_2^o) refer to the pre-technical change parameters. Asymptotic stability means that the model converges to a stationary long-run state with equalized profit rates and full adoption of the new techniques in both sectors.

Figure 1: Simulation of the Model

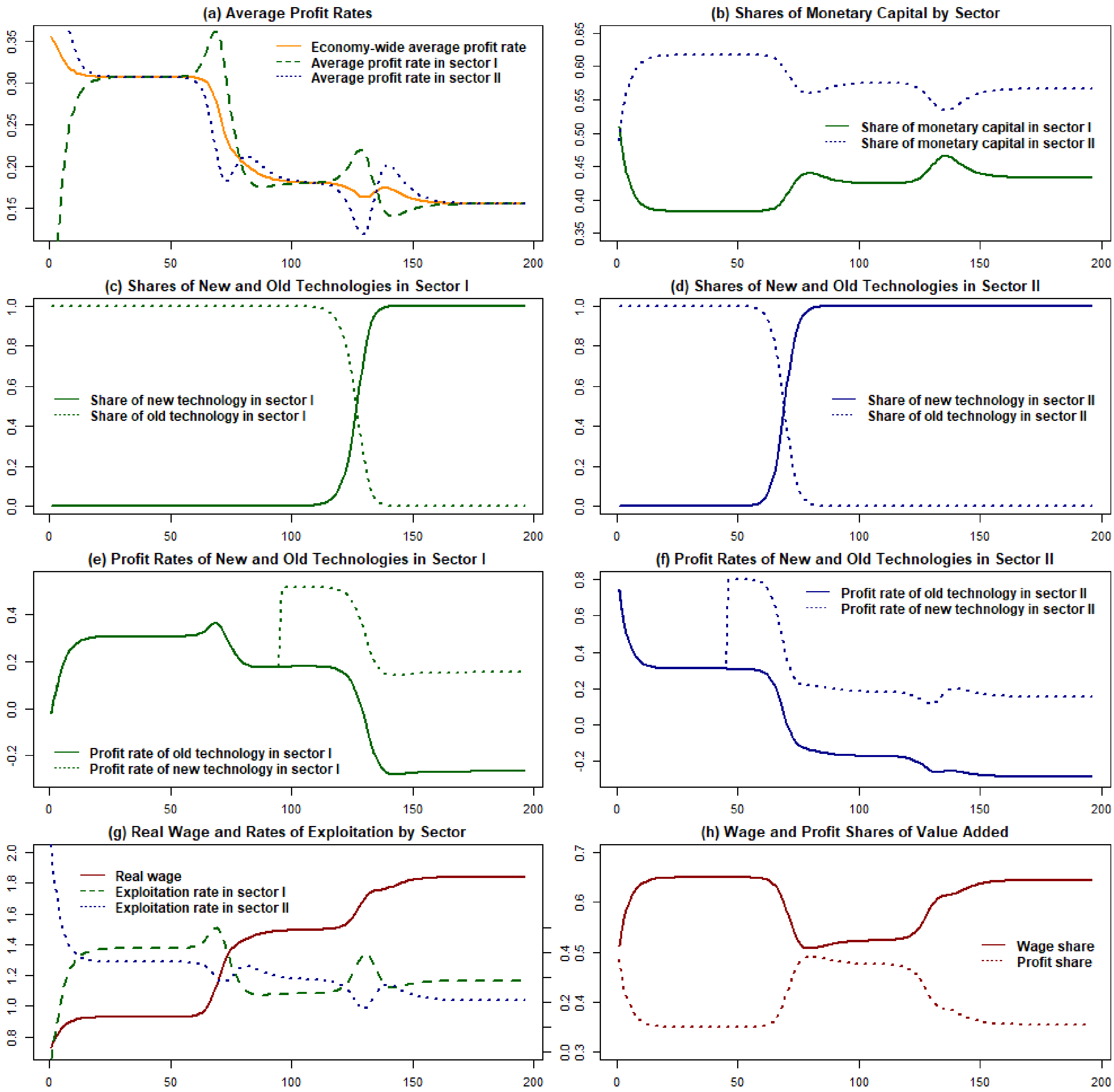


Figure 2: Labor Values as Functions of Animal Spirits (β) and Investment Sensitivity to Profits (γ_i) at Period $t = 150$

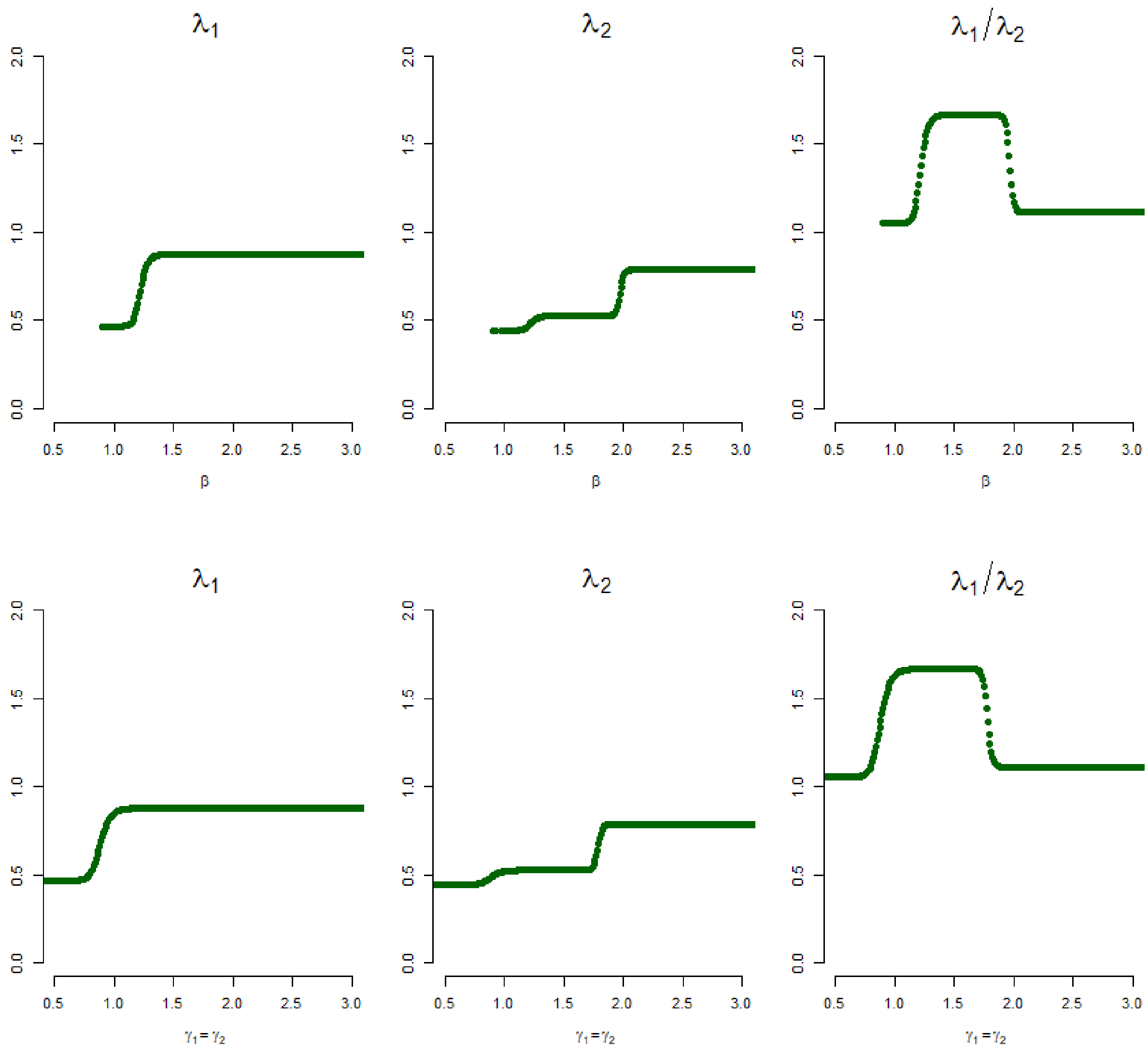


Figure 3: Long-Run States as Functions of Animal Spirits (β)

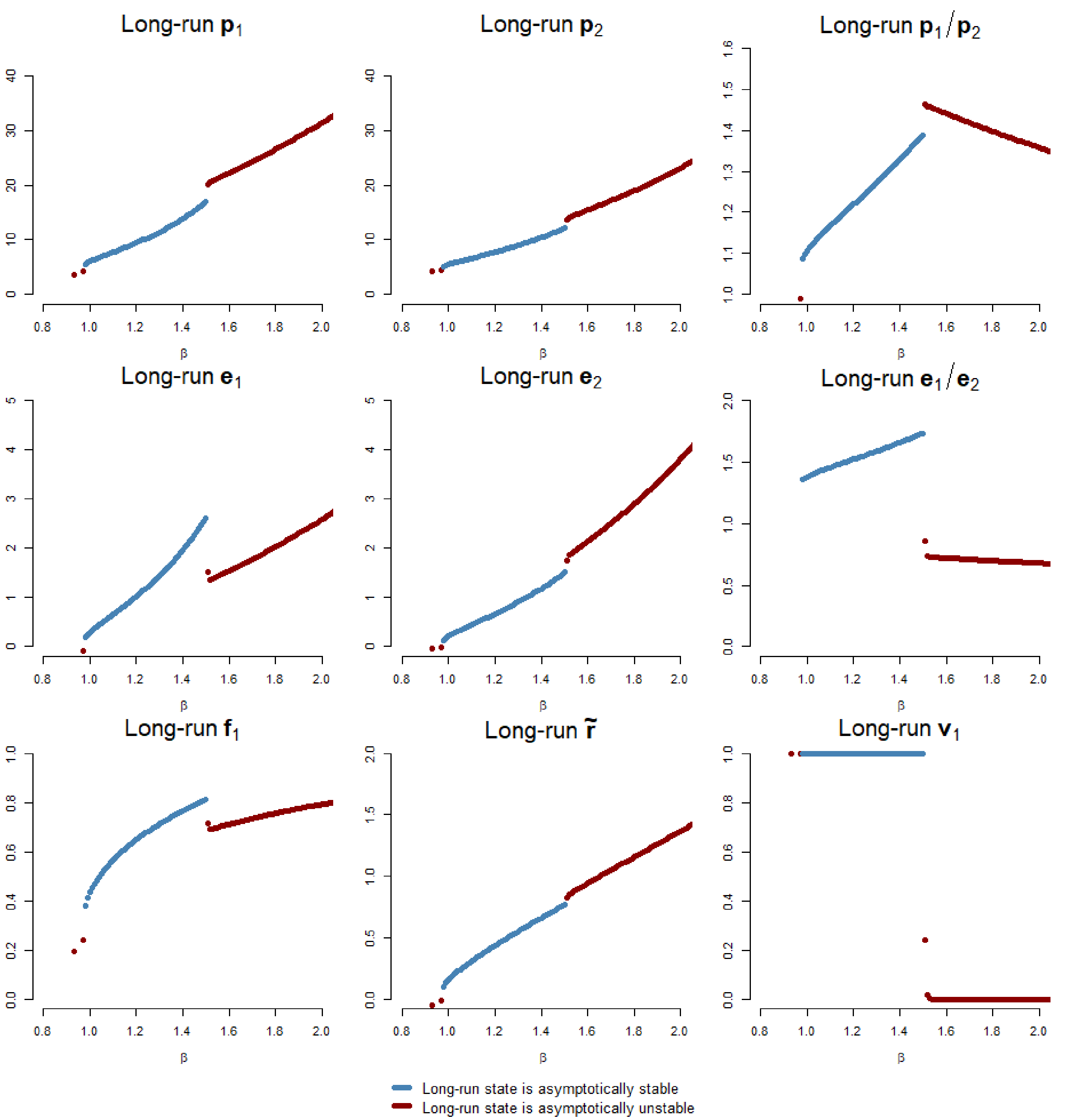


Figure 4: Long-Run States as Functions of Investment Sensitivity to Profits (γ_i)

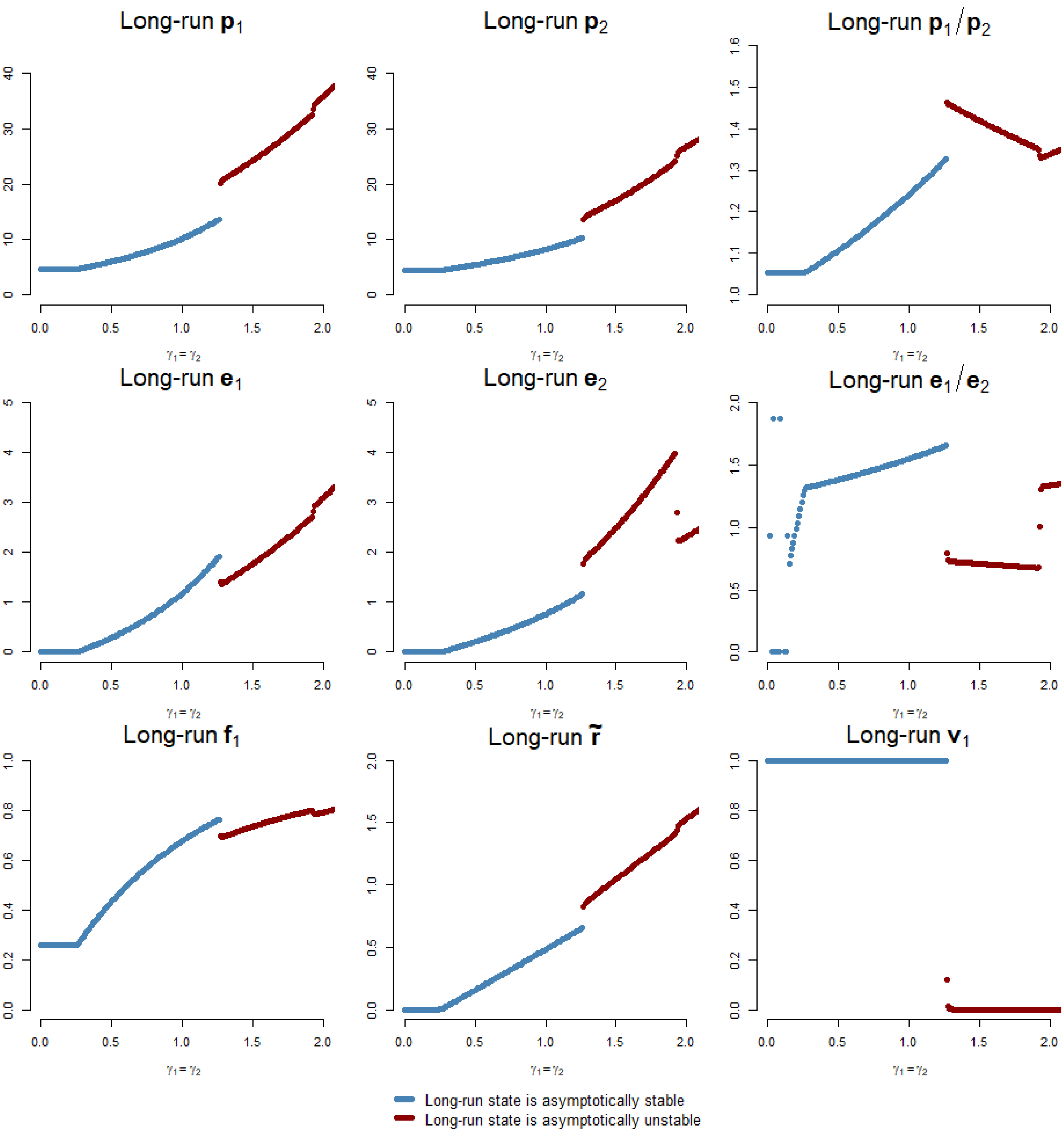
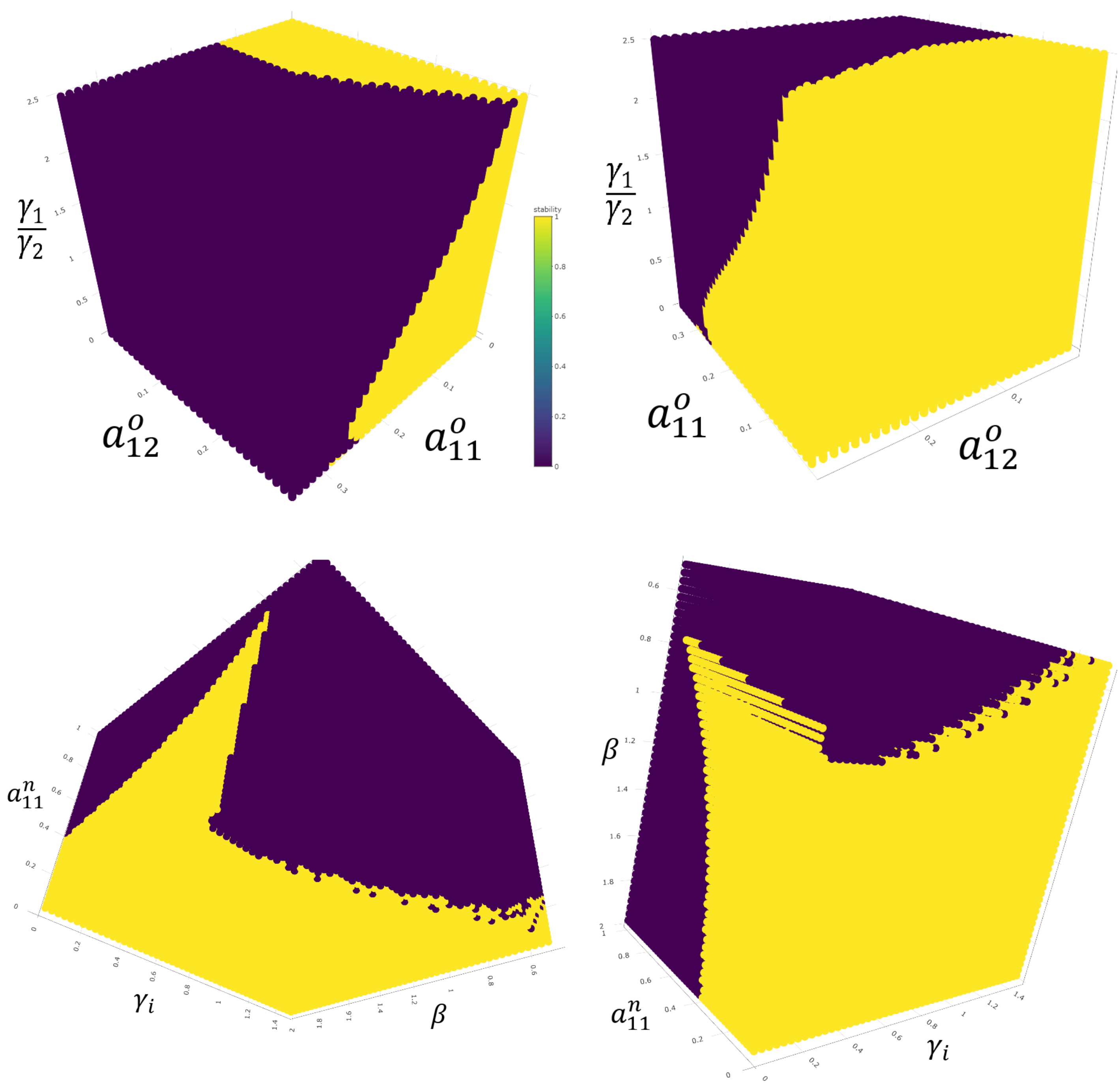


Figure 5: Stability of the Long-Run States



Note: Model simulations using 387,500 iterations under different parameters values. Brighter regions indicate asymptotic stability and darker regions indicate asymptotic instability. Figures in each row are different rotations of the same four-dimensional cube, where color is the fourth dimension.