Cambridge equation

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The Cambridge equation is a relation of logical consistency between the rate of profits (P/K), the natural rate of growth (g_n) and the propensity to save out of total profits (s_p), which states that:

$$\frac{P}{K} = \frac{1}{s_p} g_n \tag{1}$$

if a dynamic equilibrium with full employment is to be maintained through time.

When Domar's (1946) dynamic equilibrium solution – that is, exponential growth of investment at a steady rate such that total effective demand would grow hand in hand with productive capacity – was coupled with Harrod's (1948) natural growth rate $g_n = n + \lambda$, to wit, the maximum expansion rate supported by technical conditions (the growth of labour force n and of labour productivity λ), growth theory came to an *impasse*. For a given aggregate savings-to-income ratio (s) and technique in use (synthesized by capital-output ratio κ), the equilibrium relation

$$\frac{s}{\kappa} = g_n \tag{2}$$

could only be satisfied by a fluke, as all three magnitudes were considered constant. The Cambridge equation (1) was proposed to give a Keynesian answer to the Harrod–Domar dilemma: it specifies the long-run functional distribution of income between wages and profits, which produces precisely the saving ratio (s) required by equilibrium growth in equation (2).

Equation (1) is a fundamental relation of Keynesian income distribution theory (Kaldor, 1955) that links profits to savings through the ownership of the capital stock. Its logic runs as follows (Pasinetti, 1962): in any production system, wages are distributed amongst members of society in proportion to the amount of labour they contribute, whereas profits are distributed in proportion to the amount of capital they own. Thus, if ownership of capital derives from accumulated savings, in the long run, profits (P) will turn out to be distributed in proportion to contributed savings (S), for each social category.

But as long as there is a social category that derives all its income, and therefore savings, exclusively from profits, then the saving behaviour of just this group will determine the actual value of the ratio of profits to savings for the whole system. And the rate of profits (P/K) implied by this ratio of profits to savings (P/S) will be the one given by equation (1). At such a basic level of investigation, the Cambridge equation is independent of the institutional set-up of the advanced industrial society under study.

To see this, consider first a capitalist system. Capitalists may control the stock of fixed assets but not the entire financing of its creation, as workers – by saving (S_w) – own part of the capital stock through loans to the capitalists, receiving part of the profits (P_w). But as long as capitalists, differently from workers, save exclusively out of profits $S_c = s_c P_c$, then (see Pasinetti, 1974, pp. 127–8):

$$\frac{P}{S} = \frac{P_c + P_w}{S_c + S_w} = \frac{P_w}{S_w} = \frac{P_c}{S_c} = \frac{P_c}{s_c P_c} = \frac{1}{s_c} \qquad (3)$$

so that, in a dynamic equilibrium ($S = I$) with full employment, equation (3) collapses to equation (1) with $s_p = s_c$, namely, the propensity to save out of total profits coincides with (though not being identical to) the capitalists' propensity to save (s_c). Crucially, workers' propensity to save, though influencing the distribution of income between capitalists and workers (P_c/Y), does not influence the long run rate of profits (P/K) nor the functional income distribution between profits and wages (P/Y).

On the other hand, in a socialist system, where only the State accumulates, that is, $s_p = 1$, the Cambridge equation (1) establishes an equality between the natural rate of growth and a natural rate of profits:

$$\frac{P}{K} = g_n \tag{4}$$

implying that aggregate investments coincide with aggregate profits, and aggregate consumption coincides with aggregate wages, although individual wages and interest payments on individual savings are partly consumed and partly saved. In such a system, the technique in use yields the highest per-capita consumption, so that the natural rate of profits is efficient (Pasinetti, 1974).

The logic behind equation (1) renders apparent the pre-institutional character of functional income categories (wages and profits): they underlie, but do not directly correspond to, visible social categories (such as workers and capitalists) specific to an institutional set-up, exemplifying the separation between pre-institutional and institutional layers of analysis (Pasinetti, 2007).

To sum up, through the Cambridge equation, the Keynesian theory of income distribution establishes a continuity with the concept of residual income featuring in the Classical economists, but it reverses its causal direction by inverting the contrasting roles of wages and profits: if a dynamic equilibrium with full employment is to be maintained, investment requirements induced by the natural growth rate (g_n) and the propensity to save out of profits (s_p) set the share of total profits, so that it is the wages that, so to speak, become the surplus of the system. Even more importantly, when the relations are stripped down to their essentials, that is, in the purest case in which $s_p = 1$, and therefore (1) becomes $(P/K) = g_n = n + \lambda$, the natural rate of profits is determined, fundamentally, not by the 'quantity of capital' (as long asserted by mainstream economics), but by the sum of the rates of growth of labour and of the productivity of labour.

SEE ALSO:

Effective demand; *Essays in the Theory of Economic Growth*; Income distribution; Pasinetti's paradox; Wages.

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